Math 30-2: U3L4 Teacher Notes

Mutually Exclusive Events

Key Math Learnings:

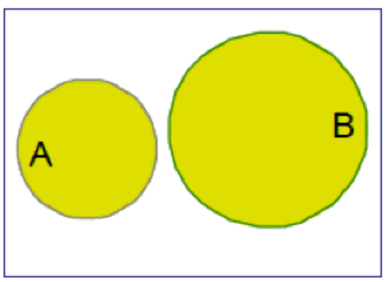
By the end of this lesson, you will learn the following concepts:

- Classify events as mutually exclusive or non-mutually exclusive, and explain the reasoning.
- Determine if two events are complementary, and explain the reasoning.
- Solve problems that involves mutually exclusive or non-mutually exclusive events.
- Solve contextual problems that involve probability of complementary events.
- Create and solve a problem that involves mutually exclusive or non-mutually exclusive events.

Mutually Exclusive and Non-Mutually Exclusive Events

Two events A and B that cannot occur at the same time are mutually exclusive events. They have no common outcomes.

Mutually Exclusive Events



***P*(*A* or *B*) = *P*(*A*) + *P*(*B*)**

$$P(A \cup B) = P(A) + P(B)$$

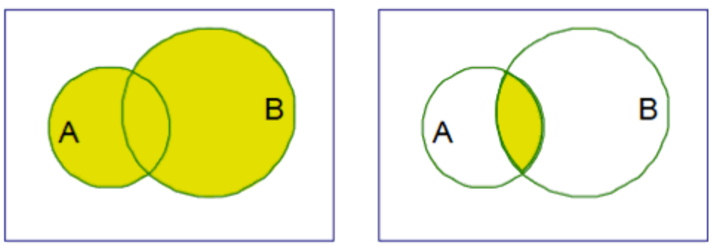
Since the sets never intersect:

***P*(*A* and *B*) = 0**

$$P(A \cap B) = 0$$

Two events A and B that are **not mutually exclusive** have some common outcomes.

Non - Mutually Exclusive Events



$P(A \cup B)$ $\quad$ $P(A \cap B)$

Addition Principle

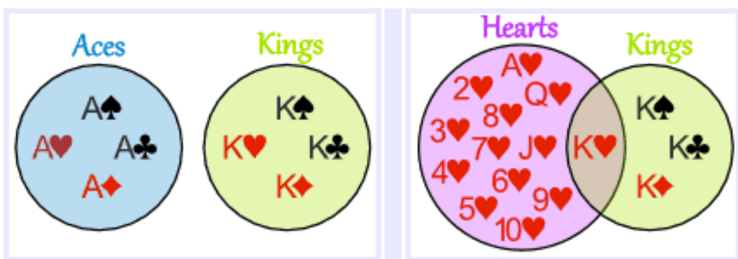
***P*(*A* or B) = *P*(*A*) + *P*(B) - *P*(*A* and B)**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Identifying Mutually Exclusive and Non-Mutually Exclusive Events

1. Using a Venn Diagram -- When using a Venn diagram you are looking for any areas of the sets to overlap. If there is no overlap, then the events are Mutually Exclusive.

For Example:



Aces and Kings are **Mutually Exclusive** (can't be both)

Hearts and Kings are **not** Mutually Exclusive (can be both)

2. Mathematically

Step 1: Add up the probabilities of the separate events (A and B).

Step 2: Compare your answer to the given "union" statement (A **U** B). If they are the same, the events are mutually exclusive. If they are different, they are not mutually exclusive.

For Example:

"If P(A) = 0.20, P(B) = 0.35 and (P **U** B) = 0.51, are A and B mutually exclusive?"

Solution:

.20 + .35 = **.55**

0.55 does not equal 0.51, so the events are ***not* mutually exclusive**.

Complementary Events are Mutually Exclusive

Two events are described as ***complementary*** if they are the only two possible outcomes.

Example

Imagine we are testing whether it rains on a particular day.

Solution:

The events "it rains" and "it doesn't rain" are complementary because:

- - only one of the two events can occur
- - no other event can occur

Therefore, these two events are ***complementary***.

Example

Consider the rolling of a die to see whether the result is *odd* or *even*.

Solution:

The events "odd" and "even" are complementary because:

- - the result must be either "odd" or "even" (not both)
- - the result cannot be anything except "odd" or "even"

Therefore, these two events are also ***complementary***.

When two events are complementary, we can use the following properties to solve problems.

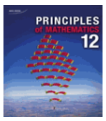
Complementary Events

If A is an event, and A' is the complementary event,

$$P(A) + P(A') = 1$$

or

$$P(A') = 1 - P(A)$$

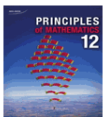


Practice Problem:

Complete “Check your Understanding” question 4 on page 176 of your textbook.

Solution:

4. a) No. e.g., 2 is both an even number and a prime number.
b) Yes. e.g., You cannot roll a sum of 10 and a roll of 7 at the same time.
c) Yes. e.g., You cannot walk and ride to school at the same time.



Practice Problem:

Complete “Check your Understanding” question 5 on page 177 of your textbook.

Solution:

5. a) $P(A) = \dfrac{n(A)}{n(C)}$

$P(A) = \dfrac{144\ 945}{389\ 045}$

$P(A) = \dfrac{28\ 989}{77\ 809}$

The probability a person who is Métis lives in Alberta or British Columbia is $\dfrac{28989}{77809}$, or about 0.373 or 37.3%.

b) $P(M) = \dfrac{n(M)}{n(C)}$

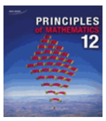
$P(M) = \dfrac{119\ 920}{389\ 045}$

$P(M) = \dfrac{23\ 984}{77\ 809}$

The probability that a person who is Métis lives in Manitoba or Saskatchewan is $\dfrac{23984}{77809}$, or about 0.308 or 30.8%.

c) Yes, because these two events are mutually exclusive, so $P(A \cap M)$ is equal to 0.

d) The odds in favour of a person who is Métis living in one of the four western provinces are 264 865 : 124 180, or 52 973 : 24 836.



Practice Problem:

Complete “Check your Understanding” question 8 on page 177 of your textbook.

Solution:

8. a) Let S represent studying and V represent playing video games.

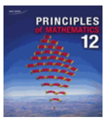
$P(S \cup V) = P(S) + P(V) - P(S \cap V)$

$0.8 = 0.4 + 0.6 - P(S \cap V)$

$P(S \cap V) = 0.2$

The probability that John will do both activities is 0.2 or 20%.

b) No. Since $P(S \cap V) \neq 0$, then $n(S \cap V) = 0$, so the sets of favourable outcomes for S and V are not disjoint.



Practice Problem:

Complete “Check your Understanding” question 9 on page 178 of your textbook.

Solution:

9. a) No. e.g., One athlete won two ore more medals at the Summer and Winter Olympics.

b) Total number of medal winners = 307

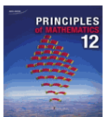
The odds in favour of a Canadian medal winner winning two or more medals at the Summer Olympics are 21 : (307 – 21) or 21 : 286.

c) $n(S \cup W) = 20 + 47 + 1$

$n(S \cup W) = 68$

Total number of medal winners = 307.

The odds in favour of the athlete having won two or more medals is 68 : (307 – 68) or 68 : 239.

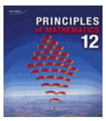


Practice Problem:

Complete “Check your Understanding” question 11 on page 178 of your textbook.

Solution:

11. e.g., There are 67 Grade 10 students that take art and 37 that take photography. If there are 87 students, how many take both? (17)



Practice Problem:

Complete "Check your Understanding" question 12 on page 179 of your textbook.

Solution:

12. a) Let *G* represent wearing glasses and *H* represent having a hearing loss.
If 68% of seniors have a hearing loss, and 10% of these people do not wear glasses, then 10% · 68%, or 6.8% of seniors have a hearing loss but do not wear glasses. This means that 61.2% of seniors wear glasses and have a hearing loss.

$P(H \setminus G) = 6.8\%$

$P(G \cap H) = 61.2\%$

$P(G \setminus H) = P(G) - P(G \cap H)$

$P(G \setminus H) = 76\% - 61.2\%$

$P(G \setminus H) = 14.8\%$

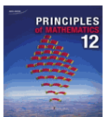
The probability this person will wear glasses and not have hearing aids is 14.8%.

b) Let *G* represent wears glasses and *H* represent having a hearing loss.

$P((G \cup H)') = 100\% - (76\% + 6.8\%)$

$P((G \cup H)') = 17.2\%$

The probability that this person will not wear glasses and not have hearing loss is 17.2%.



Practice Problem: **(KEY QUESTION)**

Complete "Check your Understanding" question 13 on page 179 of your textbook.

Solution:

13. a) Let E represent the eights, and let K represent the kings. Let O represent all cards.

$$n(E) = 4$$
$$n(K) = 4$$
$$n(E \cap K) = 0$$
$$n(E \cup K) = n(E) + n(K)$$
$$n(E \cup K) = 8$$
$$n(O) = 52$$

$$P(E \cup K) = \frac{n(E \cup K)}{n(O)}$$
$$P(E \cup K) = \frac{8}{52}$$
$$P(E \cup K) = \frac{2}{13}$$

The probability of drawing an eight or a king is $\frac{2}{13}$, or about 0.154 or 15.4%.

b) Let R represent the red cards, and let F represent the face cards. Let O represent all cards.

$n(R) = 26$

$n(F) = 12$

$n(R \cap F) = 6$

$n(R \cup F) = n(R) + n(F) - n(R \cap F)$

$n(R \cup F) = 26 + 12 - 6$

$n(R \cup F) = 32$

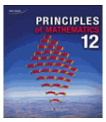
$n(O) = 52$

$P(R \cup F) = \dfrac{n(R \cup F)}{n(O)}$

$P(R \cup F) = \dfrac{32}{52}$

$P(R \cup F) = \dfrac{8}{13}$

The probability of drawing a red card or a face card is $\dfrac{8}{13}$, or about 0.615 or 61.5%.



Practice Problem:

Complete “Check your Understanding” question 14 on page 179 of your textbook.

Solution:

14. Let D represent the households that have one or more dogs, and let C represent the households that have one of more cats. Let O represent all Prairie households. $P(D) = 37\%$; $P(C) = 31\%$

a) $P(D \cup C) = 100\% - P((D \cup C)')$

$P(D \cup C) = 100\% - 47\%$

$P(D \cup C) = 53\%$

The probability a Prairie household has a cat or dog is 53%.

b) $P(D \cup C) = P(D) + P(C) - P(D \cap C)$

$53\% = 37\% + 31\,\% - P(D \cap C)$

$53\% = 68\% - P(D \cap C)$

$P(D \cap C) = 15\%$

$P(C \setminus D) = P(C) - P(D \cap C)$

$P(C \setminus D) = 31\% - 15\%$

$P(C \setminus D) = 16\%$

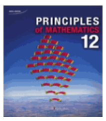
The probability that a Prairie household has one or more cats, but no dogs, is 16%.

c) $P(D \setminus C) = P(D) - P(D \cap C)$

$P(D \setminus C) = 37\% - 15\%$

$P(D \setminus C) = 22\%$

The probability that a Prairie household has one or more dogs, but no cats, is 22%.



Practice Problem:

Complete “Check your Understanding” question 16 on page 179 of your textbook.

Solution:

16. Let S represent damage to the computer’s power supply and let C represent damage to other components.

$P(S) = 0.15\%$

$P(C) = 0.30\%$

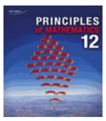
$P(S \cap C) = 0.10\%$

$P(S \cup C) = P(S) + P(C) - P(S \cap C)$

$P(S \cup C) = 0.15\% + 0.30\% - 0.10\%$

$P(S \cup C) = 0.35\%$

No. e.g., Since the probability of any form of damage is 0.35%, the computer does not need a surge protector.



Practice Problem:

Complete “Check your Understanding” question 18 on page 180 of your textbook.

Solution:

18. e.g., To determine the probability of two events that are not mutually exclusive, you must subtract the probability of both events occurring after adding the probabilities of each event. Example: Female students at a high school may play hockey or soccer. If the probability of a female student playing soccer is 62%, the probability of her playing in goal is 4%, and the probability of her either playing soccer or in goal is 64%, then the probability of her playing in goal at soccer is 62% + 4% – 64% = 2%.