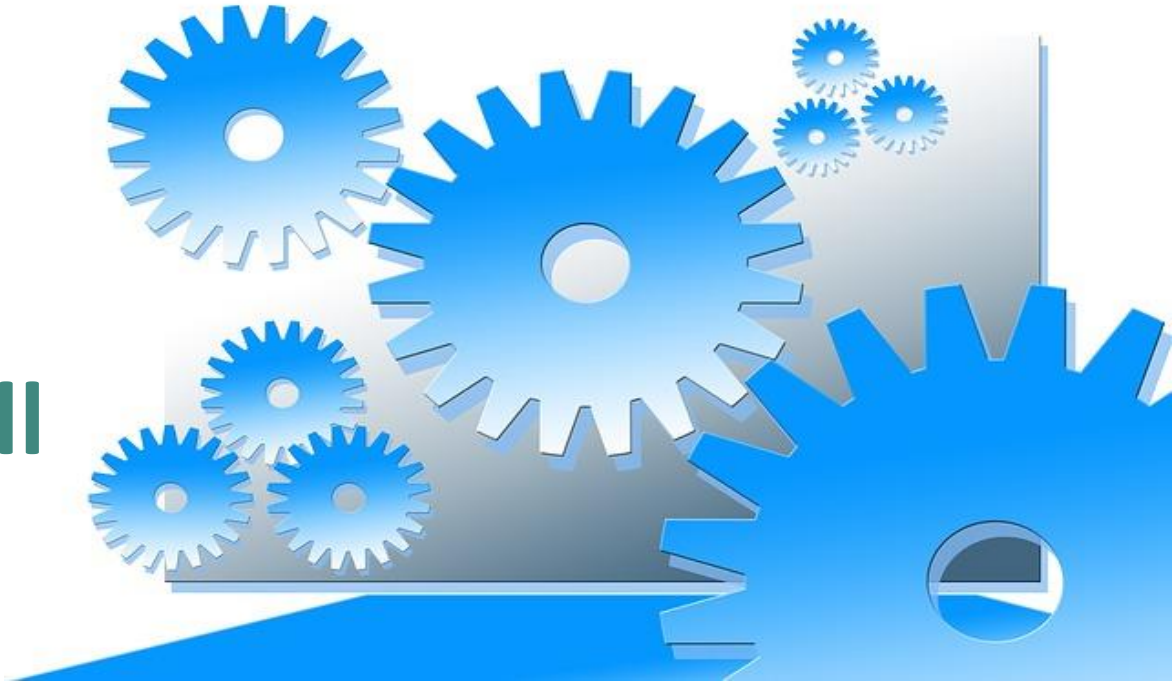
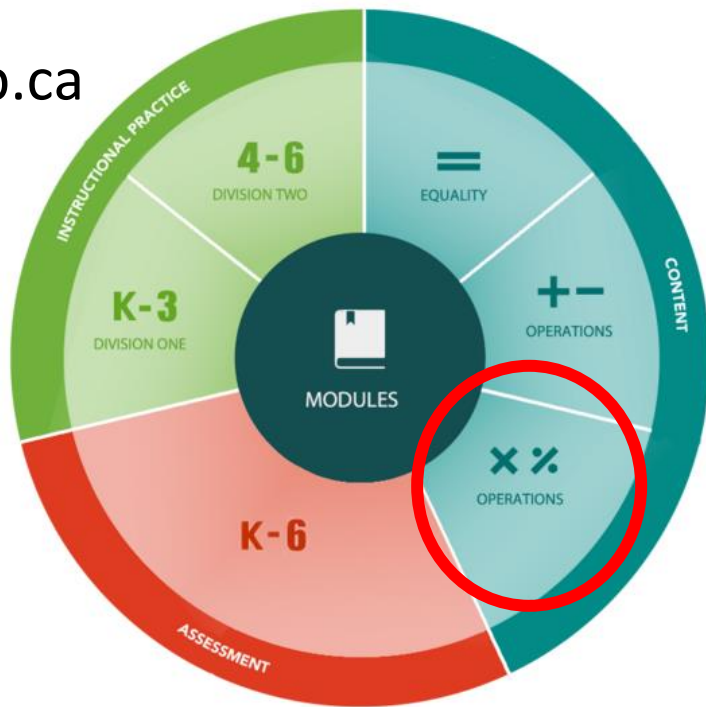


Making Multiplicative Thinking Accessible to All



Learning Guide

<http://learning.arpcdc.ab.ca>



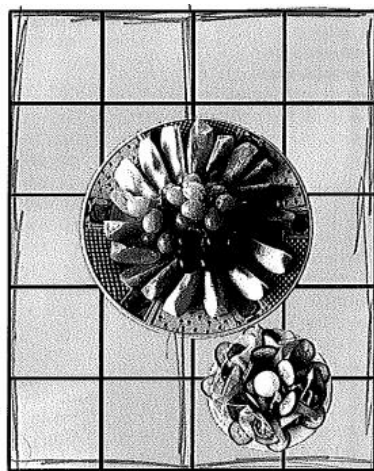
Webinar Objectives

- Develop participants' understanding of Multiplicative Thinking
- Explore a variety of multiplicative strategies
- Provide strategies for supporting students

Which is Which?

Counting
Additive Thinking
Multiplicative
Thinking

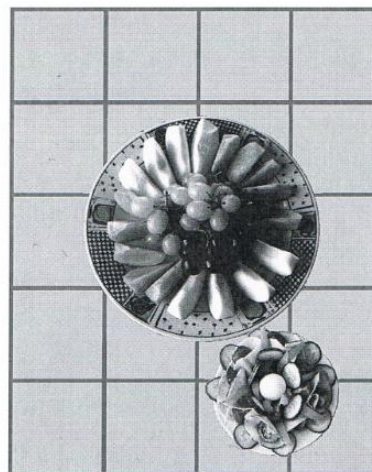
A



$$10 + 10$$

$$= 20$$

B



5

4

$$5 \times 4 = 20$$

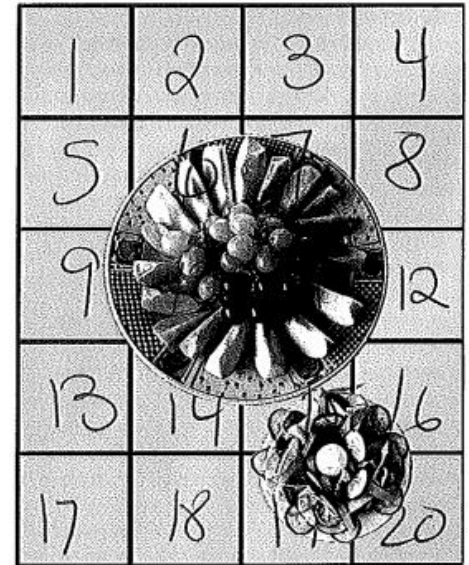
C



1	2	3	4
5	6	7	8
9			12
13	14		16
17	18	19	20

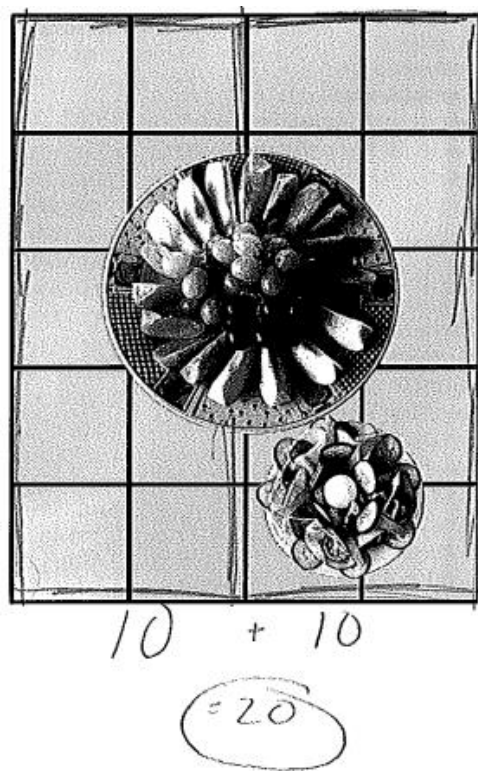


Counting



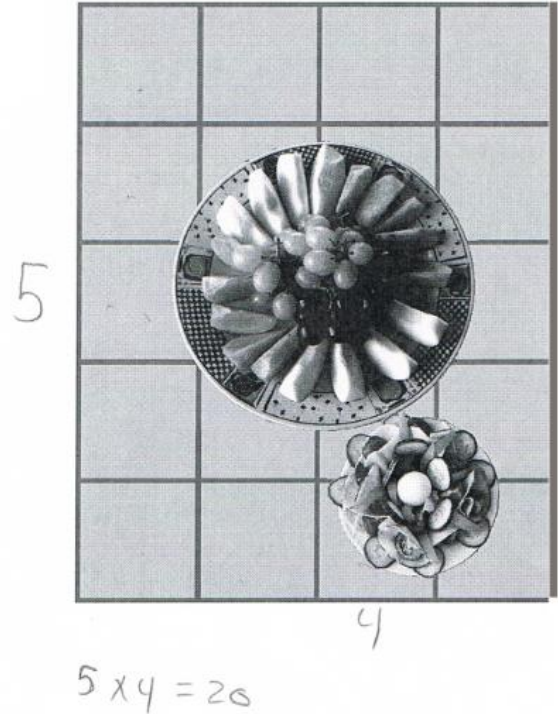
Additive Thinking

Students are able to manipulate numbers by **joining**, **separating**, and **comparing** while engaging in **flexible mathematical reasoning**.





Multiplicative Thinking

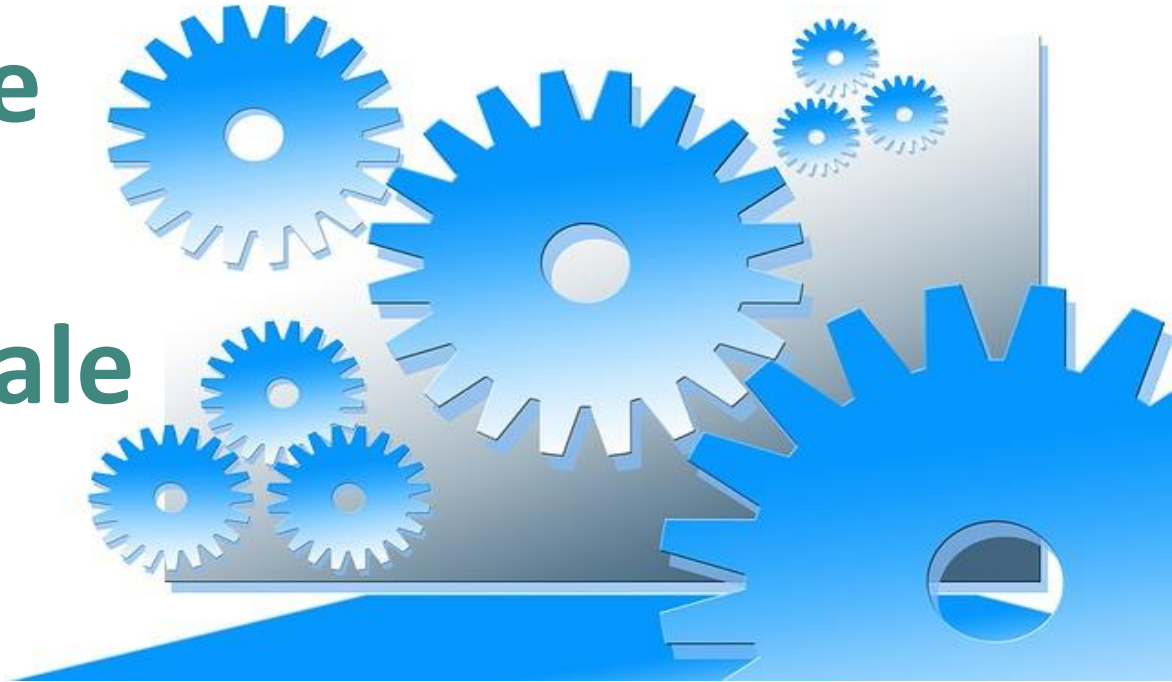


Definition

Multiplicative Thinking is

- a capacity to work flexibly with the concepts, strategies and representations of multiplication and division as they occur in a wide range of contexts. (mathematical reasoning)
- going beyond memorization of basic arithmetic skills, and
- the means to communicate multiplicative understanding effectively in a variety of ways (for example, words, diagrams, symbolic expressions, and written algorithms).

Multiplicative Thinking A Cautionary Tale





A Cautionary Tale – Part 1

My sister is tying balloons to the chairs for a birthday party. She wants to tie 4 balloons to each chair. The bag contains 77 balloons. How many chairs can she decorate?

The image shows a student's handwritten work on a grid. The grid has 10 columns and 3 rows. The numbers 1 through 20 are written in red above the columns. The numbers 4, 8, 12, 16, 20, 24, 28, 32, 36, 40 are written in black above the first row. The numbers 44, 48, 52, 56, 60, 64, 68, 72, 76, 80 are written in black above the second row. The numbers 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 are written in red above the third row. The numbers 4, 8, 12, 16, 20, 24, 28, 32, 36, 40 are circled in the first row. The numbers 44, 48, 52, 56, 60, 64, 68, 72, 76, 80 are circled in the second row. The numbers 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 are circled in the third row. The student has written "She needs 19 chairs. And she will have 1 extra balloons" in the bottom left and "And she will have" in the bottom right.

4	8	12	16	20	24	28	32	36	40
44	48	52	56	60	64	68	72	76	80
11	12	13	14	15	16	17	18	19	20

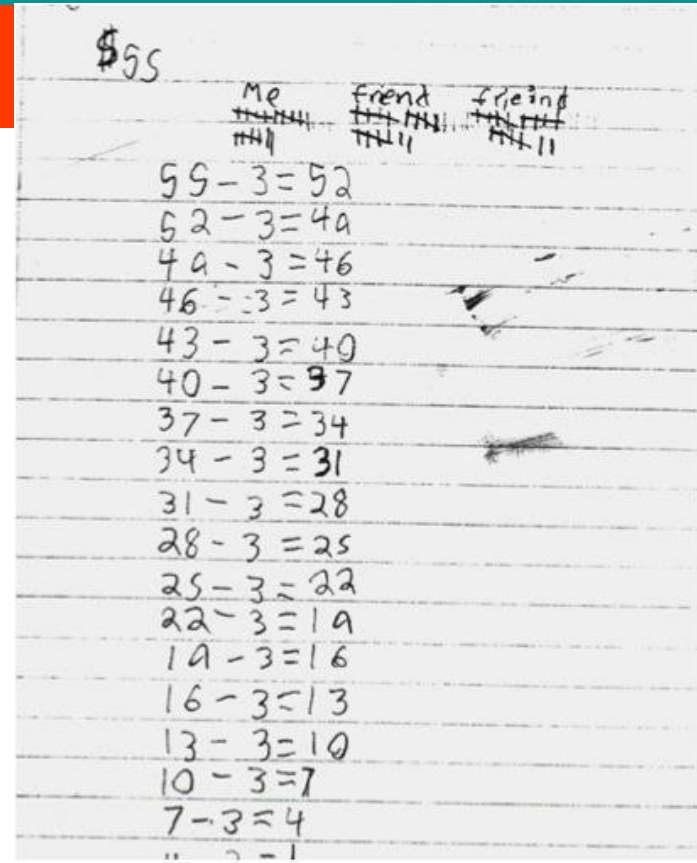
She needs 19 chairs. And she will have 1 extra balloons

And she will have



A Cautionary Tale – Part 2

I found a bike. My neighbour said he'd give me a \$40 reward for finding it. If I cleaned it up, I'd get another \$15. Can I share the money equally with my two friends?



The student says the answer is 17.

Keep in Mind

We want kids to eventually see multiplication and division as an efficient upgrade for repeated addition and subtraction.

Keep in Mind

Support kids where they are but...
make it very difficult to remain
where they are.

Multiplicative Thinking Big Idea 1





Big Idea 1: Multiplicative Thinking covers...

Big Idea 1: Multiplicative Thinking extends to...

place value, percentages, scale, proportions, rate, ratio, arrays, division, fractions, decimals, etc.

True or False

In the number 342, there are 4 tens.



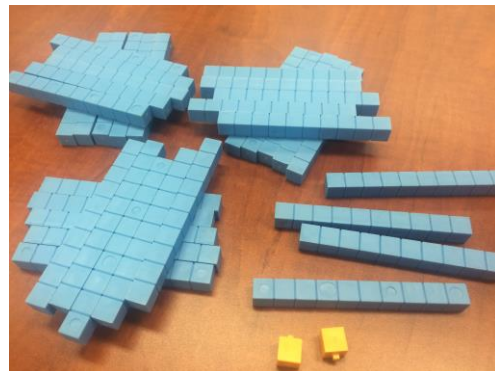
Place Value: Numbers are Composite

Whole Numbers:

- Tens, Hundreds, Thousands, etc.

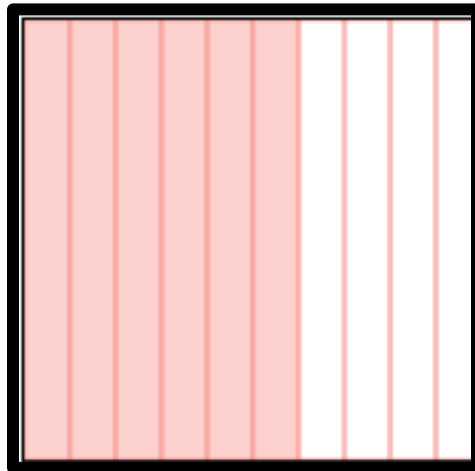
Decimals:

- Tenths, Hundredths, Thousandths, etc.



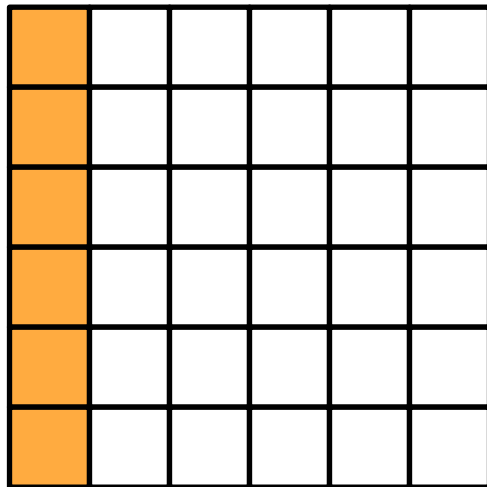
True or False

The picture below represents $\frac{2}{5}$



True or False

Both ratios $1:5$ and $1:6$ can be used to represent the picture.



T – Charts or Ratio Tables

# of Inputs	# of Outputs
1	4
2	8
4	How could I find this value?
6	?
10	?
8	?
?	36

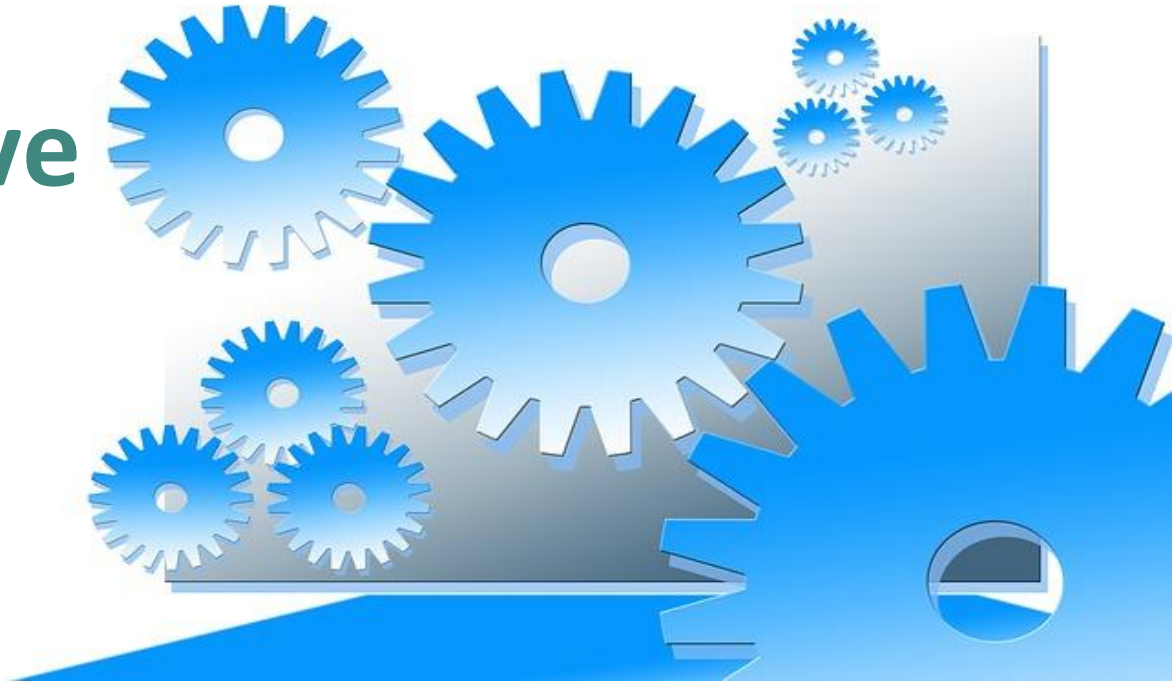
Keep in Mind

Multiplicative Thinking takes years to develop because it consists of many concepts and involves multiple ways to solve them.

When facing a problem, we need to ask:

- What is the question talking about?
- How can it be represented?
- How can we use what we know?
- Which strategies are better and why?

Multiplicative Thinking Big Idea 3



Big Idea 3

The distributive property is a powerful strategy for mental math

The Distributive Property

A number in a multiplication expression can be decomposed into two or more numbers strategically in order to make solving questions easier.

For example: 47 can be represented as $40 + 7$



The Distributive Property Video



The Distributive Property

$$5 \times 4$$



The Distributive Property

5×4



1×4

4×4

The Distributive Property

5×4



2×4



3×4

The Distributive Property

5×4



3×4



2×4

The Distributive Property

5×4



4×4



1×4

Number Sense Grade 3 N11 and N12

Understand and recall multiplication facts to 5×5 and related division facts.

True or False

If you know your 2 times tables and your 5 times tables, you can solve any multiplication fact.



5 x 5

5

2

10

$2 \times 5 = 10$

2

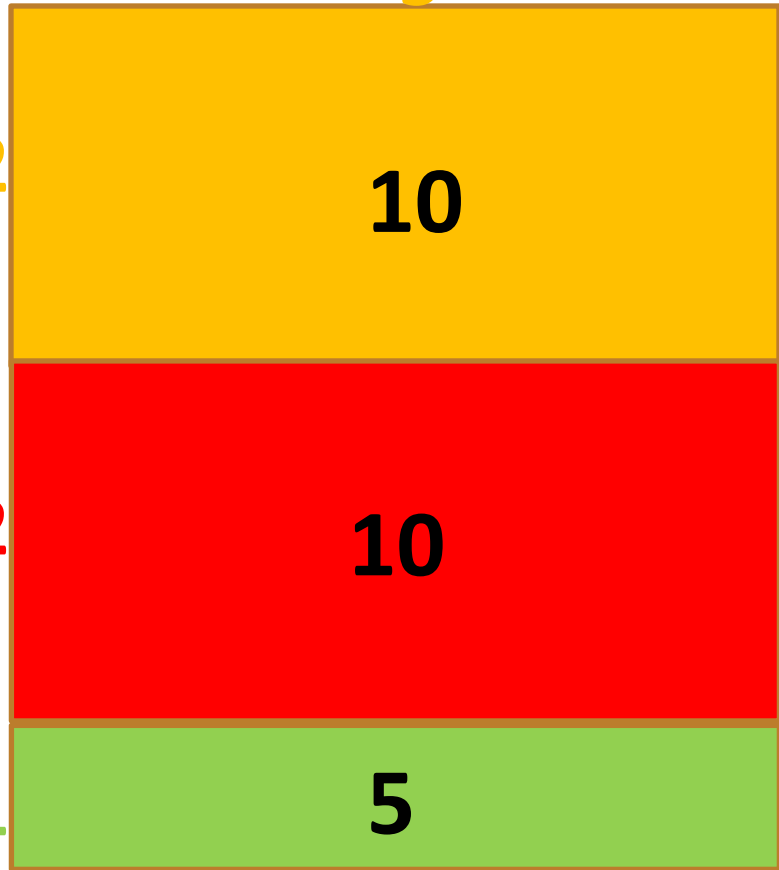
10

$(2 \times 5) + (2 \times 5) = 20$

1

5

$(2 \times 5) + (2 \times 5) + (1 \times 5) = 25$



If All They Know is 2×5 , They Can Figure Out...

3×5

4×5

5×5

...

2×10

2×15

2×20

...

4×5

8×5

12×5

...

2×6

2×7

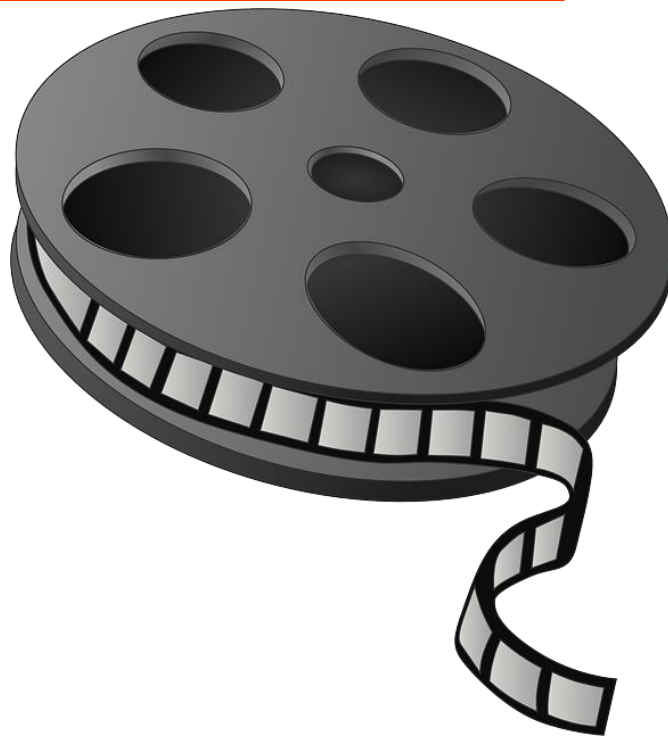
2×8

...

Any factor higher than 5 hits Grade 4 and 5 outcomes



Using the Distributive Property



Using What I Know to Solve 3×6

3

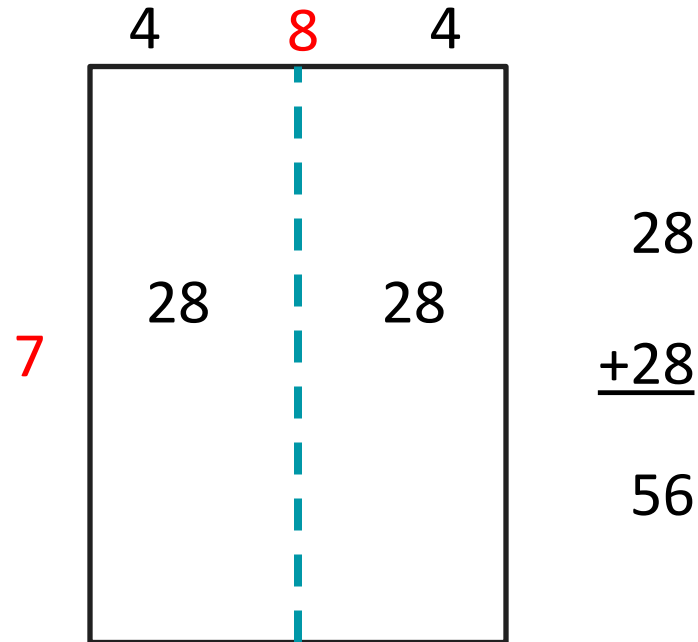
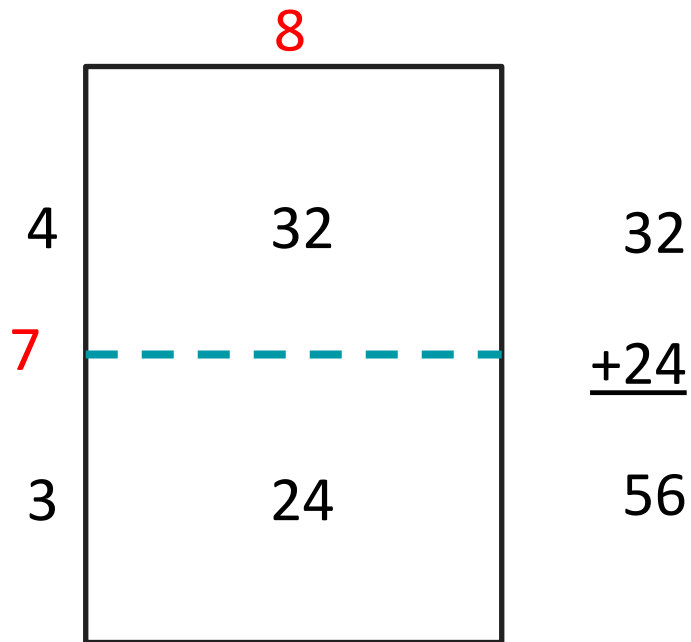
3	9	9
6		<u>+9</u>
3	9	18

3

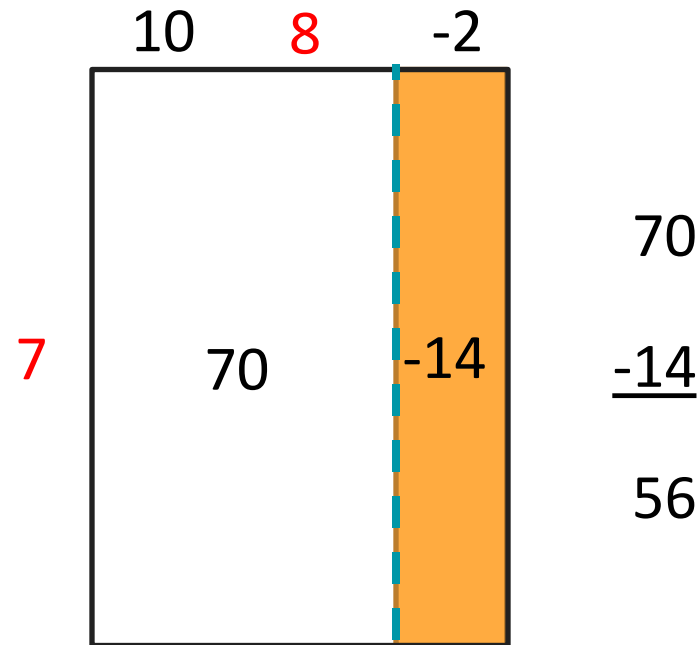
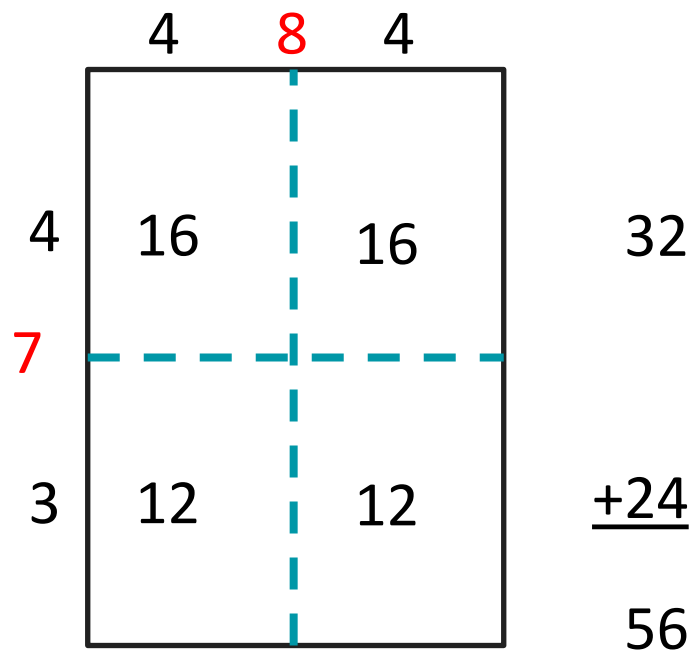
5	15	15
6		<u>+3</u>
1	3	18



Using What I Know to Solve 7×8



Using What I Know to Solve 7×8





Using the Distributive Property



Number Sense Grade 4 N6 and N7

Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) and their related division facts.



Using What I Know to Solve 23×8

Diagram illustrating the distributive property for 23×8 . The number 23 is decomposed into 20 and 3. The product is calculated as $8 \times 20 = 160$ and $8 \times 3 = 24$, which are then added together to get 184.

	20	23	3
8	160		24

$$\begin{array}{r} 160 \\ +24 \\ \hline 184 \end{array}$$

Diagram illustrating the distributive property for 23×8 . The number 8 is decomposed into 10 and -2. The product is calculated as $23 \times 10 = 230$ and $23 \times -2 = -46$, which are then added together to get 184.

	10	8	-2
23	230		-46

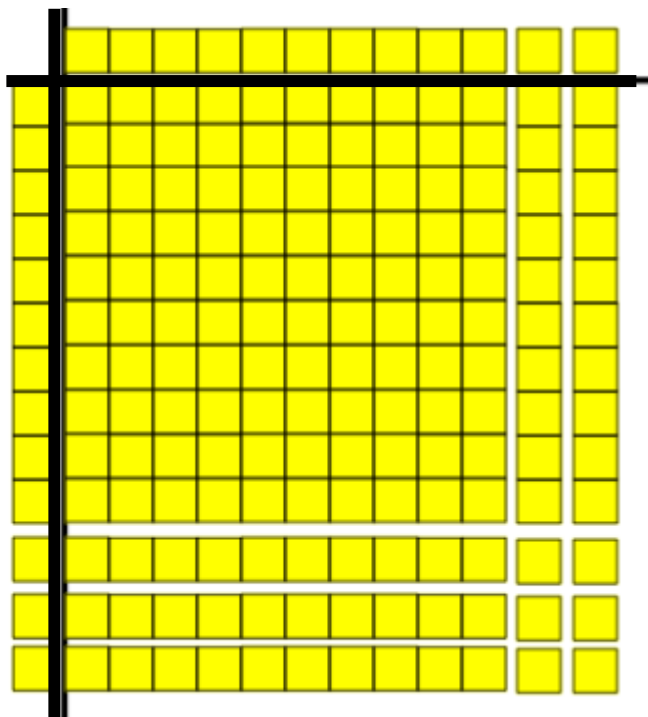
$$\begin{array}{r} 230 \\ -46 \\ \hline 184 \end{array}$$

Number Sense Grade 5 N5

Demonstrate an understanding of multiplication (2 digit by 2 digit).

Manipulatives Build Concrete Understanding

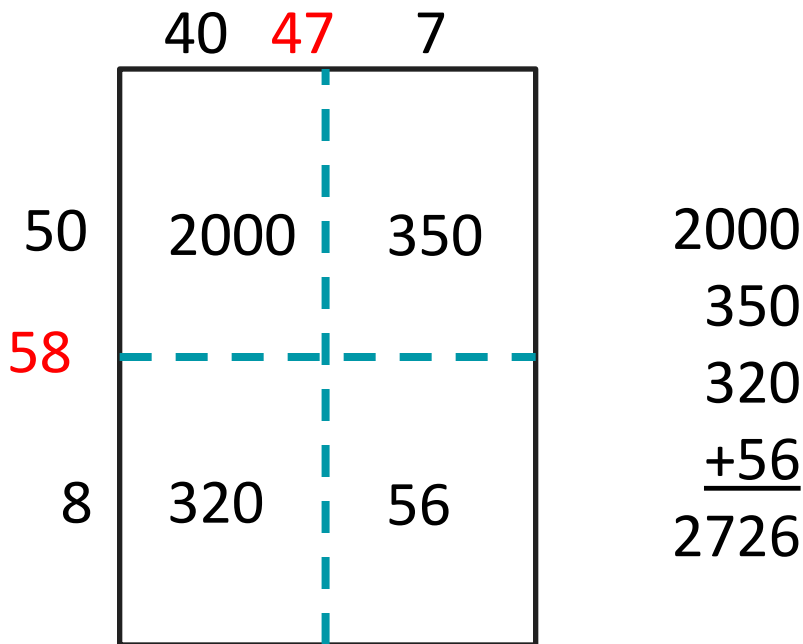
$$12 \times 13$$



	10	+ 2
10	100	20
+ 3	30	6



Using What I Know to Solve 47×58





Other Ways to Solve 36×27 Symbolically

	20	7
30	600	210
6	120	42

$$\begin{array}{r} 36 \\ \times 27 \\ \hline 42 \\ 210 \\ 120 \\ \hline 600 \end{array}$$

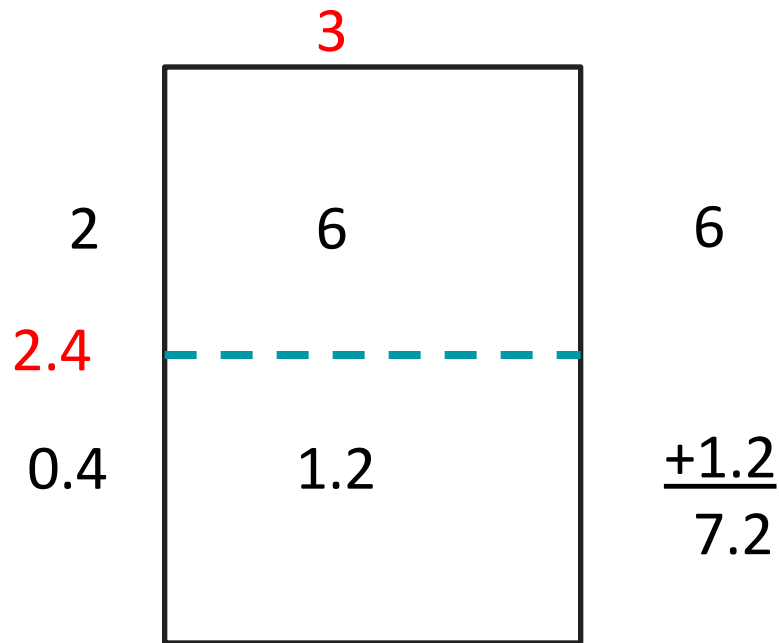
$$\begin{array}{r} & & & & 1 \\ & & & & 4 \\ & & & 36 & \\ \times & 27 & & & \\ \hline & 252 & & & \\ 720 & & & & \\ \hline \end{array}$$

Number Sense Grade 6 N8

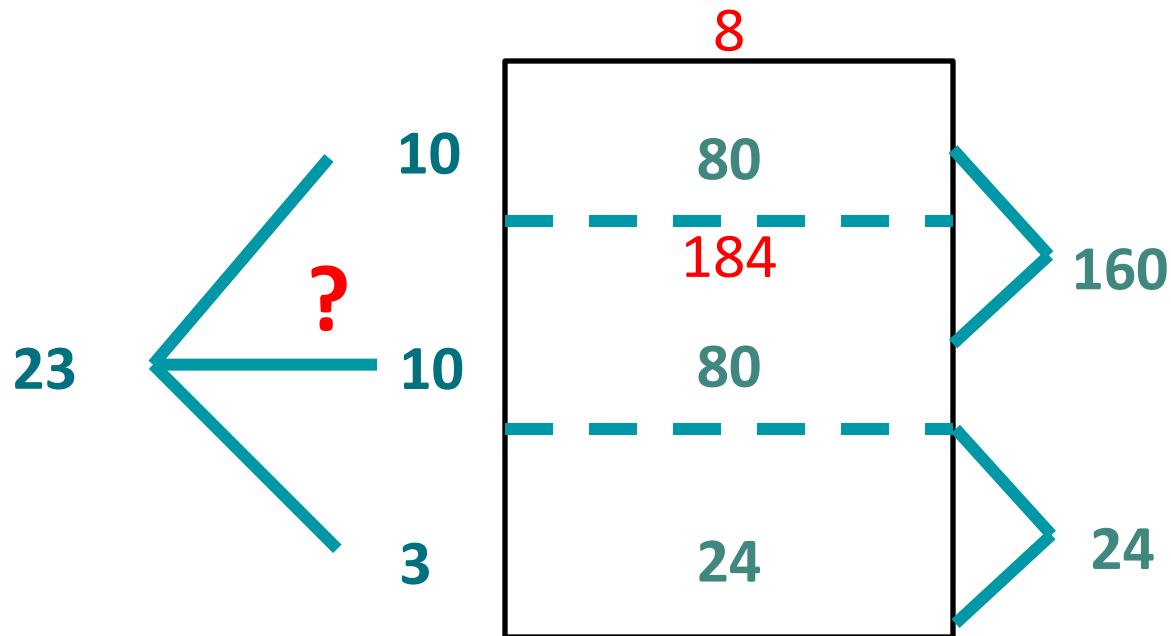
Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).



Using What I Know to Solve 3×2.4



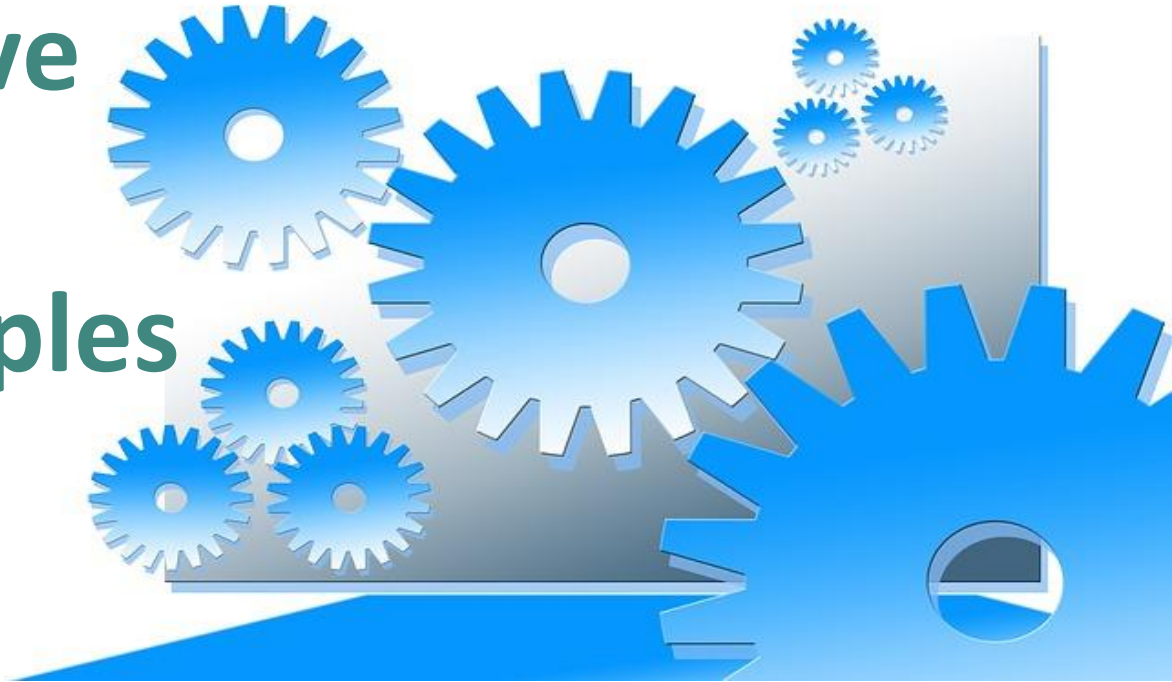
Using Blank Arrays for Division: $184 \div 8$



Keep in Mind

Known facts help students be strategic when using the distributive property.

Multiplicative Thinking Student Examples





Sweettarts

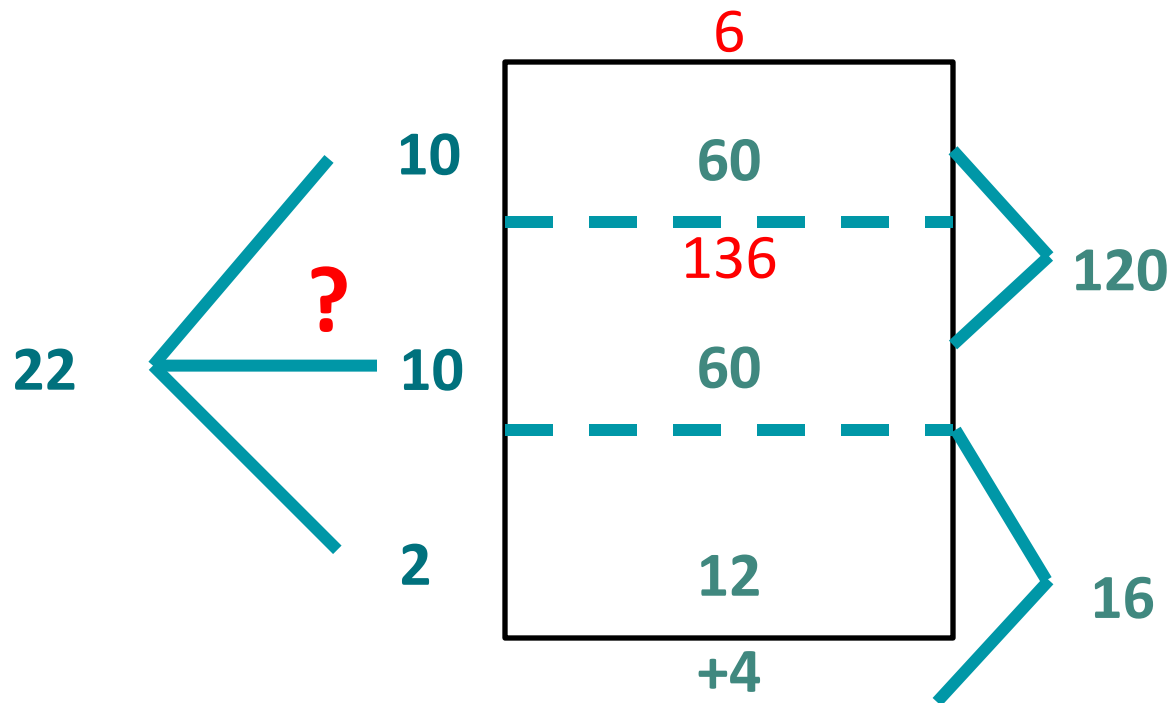
I have 136 Sweettarts to put in 6 birthday party goody bags. How many Sweettarts will go in each bag?

136 Sweettarts
6 kids birthday party
divide into 6 bags

$$\begin{array}{r} 6 \quad 12 \quad 18 \quad 24 \quad 30 \quad 36 \quad 42 \quad 48 \\ 22^{R4} \quad 54 \quad 60 \quad 66 \quad 72 \quad 78 \quad 84 \quad 90 \\ 6 \overline{)136} \quad 96 \quad 102 \quad 108 \quad 114 \quad 120 \quad 126 \\ \underline{132} \quad 4R \end{array}$$

If you counted by 6 to 132 and then you counted to 136 you count 4 to 136 then you have the answer 22^{R4}.

Using Blank Arrays for Division: $136 \div 6$





Quick Assessment

Quick Assessment			
The Answer	<input type="checkbox"/> Is Correct	<input type="checkbox"/> Is Incorrect	
	<input type="checkbox"/> Obvious <input type="checkbox"/> Inferred slightly <input type="checkbox"/> Inferred majorly	<input type="checkbox"/> Has a minor mistake <input type="checkbox"/> Has a misunderstanding	
The Strategy is a(n)	<input type="checkbox"/> Counting Strategy	<input type="checkbox"/> Additive Thinking Strategy	<input type="checkbox"/> Multiplicative Thinking Strategy
	<input type="checkbox"/> Counting on/back <input type="checkbox"/> Counting on/back	<input type="checkbox"/> Making 10 <input type="checkbox"/> Using doubles (3+3) <input type="checkbox"/> Part-Part-Whole	<input type="checkbox"/> Using Doubles (3x2) <input type="checkbox"/> Arrays <input type="checkbox"/> Part-Part-Whole <input type="checkbox"/> Known Facts
<input type="checkbox"/> Other			
Notes/Next Steps	Follow up Questions to Ask the Student		
	Follow up Steps for Student		



Multiplicative Thinking

Elementary Mathematics Professional Learning

Sweettarts

I have 136 Sweettarts to put in 6 birthday party goody bags. How many Sweettarts will go in each bag?

② I have 136 SweetTarts and I want to divided them into 6 bags. How many would each bag get?

22 22 22 22 22 22

Bag Bag Bag Bag Bag Bag

SweetTarts

22 R4

6/136

yes I was on the right track

Quick Assessment							
The Answer	<input type="checkbox"/> Is Correct	<input type="checkbox"/> Is Incorrect	The Strategy is a(n)	<input type="checkbox"/> Counting Strategy	<input type="checkbox"/> Additive Thinking Strategy	<input type="checkbox"/> Multiplicative Thinking Strategy	<input type="checkbox"/> Other
	<input type="checkbox"/> Obvious <input type="checkbox"/> Inferred slightly <input type="checkbox"/> Inferred majorly	<input type="checkbox"/> Has a minor mistake <input type="checkbox"/> Has a misunderstanding		<input type="checkbox"/> Counting <input type="checkbox"/> Counting on/back	<input type="checkbox"/> Making 10 <input type="checkbox"/> Using doubles (3+3) <input type="checkbox"/> Part-Part-Whole	<input type="checkbox"/> Using Doubles (3x2) <input type="checkbox"/> Arrays <input type="checkbox"/> Part-Part-Whole <input type="checkbox"/> Known Facts	
Notes/Next Steps:	Follow up Questions to Ask the Student			Follow up Steps for Student			



Sweettarts

I have 136 Sweettarts to put in 6 birthday party goody bags. How many Sweettarts will go in each bag?

Handwritten student work showing a division problem and a multiplication check:

$$136 \div 6 = 22$$

$$\begin{array}{r} 16 \\ \times 6 \\ \hline 96 \\ \hline 136 \end{array}$$

$$6 \times 6 = 36$$

$$16 \times 6 = 96$$

$$136 \div 6 = 22$$

There should be 22 in each bag.

Quick Assessment						
The Answer	<input type="checkbox"/> Is Correct	<input type="checkbox"/> Is Incorrect	The Strategy is a(n)	<input type="checkbox"/> Counting Strategy	<input type="checkbox"/> Additive Thinking Strategy	<input type="checkbox"/> Multiplicative Thinking Strategy
	<input type="checkbox"/> Obvious <input type="checkbox"/> Inferred slightly <input type="checkbox"/> Inferred majorly	<input type="checkbox"/> Has a minor mistake <input type="checkbox"/> Has a misunderstanding		<input type="checkbox"/> Counting <input type="checkbox"/> Counting on/back	<input type="checkbox"/> Making 10 <input type="checkbox"/> Using doubles (3+3) <input type="checkbox"/> Part-Part-Whole	<input type="checkbox"/> Using Doubles (3x2) <input type="checkbox"/> Arrays <input type="checkbox"/> Part-Part-Whole <input type="checkbox"/> Known Facts
Notes/Next Steps:	Follow up Questions to Ask the Student			Follow up Steps for Student		



Sweettarts

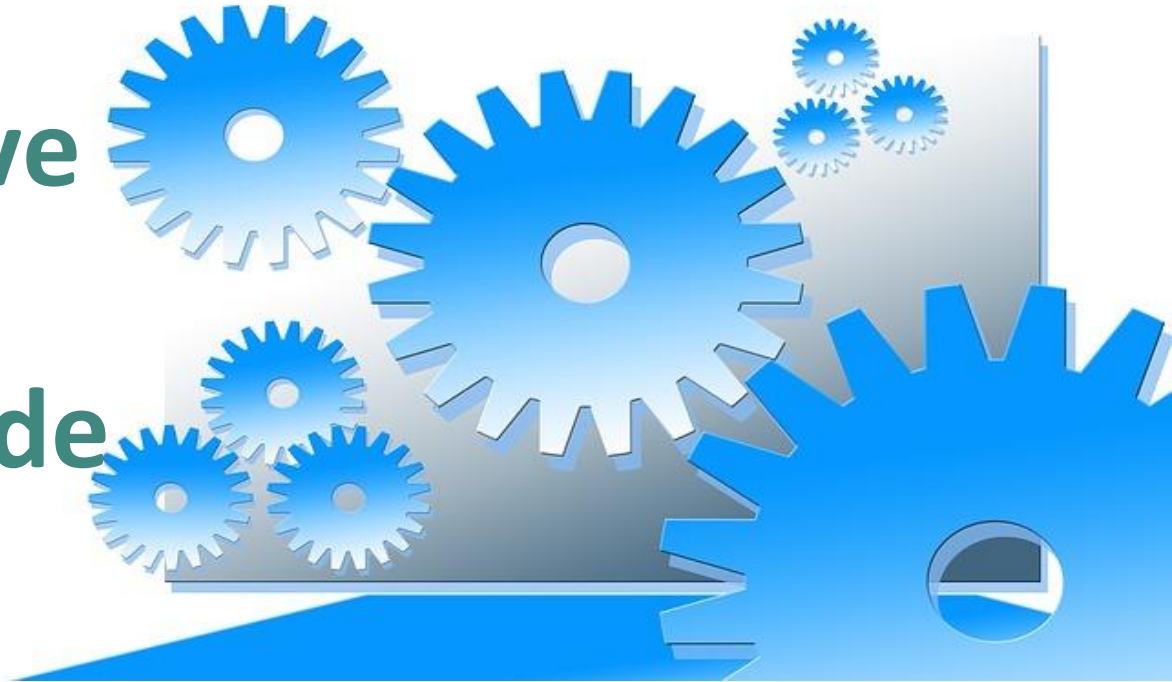
Handwritten long division on a grey background:

$$\begin{array}{r} 6 \overline{) 136} \\ \underline{-120} \\ 16 \\ \underline{-12} \\ 4 \end{array}$$

Annotations:

- A bracket on the right side of the quotient digits (2, 2) is labeled "per group".
- A bracket on the right side of the subtraction steps (-120, -12) is labeled "Used up".
- The remainder "4" is circled in red.

Multiplicative Thinking Learning Guide



LOGIN

Username

Password

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Create new account [Lost password?](#)



Elementary Mathematics Professional Learning
Apprentissage professionnel en mathématiques à l'élémentaire

Elementary Mathematics Professional Learning

Equality Webinar

English: November 2, 2015 at 1:00pm or 4:30pm

French: November 3, 2015 at 1:00pm or 4:00pm

Courses that require a login are indicated in the top menu with an asterisk ().*

For steps on how to create an account on the ARPDC Learning Portal, [please click here](#).

Welcome to the ARPDC Learning Portal

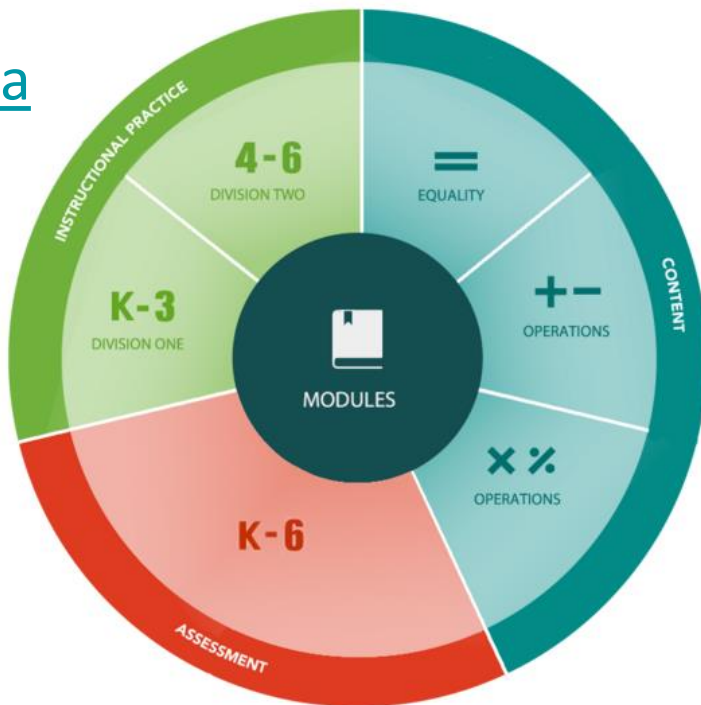
You will find a variety of resources, strategies and ideas all based in the Alberta Education context.

To facilitate access to additional PD resources, educators are invited to explore the links available by clicking on the image below:



EMPL Website Tour

<http://learning.arpcdc.ab.ca>



EMPL Opportunities



Upcoming Webinars



Assessment

February 22, 2016

- 1 pm - 2 pm
- 4 pm - 5 pm

Elementary Math Professional Learning Opportunity
Register at: <http://arpdc.ab.ca>

ARPDC



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www.erlc.ca

Final Thought...

“Go down deep enough into ANYTHING and you will find Mathematics.”

~Dean Schlicter