Math 30-2: U3L6 Teacher Notes

Independent Events

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

What are Independent Events?

Independent events are two events that **DON'T** depend on the other.

For Example:

Flipping a coin and picking a card from a deck of cards.

NOTE: Drawing an item and then drawing another item, with REPLACEMENT, are considered Independent Events

Click the icon for a video on Identifying Independent Events

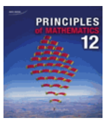
Probability of Independent Events

The probability that two independent events, A and B will both occur is the product of their individual probabilities.

Probability of Two Independent Events
P(A and B) = P(A) x P(B)
$P(A \cap B) = P(A) \times P(B)$

NOTE: Using a tree diagram is very helpful in solving problems that involve independent events.

Click the icon for a video on calculating probabilities of Independent Events



Practice Problem:

Complete “Check your Understanding” question 5 on page 198 of your textbook.

Solution:

a. If two events are independent, then $P(A \cap B) = P(A) \times P(B)$

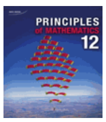
$$P(A \cap B) = P(A) \times P(B)$$
$$0.12 = 0.35 \times 0.4$$
$$0.12 \neq .14$$

Since they are not equal, the events are NOT independent.

b. If two events are independent, then $P(A \cap B) = P(A) \times P(B)$

$$P(A \cap B) = P(A) \times P(B)$$
$$0.468 = 0.720 \times 0.650$$
$$0.468 = 0.468$$

Since they are equal, the events are independent.



Practice Problem:

Complete "Check your Understanding" question 7 on page 199 of your textbook.

Solution:

These events are dependent, because the deck has 40 cards when the first card is dealt, but it has 39 cards when the second card is dealt.

Let *C* represent a club being dealt, and let *H* represent a heart being dealt.

$$P(C)=\frac{10}{40}$$

$$P(C)=\frac{1}{4}$$

$$P(H|C)=\frac{10}{39}$$

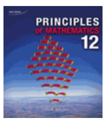
$$P(C\cap H)=P(C)\cdot P(H|C)$$

$$P(C\cap H)=\frac{1}{4}\cdot\frac{10}{39}$$

$$P(C\cap H)=\frac{10}{156}$$

$$P(C\cap H)=\frac{5}{78}$$

The probability that the first card dealt is a club and the second card dealt is a heart is 5/78, or about 0.0641.



Practice Problem:

Complete "Check your Understanding" question 8 on page 199 of your textbook.

Solution:

8. Let *A* represent the first roll, and *B* represent the second.

a) $P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = \frac{1}{6} \cdot \frac{1}{6}$

$P(A \cap B) = \frac{1}{36}$

The probability is $\frac{1}{36}$.

b) $P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = \frac{3}{6} \cdot \frac{3}{6}$

$P(A \cap B) = \frac{1}{2} \cdot \frac{1}{2}$

$P(A \cap B) = \frac{1}{4}$

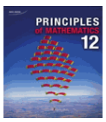
The probability is $\frac{1}{4}$.

c) $P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = \frac{5}{6} \cdot \frac{5}{6}$

$P(A \cap B) = \frac{25}{36}$

The probability is $\frac{25}{36}$.



Practice Problem:

Complete “Check your Understanding” question 10 on page 199 of your textbook.

Solution:

10. a) These two events are independent. Let H represent getting heads from tossing a coin, and let S represent spinning a six on a spinner.

$$P(H) \cdot P(S) = P(H \cap S)$$

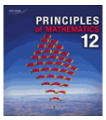
$$\frac{1}{2} \cdot P(S) = \frac{1}{12}$$

$$\frac{2}{2} \cdot P(S) = \frac{1}{12} \cdot 2$$

$$P(S) = \frac{1}{6}$$

The probability of spinning a six is $\frac{1}{6}$.

e.g., Spinner has 6 equal areas, numbered 1 to 6.



Practice Problem:

Complete “Check your Understanding” question 11 on page 199 of your textbook.

Solution:

a. No. Anne has a probability of $\left(\frac{3}{10}\right)\left(\frac{5}{9}\right) \approx 0.167$ of drawing two blue marbles, and Abby has a probability of $\left(\frac{8}{19}\right)\left(\frac{7}{18}\right) \approx 0.164$.

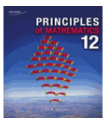
b. For Anne, there are two possible situations—a red marble from the first bag and a blue marble from the second, or a blue marble from the first bag and a red marble from the second bag. The probability is $\left(\frac{7}{10}\right)\left(\frac{5}{9}\right) + \left(\frac{3}{10}\right)\left(\frac{4}{9}\right) = \frac{47}{90}$.

For Abby, there are also two possible situations—a red marble on the first draw and a blue marble on the second draw, or a blue marble on the first draw and a red marble on the second draw. The probability is $\left(\frac{11}{19}\right)\left(\frac{8}{18}\right) + \left(\frac{8}{19}\right)\left(\frac{11}{18}\right) = \frac{88}{171}$.

Therefore, the probability that Anne and Abby will both draw one red marble and one blue marble is as follows:

$$\frac{47}{90} \times \frac{88}{171} = \frac{2068}{7695}$$

c. No. Anne has a probability of $\left(\frac{5}{10}\right)\left(\frac{5}{10}\right) = 0.25$ of drawing two red marbles, and Abby has a probability of $\left(\frac{10}{20}\right)\left(\frac{9}{19}\right) \approx 0.237$.



Practice Problem:

Complete “Check your Understanding” question 13 on page 200 of your textbook.

Solution:

13. Because the winner's ticket is returned to the draw after the first prize is awarded, the two events are independent.
Let W represent Tiegan winning the first prize, and let X represent winning the second prize.

a)

$$P(W)=\frac{5}{100} \qquad P(W\cap X)=P(W)\cdot P(X)$$

$$P(W)=\frac{1}{20} \qquad P(W\cap X)=\frac{1}{20}\cdot\frac{1}{20}$$

$$P(X)=\frac{1}{20} \qquad P(W\cap X)=\frac{1}{400}$$

The probability Tiegan wins both prizes is $\frac{1}{400}$, 0.0025 or 0.25%.

b)

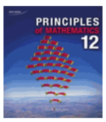
$$P(W')=1-P(W) \qquad P(W'\cap X')=P(W')\cdot P(X')$$

$$P(W')=1-\frac{1}{20} \qquad P(W'\cap X')=\frac{19}{20}\cdot\frac{19}{20}$$

$$P(W')=\frac{19}{20} \qquad P(W'\cap X')=\frac{361}{400}$$

$$P(W')=P(X')$$

The probability that Tiegan wins neither prize is $\frac{361}{400}$, 0.9025 or 90.25%.



Practice Problem: (KEY QUESTION)

Complete “Check your Understanding” question14 on page 200 of your textbook.

Solution:

14. a) e.g., Problem: What is the probability of drawing a card from a shuffled standard deck and getting a red card, then replacing it, shuffling the deck again, drawing a second card, and getting a heart?
Solution: Let R represent getting a red card and H represent getting a heart.

$$P(R \cap H) = P(R) \cdot P(H)$$

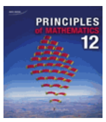
$$P(R \cap H) = \frac{26}{52} \cdot \frac{13}{52}$$

$$P(R \cap H) = \frac{1}{2} \cdot \frac{1}{4}$$

$$P(R \cap H) = \frac{1}{8}$$

The probability is $\frac{1}{8}$, 0.125 or 12.5%.

b) e.g., Problem: What is the probability of drawing a card from a shuffled standard deck and getting a red card, then drawing a second card without replacing the first one, and getting a spade?



Practice Problem:

Complete “Check your Understanding” question 16 on page 200 of your textbook.

Solution:

16. a) The formula is $P(A \cap B) = P(A) \cdot P(B)$ only when A and B are independent events.

b) e.g. Drawing two red marbles from a bag containing 5 red and 15 blue marbles, with replacement:
A = {red on 1st draw} and B = {red on 2nd draw} are independent events, so

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = \frac{1}{4} \cdot \frac{1}{4}$$

$$P(A \cap B) = \frac{1}{16}$$

c) e.g., Drawing two red marbles from a bag containing 5 red and 15 blue marbles, without replacement:
A = {red on 1st draw} and B = {red on 2nd draw} are dependent events so

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(A \cap B) = \frac{1}{4} \cdot \frac{4}{19}$$

$$P(A \cap B) = \frac{1}{19}$$