Math 30-2: U2L4 Teacher Notes

Permutations When Objects are Identical

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

4.1 Represent and solve counting problems, using a graphic organizer.

4.3 Identify and explain assumptions made in solving a counting problem.

5.2 Determine, with or without technology, the value of a factorial.

5.3 Simplify a numeric or an algebraic fraction that contains factorials in both the numerator and denominator.

5.6 Determine the number of permutations of n elements taken n at a time where some elements are not distinct.

5.7 Explain, using examples, the effect on the total number of permutations of n elements when two or more elements are identical.

Permutations with Some Identical Elements

Can a choice, or item, be repeated or used more than once? The rule of thumb is: **It can't be repeated unless you are told otherwise or unless the items that are being arranged, are numbers.**

Permutations with Repetition

The number of different permutations of n objects, where there are n_1 indistinguishable objects of style 1, n_2 indistinguishable objects of style 2, ..., and n_k indistinguishable objects of style k, is ;

$$\frac{n!}{n_1! \bullet n_2! \bullet \ldots n_k!}$$

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Example

How many different ways can you rewrite the word LIKE?

Solution:

Let's list all the possible arrangements. There are not identical letters. Therefore,

LIKE	LIEK	LKIE	LKEI	LEIK	LEKI
ILKE	ILEK	IELK	IEKL	IKLE	IKEL
KLIE	KLEI	KILE	KIEL	KELI	KEIL
ELIK	ELKI	EILK	EIKL	EKLI	EKIL

Therefore there are 24 different permutations $_4P_4 = 4!$

Sometimes you will deal with permutations in which some items are identical.

Try it with the word FALL

Keep track of the common letters by using L_1 and L_2

FAL_1L_2	AFL_1L_2	L_1AFL_2	L_2AL_1F
FAL_2L_1	AFL_2L_1	L_1AL_2F	L_2AFL_1
FL_1L_2A	AL_1L_2F	L_1FL_2A	L_2L_1FA
FL_2L_1A	AL_2L_1F	L_1L_2FA	L_2FL_1A
FL_1AL_2	AL_1FL_2	L_1FAL_2	L_2L_1AF
FL_2AL_1	AL_2FL_1	L_1L_2AF	L_2FAL_1

If the two L's trade places, there are ${}_2P_2 = 2!$ ways that they can be arranged.

Therefore the total number of ways that the letters can be arranged is now expressed as

$$4! / 2! = 24 / 2 = 12$$

Divide the total number of permutations by the number of ways that you can arrange the identical letters

Try it with the word SISS

From previous knowledge we can see that we take

$4! / 3!$ Since 3! Is the number of ways that the S can be arranged.

$24 / 6 = 4$

The number of permutations of a set of *n* items of which *a* are identical is

$$\frac{n!}{a!}$$

For Example:

How many arrangements are there for the word "bookkeeper".

Solution:

$10!$ Start with the total number of arrangements of 10 letters

$\frac{10!}{3!\,2!\,2!}$ Now divide out each letter that is similar by the number of similar arrangements. There are 3 e's, 2 o's and 2 k's

For a set of n objects containing a identical elements, b identical objects, c identical objects, there are

$$\frac{n!}{a!\,b!\,c!\ldots}$$ **permutations**

For Example

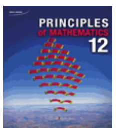
Shawna is looking at all of her cars in the driveway. She has 3 mustangs, 2 motorcycles, 4 corvettes and 3 porsche's. In how many ways can she arrange her cars?

Solution:

There are 12 cars in total, with 3 mustangs, 2 motorcycles, 4 corvettes and 3 porsches.

$$\frac{12!}{3!\ 2!\ 4!\ 3!} = 277\ 200$$

Shawna can arrange her cars 277 200 different ways.



Practice Problem:

Complete “Practising” question 2 on page 104 of your textbook.

Solution:

Let A represent the arrangement of 6 flags:

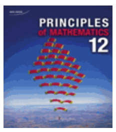
$$A = \frac{6!}{2!2!2!}$$

$$A = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$$

$$A = 6 \cdot 5 \cdot 3$$

$$A = 90$$

There are 90 different signals that can be made from the 6 flags hung in a vertical line.



Practice Problem:

Complete “Practising” question 5 on page 105 of your textbook.

Solution:

5. Let C represent the number of ways:

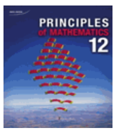
$$C = \frac{9!}{2!3!4!}$$

$$C = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$C = 9 \cdot 7 \cdot 5 \cdot 4$$

$$C = 1260$$

There are 1260 ways that Norm can distribute 1 cookie to each grandchild.



Practice Problem:

Complete “Practising” question 6 on page 105 of your textbook.

Solution:

6. a) Let A represent the number of arrangements:

$A = 5!$

$A = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$A = 120$

There are 120 different arrangements that can be made using all the letters.

b) Let A represent the number of arrangements:

$$A = \frac{7!}{2!}$$

$$A = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!}$$

$$A = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$$

$$A = 2520$$

There are 2520 different arrangements that can be made using all the letters.

c) Let A represent the number of arrangements:

$$A = \frac{8!}{2!}$$

$$A = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!}$$

$$A = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$$

$$A = 20160$$

There are 20 160 different arrangements that can be made using all the letters.

d) Let A represent the number of arrangements:

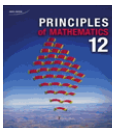
$$A = \frac{12!}{2!3!}$$

$$A = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$$

$$A = 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$A = 39916800$$

There are 39 916 800 different arrangements that can be made using all the letters.



Practice Problem: **(KEY QUESTION)**

Complete “Practising” question 7 on page 105 of your textbook.

Solution:

7. a) Let A represent the number of arrangements:

$$A = \frac{15!}{5!5!5!}$$

$$A = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$A = 14 \cdot 13 \cdot 11 \cdot 9 \cdot 7 \cdot 6$$

$$A = 756\,756$$

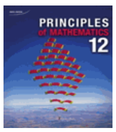
There are 756 756 different ways he can arrange the books on the shelf.

b) Group the sets of 5 together.

$A = 3 \cdot 2 \cdot 1$

$A = 6$

There are 6 ways he can arrange the books.

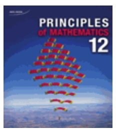


Practice Problem:

Complete “Practising” question 8 on page 105 of your textbook.

Solution:

8. e.g., A shish kabob skewer has 4 pieces of beef, 2 pieces of green pepper, and 1 piece each of mushroom and onion. How many different combinations are possible?



Practice Problem:

Complete “Practising” question 9 on page 105 of your textbook.

Solution:

9. a) Let R represent the number of routes:

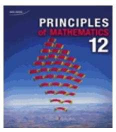
$$R = \frac{9!}{5!4!}$$

$$R = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$R = 7 \cdot 6 \cdot 4 \cdot 3$$

$$R = 126$$

There are 126 routes travelling from point A to point B if you travel only south or east.

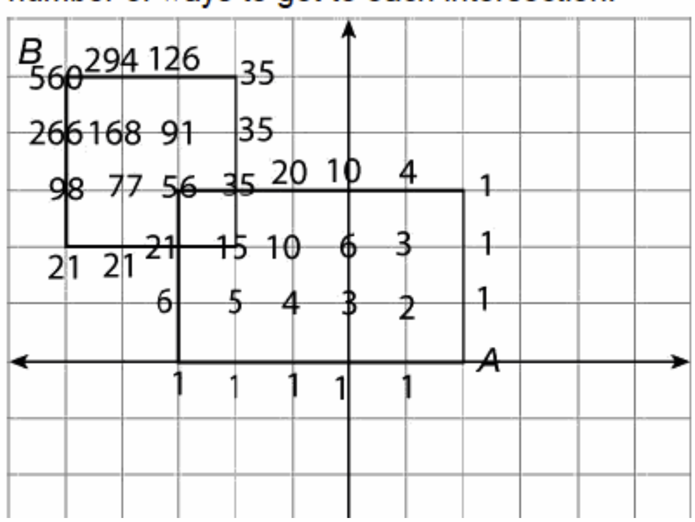


Practice Problem:

Complete “Practising” question 11 on page 105 of your textbook.

Solution:

11. a) e.g., I drew the following diagram to show the number of ways to get to each intersection.



The sum of the numbers on the top right and bottom left corners of each block is equal to the number of routes to the top left corner of each block. There are 560 different routes from A to B, if you travel only north or west.

b) I need to go north twice and west four times, for a total of 6 moves, to travel the first 2 by 4 block of the route. I need to go north once and west once, for a total of 2 moves, to travel the next 1 by 1 block of the route. I need to go north twice and west twice, for a total of 4 moves, to travel the last 2 by 2 block of the route.

Let R represent the number of routes:

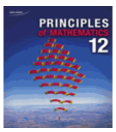
$$R = \frac{6!}{4!2!} \cdot 2! \cdot \frac{4!}{2!2!}$$

$$R = \left(\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}\right) \cdot (2 \cdot 1) \cdot \left(\frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1}\right)$$

$$R = (5 \cdot 3) \cdot (2) \cdot (6)$$

$$R = 180$$

There are 180 different routes from A to B, if you travel only north or west.



Practice Problem:

Complete "Practising" question 13 on page 106 of your textbook.

Solution:

13. a) Let P represent the number of permutations:

$P = 7!$

$P = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$P = 5040$

There are 5040 different arrangements possible for the new totem pole.

b) Let P represent the number of permutations:

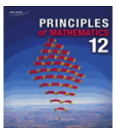
$$P = \frac{7!}{2!2!}$$

$$P = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1}$$

$$P = 7 \cdot 6 \cdot 5 \cdot 3 \cdot 2 \cdot 1$$

$$P = 1260$$

There are 1260 different arrangements possible for the new totem pole.



Practice Problem:

Complete “Practising” question 15 on page 107 of your textbook.

Solution:

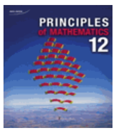
15. a) e.g., I am assuming that the coins of the same denomination are considered identical objects. Let *A* represent the number of arrangements:

$$A = \frac{9!}{4!3!2!}$$

$$A = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 2}$$

$$A = 1260$$

There are 1260 ways the 9 coins can be arranged in a line.



Practice Problem:

Complete “Practising” question 17 on page 107 of your textbook.

Solution:

17. a) Let P represent the number of permutations:

$$P = 1 \cdot \frac{8!}{3!3!2!} \cdot 1$$

$$P = 1 \cdot \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} \cdot 1$$

$$P = 1 \cdot 8 \cdot 7 \cdot 5 \cdot 2 \cdot 1$$

$$P = 560$$

There are 560 permutations possible if you must start with A and end with C.

b) e.g., If you start by putting the I's in the first and second positions, and then in the second and third positions, and so on and so forth up until you put them i the ninth and tenth positions, there are 9 different arrangements of the I's just on their own. The number o different arrangements of all the letters in each of these 9 arrangements is the number of ways to organize the other 8 letters. Since the other 8 letters are always the same, the number of permutations of the letters for eacl arrangement of the I's is the same. Let P represent the number of permutations:

$$P = 9\left(\frac{8!}{3!3!}\right)$$

$$P = 9\left(\frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot3\cdot2\cdot1}\right)$$

$$P = 9(8\cdot7\cdot5\cdot4)$$

$$P = 9(1120)$$

$$P = 10\,080$$

There are 10 080 permutations possible if the two I's must be together.