Math 30-2: U3L5 Teacher Notes

Conditional Probabilities (Dependent Events)

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

What are Dependent Events?

Events whose outcomes are affected by each other. How can I identify a dependent event from one which are independent?

Step 1: *Ask yourself, is it possible for the events to occur in any order?* If no (the steps must be performed in a certain order), go to Step 3a. If yes (the steps can be performed in any order), go to Step 2. If you are unsure, go to Step 2.

Some examples of events that can clearly be performed in any order are:

Tossing a coin, then rolling a die

Purchasing a car, then purchasing a coat

Drawing cards from a deck

Step 2: *Ask yourself, does one event in any way affect the outcome (or the odds) of the other event?* If yes, go to step 3a, if no, go to Step 3b.

Some examples of events that affect the odds or probability of the next event include:

Choosing a card, not replacing it, then choosing another (because the odds of choosing the first card are 1/52, but if you do not replace it, your are changing the odds to 1/51 for the next draw)

Choosing anything and not replacing it, then choosing another (i.e. choosing bingo balls, raffle tickets)

Some examples of events that do not affect the odds or probability of the next event occurring are:

Choosing a card and replacing it, then choosing another card (because the odds of choosing the first card are 1/52, and the odds of choosing the second card are 1/52)

Choosing anything, as long as you put the items back

Step 3a: You're done–the event is **dependent**.

Step 3b: You're done–the event is **independent**.

- **NOTE: Probabilities of Independent Events will be studied in Lesson 6.**

What Is Conditional Probability?

If two events are dependent on each other then the probability is conditional, the probability of an event occurring given that another event has already occurred. The formula for conditional probabilities is:

Conditional Probability Formula
$$\mathrm{P}(A \text{ and } B) = \mathrm{P}(A) \bullet \mathrm{P}(A \mid B)$$ $$P(A \cap B) = P(A) \times P(A \mid B)$$

The notation for conditional probability is **P(*A* | *B*)**. It is read as "the probability of event *B* will occur, given that event *A* has already occured".

Click the icon to link to lesson on Conditional Probabilities.

Example:

A bag contains 5 blue hats and 6 yellow hats. Two hats are drawn from the bag, one after the other, without replacement. The probability, to the nearest hundredth, that the first hat drawn is blue and the second hat drawn is yellow is ____________.

Solution:

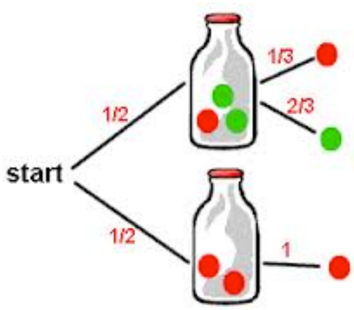
Let B represent drawing a blue hat and Y represent drawing a yellow hat. These events are dependent since there is **NO** replacement.

$$P(B \cap Y) = P(B) \times P(B \mid Y)$$
$$P(B \cap Y) = \frac{5}{11} \times \frac{6}{10}$$
$$P(B \cap Y) = \frac{30}{110} = \frac{3}{11}$$

Using Graphic Organizers to Help you Solve Problems

Being able to differentiate between Independent and Dependent events can sometimes be challenging. Using a graphic organizer such as a chart, a tree diagram or a Venn diagram can be helpful to help you sort necessary information.

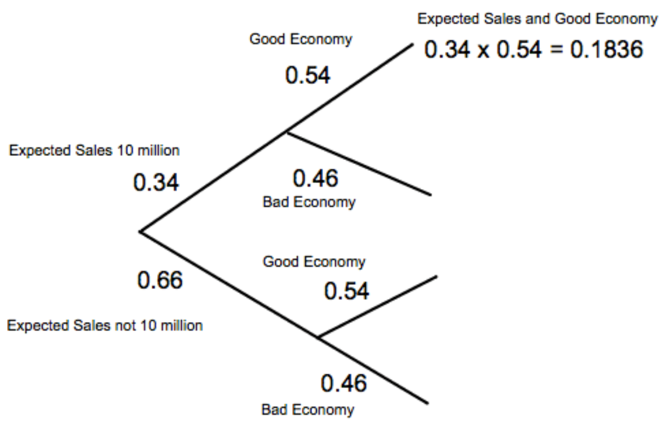
	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

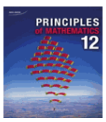


Example:

In Company X, they based their expected sales on whether it is a good economy and past statistics. In a good economy, the expected sales would be 34% likely to be $10 million. They predict that a good economy is 54% likely this year. Suppose that this year the economy is good. Determine the probability that the expected sales will be $10 million dollars.

Solution:





Practice Problem:

Complete “Check your Understanding” question 4a on page 189 of your textbook.

Solution:

We have two situations. Lexie can pull either two black socks or two white socks to make her pair.

Let *B* represent Lexie pulling a black sock from her drawer.

$$P(B) = \frac{6}{14} \qquad P(B \cap B) = P(B) \cdot P(B \mid B)$$

$$P(B) = \frac{3}{7} \qquad P(B \cap B) = \frac{3}{7} \cdot \frac{5}{13}$$

$$P(B \mid B) = \frac{5}{13} \qquad P(B \cap B) = \frac{15}{91}$$

The probability of drawing two black socks is 15/91, or about 0.165.

Let W represent Lexie pulling a white sock from her drawer.

$$P(W)=\frac{8}{14} \qquad P(W\cap W)=P(W)\cdot P(W|W)$$

$$P(W)=\frac{4}{7} \qquad P(W\cap W)=\frac{4}{7}\cdot\frac{7}{13}$$

$$P(W|W)=\frac{7}{13} \qquad P(W\cap W)=\frac{4}{13}$$

The probability of drawing two socks is 4/13, or about 0.308.

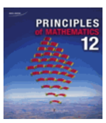
Let *A* represent drawing a pair of socks.

$$P(A)=P(B\cap B)+P(W\cap W)$$

$$P(A)=\frac{15}{91}+\frac{4}{13}$$

$$P(A)=\frac{43}{91}$$

The probability of drawing a pair of socks is 43/91, or about 0.473.



Practice Problem:

Complete “Check your Understanding” question 5 on page 189 of your textbook.

Solution:

a) Let A represent a student who plans to attend UBC, and let O represent all graduating students.

$$P(A)=\frac{n(A)}{n(O)}$$

$$P(A)=\frac{30+50}{80+110}$$

$$P(A)=\frac{80}{190}$$

$$P(A)=\frac{8}{19}$$

The probability that a graduating student will attend UBC is 8/19, or about 0.421.

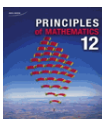
b) Let A represent a student who plans to attend UBC, and let F represent a female student.

$$P(F|A)=\frac{n(F)}{n(A)}$$

$$P(F|A)=\frac{50}{80}$$

$$P(F|A)=\frac{5}{8}$$

The probability that the student is female is 5/8, or about 0.625.

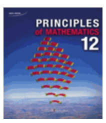


Practice Problem:

Complete “Check your Understanding” question 7 on page 189 of your textbook.

Solution:

$$
\begin{aligned}
P\,(\text{both loonies}) &= \left(\frac{4}{12}\right)\left(\frac{3}{11}\right)\\
&= \frac{12}{132}\\
&\approx 0.091
\end{aligned}
$$



Practice Problem:

Complete “Check your Understanding” question 9 on page 189 of your textbook.

Solution:

Let N represent a nice day, and let R represent a rainy day. Let J represent Ian joggin 8 km.

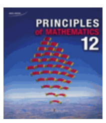
$$P(N \cap J) = P(N) \cdot P(J \mid N)$$
$$P(N \cap J) = 0.70 \cdot 0.85$$
$$P(N \cap J) = 0.595$$

$$P(R \cap J) = P(R) \cdot P(J \mid R)$$
$$P(R \cap J) = 0.30 \cdot 0.40$$
$$P(R \cap J) = 0.12$$

Add the two probabilities together.

$$P(J) = 0.595 + 0.12$$
$$P(J) = 0.715$$

The probability Ian will jog for 8km tomorrow is 0.715.



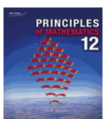
Practice Problem:

Complete “Check your Understanding” question 11 on page 190 of your textbook.

Solution:

Answer may vary. Here is a possible solution.

A student selected at random goes to a fast food outlet that particular day. What is the probability that the student had more than 1 h for lunch.



Practice Problem:

Complete “Check your Understanding” question 13 on page 190 of your textbook.

Solution:

Let F represent tires lasting 5 years, and let S represent tires lasting 6 years. Since all tires taht have lasted 6 years have also lasted 5 years, it is true that

$$P(F \cap S) = P(S).$$

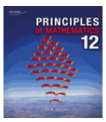
$$P(S|F) = \frac{P(F \cap S)}{P(F)}$$

$$P(S|F) = \frac{P(S)}{P(F)}$$

$$P(S|F) = \frac{0.5}{0.8}$$

$$P(S|F) = 0.625$$

The probability that tires have lasted 5 years will last 6 years is 0.625

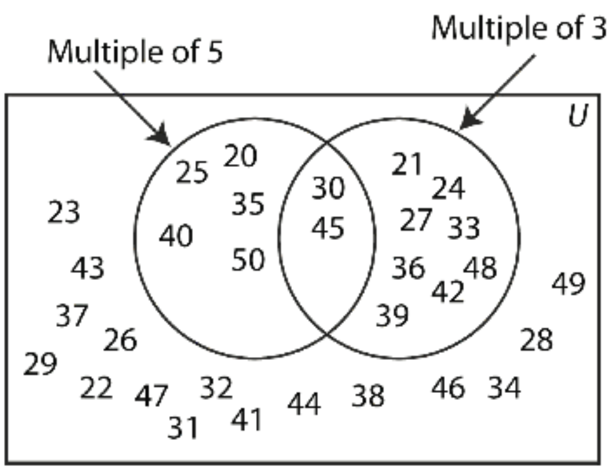


Practice Problem:

Complete “Check your Understanding” question 16 on page 191 of your textbook.

Solution:

a) The probability that a multiple of 5 will also be a multiple of 3 is 2/7, or about 0.286.



Solution:

b) Let *F* represent a number being a multiple of 5, and let *T* represent a number being a multiple of 3.

F: {20, 25, 30, 35, 40, 45, 50}
T: {21. 24. 27. 30, 33, 36, 39, 42, 45, 48}

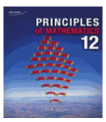
Notice that 30 and 45 are in both sets.

$$P(T|F) = \frac{P(T \cap F)}{P(F)}$$

$$P(T|F) = \frac{\left(\frac{2}{31}\right)}{\left(\frac{7}{31}\right)}$$

$$P(T|F) = \frac{2}{7}$$

The probability that a multiple of 5 will also be a multiple of 3 is 2/7 or about 0.286



Practice Problem:

Complete "Check your Understanding" question 18 on page 191 of your textbook.

Solution:

18. Let D represent a chip being defective, and let N represent a chip not being defective. Let O represent exactly one of the chips being defective.

a) There is only one way to draw two defective chips.

$$P(D) = \frac{3}{150} \qquad P(D|D) = \frac{2}{149}$$

$$P(D \cap D) = P(D) \cdot P(D|D)$$

$$P(D \cap D) = \frac{3}{150} \cdot \frac{2}{149}$$

$$P(D \cap D) = \frac{6}{22\,350}$$

$$P(D \cap D) = \frac{1}{3725}$$

The probability of drawing two defective chips is $\frac{1}{3725}$ or about 0.000 268.

b) There is only one way to draw two non-defective chips.

$$P(N) = \frac{147}{150} \qquad P(N|N) = \frac{146}{149}$$

$$P(N \cap N) = P(N) \cdot P(N|N)$$

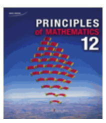
$$P(N \cap N) = \frac{147}{150} \cdot \frac{146}{149}$$

$$P(N \cap N) = \frac{21\,462}{22\,350}$$

$$P(N \cap N) = \frac{3577}{3725}$$

The probability of drawing two non-defective chips is $\frac{3577}{3725}$ or about 0.960.

c) In this case, either one defective chip is drawn first and then one non-defective chip is drawn, or vice versa. The probability of drawing exactly one defective chip is the sum of the probabilities of these events.



Practice Problem:

Complete "Check your Understanding" question 21 on page 191 of your textbook.

Solution:

Answers will vary. Here is an example.

The probability of event A and B both occuring is the probability A occurs, multiplied by the probability B occurs given that A occurs.

Example: If a 6-sided fair die is rolled twice,

P(rolling a 4 the first time and a total > 7)
= P(rolling a 4 the first time)
· P(a total > 7 | rolling a 4 the first time)

or $\frac{1}{12} = \frac{1}{6} \cdot \frac{1}{2}$.

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