Math 30-2: U3L3 Teacher Notes

Probabilities Using Counting Methods

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Solve a contextual problem that involves probability and permutations.
- Solve a contextual problem that involves combinations and probability.

You may be able to use the Fundamental Counting Principle and techniques using permutations and combinations to solve probability. Let's look at a few examples of when to use each method.

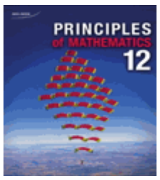
Permutations are used when order is important in the outcomes

Combinations are used when order is NOT important in the outcomes.

Click here to watch a Youtube video on The Fundamental Counting Principle of Probability (Numbers Application)

Click here to watch a Youtube video on Probability Permutations part 1 lesson

Click here to watch a Youtube video on Probability Permutations part 2 lesson



Practice Problem:

Complete "Check your Understanding" question 4a on page 159 of your textbook.

Solution:

4. a) Let *B* represent the possibility that only boys will be on the trip, and let *O* represent all possibilities.

$$n(B) = {}_5C_4$$

$$n(B) = \frac{5!}{(5-4)!4!}$$

$$n(B) = \frac{5!}{1! \cdot 4!}$$

$$n(B) = \frac{5 \cdot 4!}{1 \cdot 4!}$$

$$n(B) = 5$$

The number of possibilities of there being all boys on the trip is 5.

$$n(O) = {}_{11}C_4$$

$$n(O) = \frac{11!}{(11-4)!4!}$$

$$n(O) = \frac{11!}{7! \cdot 4!}$$

$$n(O) = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 4!}$$

$$n(O) = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2}$$

$$n(O) = 11 \cdot 10 \cdot 3$$

$$n(O) = 330$$

The total number of possibilities for selecting 4 students for the trip is 330.

$$P(B) = \frac{n(B)}{n(O)}$$

$$P(B) = \frac{5}{330}$$

$$P(B) = \frac{1}{66}$$

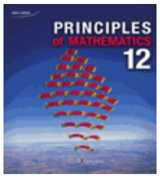
The total number of possibilities for selecting 4 students for the trip is 330.

$$P(B) = \frac{n(B)}{n(O)}$$

$$P(B) = \frac{5}{330}$$

$$P(B) = \frac{1}{66}$$

Therefore, the probability that only boys will be on the trip is $\frac{1}{66}$ or 0.0152 or 1.52%.



Practice Problem:

Complete “Check your Understanding” question 4c on page 159 of your textbook.

Solution:

c) Let *D* represent that there are more girls than boys on the trip. This is true if 4 girls and no boys are selected and if 3 girls and 1 boy are selected.

$$n(D) = {}_6C_4 + {}_6C_3 \cdot {}_5C_1$$

$$n(D) = \frac{6!}{(6-4)!4!} + \frac{6!}{(6-3)!3!} \cdot \frac{5!}{(5-1)!1!}$$

$$n(D) = \frac{6!}{2!4!} + \frac{6!}{3!3!} \cdot \frac{5!}{4!1!}$$

$$n(D) = \frac{6 \cdot 5 \cdot 4!}{2!4!} + \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!3!} \cdot \frac{5 \cdot 4!}{4!}$$

$$n(D) = 15 + 20 \cdot 5$$

$$n(D) = 115$$

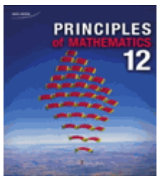
There are 115 ways in which there can be more girls than boys on the trip.

$$P(D) = \frac{n(D)}{n(O)}$$

$$P(D) = \frac{115}{330}$$

$$P(D) = \frac{23}{66}$$

The probability that more girls than boys will go is $\frac{23}{66}$ or about 0.348 or 34.8%.



Practice Problem:

Complete "Check your Understanding" question 5a on page 159 of your textbook.

Solution:

5. Let S represent a password that contains S and Q, and let W represent all possible passwords.

a) The number of possible passwords containing S and Q is ${}_2P_2 \cdot {}_{10}P_3$, or $2! \cdot {}_{10}P_3$. Order in passwords is important, so permutations are used.

$$n(S) = 2! \cdot {}_{10}P_3$$

$$n(S) = 2! \cdot \frac{10!}{(10-3)!}$$

$$n(S) = 2! \cdot \frac{10!}{7!}$$

$$n(S) = (2 \cdot 1)\left(\frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}\right)$$

$$n(S) = 2 \cdot 10 \cdot 9 \cdot 8$$

$$n(S) = 1440$$

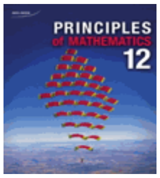
The total number of passwords of this form is ${}_{26}P_2 \cdot {}_{10}P_3$.

$$n(W) = {}_{26}P_2 \cdot {}_{10}P_3$$
$$n(W) = \frac{26!}{(26-2)!} \cdot \frac{10!}{(10-3)!}$$
$$n(W) = \frac{26!}{24!} \cdot \frac{10!}{7!}$$
$$n(W) = \frac{26 \cdot 25 \cdot 24!}{24!} \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}$$
$$n(W) = 26 \cdot 25 \cdot 10 \cdot 9 \cdot 8$$
$$n(W) = 468\ 000$$

Now determine the probability.

$$P(S) = \frac{n(S)}{n(W)}$$
$$P(S) = \frac{1440}{468000}$$
$$P(S) = \frac{1}{325}$$

The probability that a password chosen at random will include S and Q is $\frac{1}{325}$, or about 0.003 08 or 0.308%.



Practice Problem: **(KEY QUESTION)**

Complete "Check your Understanding" question 8 on page 160 of your textbook.

Solution:

8. a) Let Y represent Yuko, Luigi and Justin being chosen, and let T represent all of the ways that treasurer, secretary and liaison can be chosen. In this case, there is only 1 way to achieve the favourable outcome. The number of ways to choose treasurer, secretary and liaison is ${}_{15}P_3$.

$$n(T) = {}_{15}P_3$$

$$n(T) = \frac{15!}{(15-3)!}$$

$$n(T) = \frac{15!}{12!}$$

$$n(T) = \frac{15 \cdot 14 \cdot 13 \cdot 12!}{12!}$$

$$n(T) = 15 \cdot 14 \cdot 13$$

$$n(T) = 2730$$

Now determine the probability.

$$P(Y) = \frac{n(Y)}{n(T)}$$

$$P(Y) = \frac{1}{2730}$$

The probability is $\frac{1}{2730}$, or about 0.000 366 or 0.0366%.

b) Let Y represent Yuko, Luigi and Justin being chosen, and let L represent all of that ways that three students can be chosen. Again, there is only 1 way to achieve the favourable outcome. The number of ways to choose 3 people from 15 to clean up is ${}_{15}C_3$.

$$n(L) = {}_{15}C_3$$

$$n(L) = \frac{15!}{(15-3)! \cdot 3!}$$

$$n(L) = \frac{15!}{12! \cdot 3!}$$

$$n(L) = \frac{15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 3 \cdot 2 \cdot 1}$$

$$n(L) = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2}$$

$$n(L) = \frac{2730}{6}$$

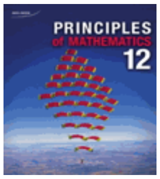
$$n(L) = 455$$

Now determine the probability.

$$P(Y) = \frac{n(Y)}{n(L)}$$

$$P(Y) = \frac{1}{455}$$

The probability that Yuko, Luigi and Justin will be picked is $\frac{1}{455}$, or about 0.0022 or 0.220%.



Practice Problem: **(KEY QUESTION)**

Complete "Check your Understanding" question 10 on page 160 of your textbook.

Solution:

10. Let T represent three girls and two boys being chosen to form a subcommittee, and let S represent all possible subcommittees.

In this example, order is not important. The number of ways to arrange three girls and two boys from 16 girls and 7 boys is ${}_{16}C_3 \cdot {}_{7}C_2$.

$$n(T) = {}_{16}C_3 \cdot {}_{7}C_2$$

$$n(T) = \frac{16!}{(16-3)! \cdot 3!} \cdot \frac{7!}{(7-2)! \cdot 2!}$$

$$n(T) = \frac{16!}{13! \cdot 3!} \cdot \frac{7!}{5! \cdot 2!}$$

$$n(T) = \frac{16 \cdot 15 \cdot 14 \cdot 13!}{13! \cdot 3 \cdot 2 \cdot 1} \cdot \frac{7 \cdot 6 \cdot 5!}{5! \cdot 2 \cdot 1}$$

$$n(T) = \frac{16 \cdot 15 \cdot 14}{3 \cdot 2} \cdot \frac{7 \cdot 6}{2}$$

$$n(T) = \frac{3360}{6} \cdot \frac{42}{2}$$

$$n(T) = 560 \cdot 21$$

$$n(T) = 11\,760$$

The number of ways to arrange 23 people in a five-person committee is ${}_{23}C_5$.

$$n(S) = {}_{23}C_5$$

$$n(S) = \frac{23!}{(23-5)! \cdot 5!}$$

$$n(S) = \frac{23!}{18! \cdot 5!}$$

$$n(S) = \frac{23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18!}{18! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(S) = \frac{23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{5 \cdot 4 \cdot 3 \cdot 2}$$

$$n(S) = \frac{4\ 037\ 880}{120}$$

$$n(S) = 33\ 649$$

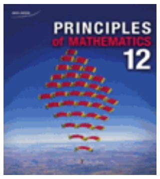
Now determine the probability.

$$P(T) = \frac{n(T)}{n(S)}$$

$$P(T) = \frac{11\,760}{33\,649}$$

$$P(T) = \frac{1680}{4807}$$

The odds in favour that the committee will contain 3 girls and 2 boys is 1680 : 4807 – 1680 or 1680 : 3127.

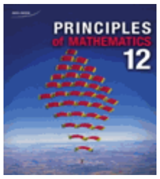


Practice Problem:

Complete “Check your Understanding” question 11 on page 160 of your textbook.

Solution:

11. The total number of outcomes is 2^4, or 16. There is only one option where no coins land on tails (all land on head). Therefore, the probability that at least one coin lands tails is $\frac{15}{16}$, or 0.9375 or 93.75%.



Practice Problem:

Complete "Check your Understanding" question 12 on page 160 of your textbook.

Solution:

12. Let B represent Bilyana and Bojana sitting together, and let S represent all possible seating arrangements.

a) The number of ways to seat Bilyana and Bojana together is $4 \cdot {}_2P_2$, or $4 \cdot 2!$. The number of ways to seat the 3 other people is ${}_3P_3$, or $3!$. The number of ways to seat the 5 friends in the row is $4 \cdot 2! \cdot 3!$, or 48.
There are 5! or 120 ways to arrange five people.

$$P(B) = \frac{n(B)}{n(S)}$$

$$P(B) = \frac{48}{120}$$

$$P(B) = \frac{2}{5}$$

The probability that Bilyana and Bojana are sitting together is $\frac{2}{5}$, 0.4 or 40%.

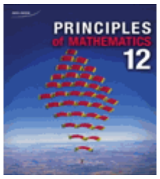
b) The probability that Bilyana and Bojana are not sitting together is the complement of the probability that they are sitting together

$P(B') = 1 - P(B)$

$P(B') = 1 - \frac{2}{5}$

$P(B') = \frac{3}{5}$

The probability Bilyana and Bojana are not sitting together is $\frac{3}{5}$, 0.6 or 60%.



Practice Problem:

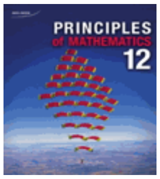
Complete “Check your Understanding” question 16 on page 161 of your textbook.

Solution:

16. Let D represent an arrangement of the eight dogs such that the bearded collie and the sheltie are together. Let O represent all arrangements.

The number of ways to place the bearded collie and the sheltie together is $7 \cdot 2!$. The number of ways to place the other 6 dogs is $6!$. Therefore, the total number of ways to seat all the dogs, with the collie and sheltie together, is $7 \cdot 2! \cdot 6!$, or $2 \cdot 7!$.

There are $8!$ ways to arrange eight dogs.



Practice Problem:

Complete "Check your Understanding" question 18 on page 161 of your textbook.

Solution:

18. e.g., I would use permutations in a problem where the order of the items was important, and use combinations in a problem where order was not important. Permutations: Determine the probability that two items are next to each other in a lineup of seven different items that has been placed in a random order. Combinations: Determine the probability that, given eight books, four of which are about math, if I choose five of the eight books, I choose three math books.