Math 30-2: U2L6 Teacher Notes

Combinations

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

4.1 Represent and solve counting problems, using a graphic organizer.

5.2 Determine, with or without technology, the value of a factorial.

5.5 Determine the number of permutations of n elements taken r at a time.

5.8 Generalize strategies for determining the number of permutations of n elements taken r at a time.

6.1 Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations.

6.2 Determine the number of combinations of n elements taken r at a time.

6.3 Generalize strategies for determining the number of combinations of n elements taken r at a time.

What is a Combination?

When ORDER of selection is of NO importance, you are working with combinations!

A **combination** is a selection of a group of objects taken from a larger pool where the kinds of objects selected is of importance, but **the order in which they are selected is not.**

There are two different notations used for combinations.

One of them you are already used to, since it is similar to permuation notation

$${}_nC_r \quad \text{and} \quad \binom{n}{r}$$

Two different notations that mean the same thing.

Notice that there is no line between the n and the r (this is a very common error. People often write it incorrectly as a fraciton, such as $\left(\frac{n}{r}\right)$ which you **SHOULD NOT!!**)

Click the icon to watch a Youtube video on Introducing Combinations.

For Example: Evaluate $\binom{7}{3}$

Solution:

Use the ${}_nC_r$ wher n = 7 and r = 3

$$
\begin{aligned}
{}_nC_r &= \frac{n!}{(n-r)!\,r!} \\
{}_7C_3 &= \frac{7!}{(7-3)!\,3!} \\
&= \frac{7!}{4!\,3!} \\
&= 35
\end{aligned}
$$

On the TI-83 this can be done easily by doing the following:

- Press **7** first, then press **MATH** and
- Cursor across to **PRB** (which stands for probability)
- Choose **3: nCr** and press **ENTER**, then press **3** and then **ENTER**
- You should get the value 35.

There are simple type of combinations, ones that have no restrictions on the number of groups.

For Example:

A coach must choose five starters from a team of 11 players. Each player can play any position.

How many different ways can the coach choose the starters?

Solution:

For Example:

Eddie selected 6 books from the library. However, he can only check out 4 books at a time. How many possible selections can he make?

Solution:

Some combinations require you to select specific item(s) to the group.

For Example:

A school committee of 6 is to be formed from 10 students. How many committees can be formed if Sam must be on the committee?

Solution:

For Example:

There are 7 possible toppings for a salad, but you only want 5 toppings, one of which must be tomatoes. How many different selections can be made?

Solution:

For Example:

From a standard deck of 52 playing cards, a 5 card hand is dealt. How many distinct five card hands are there if the queen of hearts and the three of spades must be in the hand?

Solution:

Some combinations require you to use the fundamental counting principle. When presented with multiple groups of items from which you are required to make a selection, you will MULTIPLY the separate cases together. The word AND is either written in the question or is implied

For Example:

A committee of 3 boys and 4 girls is to be formed from a group of 8 boys and 9 girls. How many committees are possible?

Solution:

For Example:

If a case of blenders contain 10 working blenders and 2 defective blenders, how many ways can you take out 4 blenders that work and one that doesn't?

Solution:

For Example:

If a student must select two courses from Group A, two courses from Group B, and one course from Group C, how many combinations are there?

Solution:

Group A	Group B	Group C
Math 30 Chemistry 30 Physics 30 Biology 30	English 30 Social 30	Math 31 Science 30 French 30

At Least/ At Most Questions
These questions will require you to ADD all the possible cases together. The word OR is either used or implied.

For Example:

A committee of 5 people is to be formed from a group of 5 men and 8 women. How many possible committees can be formed if at least 4 women are on the committee?

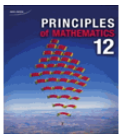
Solution:

For Example:

From a standard deck of 52 cards, a 4 card hand is dealt. How many distinct hands can be formed if there is at most 2 queens?

Solution:

Click the icon to watch a KhanAcademy video on a Combination Example.



Practice Problem:

Complete “Practising” question 1 on page 118 of your textbook.

Solution:

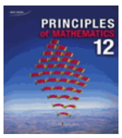
1. a)

Flavour 1	Flavour 2
vanilla	strawberry
vanilla	chocolate
vanilla	butterscotch
strawberry	vanilla
strawberry	chocolate
strawberry	butterscotch
chocolate	vanilla
chocolate	strawberry
chocolate	butterscotch
butterscotch	vanilla
butterscotch	strawberry
butterscotch	chocolate

b)

Flavour 1	Flavour 2
vanilla	strawberry
vanilla	chocolate
vanilla	butterscotch
strawberry	chocolate
strawberry	butterscotch
chocolate	butterscotch

c) The number of two-flavour permutations is double the number of two-flavour combinations because each two-flavour combination can be written in two different ways.



Practice Problem:

Complete “Practising” question 2 on page 118 of your textbook.

Solution:

2. a) Let C represent the number of committees:

$$C = {}_5C_3$$

$$C = \frac{5!}{3!(5-3)!}$$

$$C = \frac{5!}{3! \cdot 2!}$$

$$C = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1}$$

$$C = \frac{5 \cdot 4}{2 \cdot 1}$$

$$C = 5 \cdot 2$$

$C = 10$ There are 10 possible committees.

b) Let C represent the number of committees:

$$C = {}_5C_2$$

$$C = \frac{5!}{2!(5-2)!}$$

$$C = \frac{5!}{2! \cdot 3!}$$

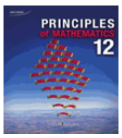
$$C = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!}$$

$$C = \frac{5 \cdot 4}{2 \cdot 1}$$

$$C = 5 \cdot 2$$

$C = 10$ There are 10 possible committees.

c) e.g., My answers for parts a) and b) are the same. This occurred because the sum of 2 and 3 is 5.



Practice Problem:

Complete "Practising" question 4 on page 118 of your textbook.

Solution:

a)

$$ {}_5C_3 = \frac{5!}{3!(5-3)!} $$
$$ {}_5C_3 = \frac{5!}{3!2!} $$
$$ {}_5C_3 = \frac{5\cdot 4\cdot 3!}{3!2\cdot 1} $$
$$ {}_5C_3 = \frac{5\cdot 4}{2\cdot 1} $$
$$ {}_5C_3 = 5\cdot 2 $$
$$ {}_5C_3 = 10 $$

b)

$$ {}_9C_8 = \frac{9!}{8!(9-8)!} $$
$$ {}_9C_8 = \frac{9!}{8!1!} $$
$$ {}_9C_8 = \frac{9\cdot 8!}{8!1} $$
$$ {}_9C_8 = \frac{9}{1} $$
$$ {}_9C_8 = 9 $$

c)

$$ {}_6C_4 = \frac{6!}{4!(6-4)!} $$
$$ {}_6C_4 = \frac{6!}{4!2!} $$
$$ {}_6C_4 = \frac{6\cdot 5\cdot 4!}{4!2\cdot 1} $$
$$ {}_6C_4 = \frac{6\cdot 5}{2\cdot 1} $$
$$ {}_6C_4 = 3\cdot 5 $$
$$ {}_6C_4 = 15 $$

d) $_{10}C_0 = \dfrac{10!}{0!(10-0)!}$

$_{10}C_0 = \dfrac{10!}{0!10!}$

$_{10}C_0 = \dfrac{1}{1}$

$_{10}C_0 = 1$

e) $_{12}C_6 = \dfrac{12!}{6!(12-6)!}$

$_{12}C_6 = \dfrac{12!}{6!6!}$

$_{12}C_6 = \dfrac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6!}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 6!}$

$_{12}C_6 = \dfrac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$

$_{12}C_6 = 2\cdot 11\cdot 5\cdot 3\cdot 2\cdot \dfrac{7}{5}$

$_{12}C_6 = 924$

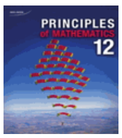
f) $_{8}C_1 = \dfrac{8!}{1!(8-1)!}$

$_{8}C_1 = \dfrac{8!}{1!7!}$

$_{8}C_1 = \dfrac{8\cdot 7!}{1\cdot 7!}$

$_{8}C_1 = \dfrac{8}{1}$

$_{8}C_1 = 8$



Practice Problem:

Complete "Practising" question 5 on page 118 of your textbook.

Solution:

5. Let C represent the number of combinations:

$$C = {}_{10}C_6$$

$$C = \frac{10!}{6!(10-6)!}$$

$$C = \frac{10!}{6!4!}$$

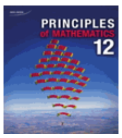
$$C = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!4 \cdot 3 \cdot 2 \cdot 1}$$

$$C = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$C = 5 \cdot 3 \cdot 2 \cdot 7$$

$$C = 210$$

There are 210 ways 6 players can be chosen to start a volleyball game from a team of 10.



Practice Problem: (KEY QUESTION)

Complete “Practising” question 8 on page 118 of your textbook.

Solution:

8. a) The problem involves combinations e.g., because it does not state that the order of the starting line matters.

b) Let L represent the number of different lineups, n = 14 and r = 8 because Connie must be the pitcher of the starting lineup.

$$L = {}_nC_r$$

$$L = {}_{14}C_8$$

$$L = \frac{14!}{8!(14-8)!}$$

$$L = \frac{14!}{8!6!}$$

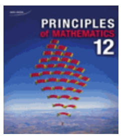
$$L = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8!6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$L = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$L = 7 \cdot 13 \cdot 11 \cdot 3$$

$$L = 3003$$

There are 3003 ways that the coach can choose his starting lineup of 9 players, if Connie must be the pitcher.



Practice Problem:

Complete "Practising" question 9 on page 118 of your textbook.

Solution:

9. a) Yes, I do agree.

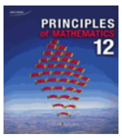
e.g.,

LS	RS
${}_6C_2$	${}_6C_4$
$\frac{6!}{2!(6-2)!}$	$\frac{6!}{4!(6-4)!}$
$\frac{6!}{2!\cdot 4!}$	$\frac{6!}{4!\cdot 2!}$
$\frac{6\cdot 5\cdot 4!}{2\cdot 1\cdot 4!}$	$\frac{6\cdot 5\cdot 4!}{4!\cdot 2\cdot 1}$
$\frac{6\cdot 5}{2\cdot 1}$	$\frac{6\cdot 5}{2\cdot 1}$
$3\cdot 5$	$3\cdot 5$
15	15

LS = RS

b) e.g., Some other cases with the same relationship as part a) are ${}_8C_1 = {}_8C_7$, ${}_6C_0 = {}_6C_6$, and ${}_{12}C_7 = {}_{12}C_5$. I notice that if you have two combinations with the same *n*, and the sum of the *r*'s for those combinations is equal to *n*, then the value of the combinations will be the same.

c) e.g., $\binom{n}{r} = \binom{n}{n-r}$



Practice Problem:

Complete “Practising” question 11 on page 119 of your textbook.

Solution:

11. a) Let C represent the number of committees:

$C = {}_{10}C_5$

$C = \dfrac{10!}{5!(10-5)!}$

$C = \dfrac{10!}{5!5!}$

$C = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5!}$

$C = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$C = 2 \cdot 3 \cdot 2 \cdot 7 \cdot 3$

$C = 252$

There are 252 committees that can be formed if there are no conditions.

b)

Let W represent the number of combinations for the women:

$W = {}_6C_3$

$W = \dfrac{6!}{3!(6-3)!}$

$W = \dfrac{6!}{3! \cdot 3!}$

$W = \dfrac{6 \cdot 5 \cdot 4 \cdot 3!}{3 \cdot 2 \cdot 1 \cdot 3!}$

$W = \dfrac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}$

$W = 2 \cdot 5 \cdot 2$

$W = 20$

Let M represent the numbe of combinations for the mei

$M = {}_4C_2$

$M = \dfrac{4!}{2!(4-2)!}$

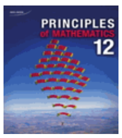
$M = \dfrac{4!}{2! \cdot 2!}$

$M = \dfrac{4 \cdot 3 \cdot 2!}{2 \cdot 1 \cdot 2!}$

$M = \dfrac{4 \cdot 3}{2 \cdot 1}$

$M = 2 \cdot 3$

$M = 6$

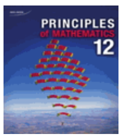


Practice Problem:

Complete “Practising” question 13 on page 119 of your textbook.

Solution:

13. a) i) 5 objects, 3 in each combination
ii) 10 objects, 2 in each combination
iii) 5 objects, 3 in each combination
b) e.g., **i)** How many ways can you choose 3 coins from a bag containing a penny, a nickel, a dime, a quarter, and a loonie?



Practice Problem:

Complete "Practising" question 16 on page 120 of your textbook.

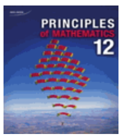
Solution:

16. a) 1, e.g., the player can only win if the six numbers they choose are the same and in the same order as the six numbers drawn.

b)
$$_{66}C_6 = \frac{66!}{6!(66-6)!}$$
$$_{66}C_6 = \frac{66!}{6!60!}$$
$$_{66}C_6 = \frac{66 \cdot 65 \cdot 64 \cdot 63 \cdot 62 \cdot 61 \cdot 60!}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 60!}$$
$$_{66}C_6 = \frac{66 \cdot 65 \cdot 64 \cdot 63 \cdot 62 \cdot 61}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$
$$_{66}C_6 = 11 \cdot 13 \cdot 16 \cdot 21 \cdot 31 \cdot 61$$
$$_{66}C_6 = 90\,858\,768$$

There are 90 858 768 different ways the player can win.

c) e.g., No. Even if everyone in the city plays, it is very unlikely that anyone will win since each player only has a 1 in 90 858 767 chance of winning.



Practice Problem:

Complete “Practising” question 17 on page 120 of your textbook.

Solution:

17. e.g., The number of sides in a polygon is equal to the number of vertices, the number of vertices = *n*. From each vertex, *v*, a diagonal is formed by a line segment intersecting a vertex that is not directly beside *v*. Thus, the number of vertices that will make a diagonal with *v* in an *n*-sided polygon is $n - 2$. Trying $r = 2$ in ${}_nC_r$ with the values from the polygons on the side of the textbook page, there is a pattern: (*d* = number of diagonals)

n	r	${}_nC_2$	d
3	2	3	0
4	2	6	2
5	2	10	5
6	2	15	9

Notice that $n + d = {}_nC_2$.
Rearranging, $d = {}_nC_2 - n$. Thus, the number of diagonals for an n-sided polygon can be determined using ${}_nC_2 - n$.

August 06, 2013