Math 30-2: U2L7 Teacher Notes

Solving Counting Problems

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

4.1 Represent and solve counting problems, using a graphic organizer.

4.3 Identify and explain assumptions made in solving a counting problem.

4.4 Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning

5.2 Determine, with or without technology, the value of a factorial.

5.3 Simplify a numeric or an algebraic fraction that contains factorials in both the numerator and denominator.

5.5 Determine the number of permutations of n elements taken r at a time.

6.1 Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations.

6.2 Determine the number of combinations of n elements taken r at a time.

The best way to practice application type of questions by example. Here are some videos and some examples that will help you through the process.

Click the icon to watch a Youtube video on Combinations Application problems.

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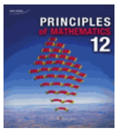
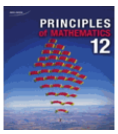


Practice Problem:

Complete “Practising” question 1 on page 126 of your textbook.

Solution:

1. a) This situation involves combinations because the order of the 3 toppings on the pizza does not matter.
b) This situation involves permutations because the three spots for the candidates who are selected are all different so order matters.
c) This situation involves permutations because for a group of 3 numbers, there are different ways to roll those three numbers because of the different colours of the dice.
d) This situation involves combinations because the 5 children who are selected are all in the same position. No information is stated in the question about positions the children may play, so I can only assume that they are not playing in specific positions.

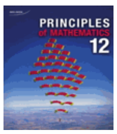


Practice Problem:

Complete “Practising” question 2 on page 126 of your textbook.

Solution:

2. e.g., Situation A involves combinations and situation B involves permutations. For situation A, order does not matter since the 3 people who are selected will all be considered equals. For situation B, this is not the case. Each of the 3 people who are selected will have a different position with a different amount of power and different roles.



Practice Problem:

Complete “Practising” question 5 on page 126 of your textbook.

Solution:

5. a) $_{200}P_5 = \frac{200!}{195!}$

$_{200}P_5 = \frac{200 \cdot 199 \cdot 198 \cdot 197 \cdot 196 \cdot 195!}{195!}$

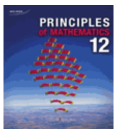
$_{200}P_5 = 200 \cdot 199 \cdot 198 \cdot 197 \cdot 196$

$_{200}P_5 = 304\ 278\ 004\ 800$

There are 304 278 004 800 ways that the top five cash prizes can be awarded if each ticket is not replaced when drawn.

b) $(200)^5 = 320\ 000\ 000\ 000$

There are 320 000 000 000 ways that the top five cash prizes can be awarded if each ticket is replaced when drawn.



Practice Problem:

Complete "Practising" question 6 on page 126 of your textbook.

Solution:

6. ${}_2C_1 \cdot {}_6C_2 \cdot {}_4C_2$

$$= \left(\frac{2!}{1!(2-1)!}\right) \cdot \left(\frac{6!}{2!(6-2)!}\right) \cdot \left(\frac{4!}{2!(4-2)!}\right)$$

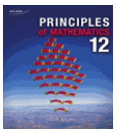
$$= 2 \cdot \left(\frac{6 \cdot 5 \cdot 4!}{2! \cdot 4!}\right) \cdot \left(\frac{4 \cdot 3 \cdot 2!}{2! \cdot 2!}\right)$$

$$= 2 \cdot \left(\frac{6 \cdot 5}{2}\right) \cdot \left(\frac{4 \cdot 3}{2}\right)$$

$$= 2 \cdot 15 \cdot 6$$

$$= 180$$

There are 180 ways that the 5 starting positions on the basketball team can be filled.



Practice Problem:

Complete “Practising” question 7 on page 126 of your textbook.

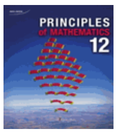
Solution:

7. $$\frac{10!}{2!\cdot 2!\cdot 2!\cdot 2!\cdot 2!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 2\cdot 2\cdot 2\cdot 2}$$

$$\frac{10!}{2!\cdot 2!\cdot 2!\cdot 2!\cdot 2!} = 10\cdot 9\cdot 7\cdot 6\cdot 5\cdot 3\cdot 2\cdot 1$$

$$\frac{10!}{2!\cdot 2!\cdot 2!\cdot 2!\cdot 2!} = 113400$$

There are 113 400 ways that the five different pairs of identical teddy bears can be arranged.



Practice Problem:

Complete "Practising" question 9 on page 127 of your textbook.

Solution:

9. e.g., First make a table to show the number of ways the two cabin cruisers can be arranged next to each other.

	CC 1	**CC 2**
Arrangement 1	1	2
Arrangement 2	2	3
Arrangement 3	3	4
Arrangement 4	4	5
Arrangement 5	5	6
Arrangement 6	2	1
Arrangement 7	3	2
Arrangement 8	4	3
Arrangement 9	5	4
Arrangement 10	6	5

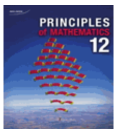
For each of these arrangements, the number of ways the six boats can dock is the number of ways that the other four boats can dock. Let D represent the total number of ways that the boats can dock:

$D = 10 \cdot 4!$

$D = 10 \cdot 24$

$D = 240$

There are 240 ways that the six boats can dock.



Practice Problem: **(KEY QUESTION)**

Complete “Practising” question 10 on page 127 of your textbook.

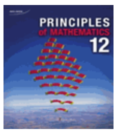
Solution:

10. e.g., Each row of seats is different, and within a row, the seats are assumed to be different. Therefore, there are 10 different people being seated in 10 different spots. Let A represent the number of seating arrangements:

$A = 10!$

$A = 3\ 628\ 800$

There are 3 628 800 ways that the 10 players can sit in the van.



Practice Problem:

Complete "Practising" question 11 on page 127 of your textbook.

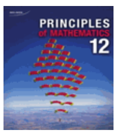
Solution:

11. a) $\frac{5!}{2!} = 60$

There are 60 different arrangements that are possible for the letters if there are no conditions.

b) 3! = 6

There are 6 different arrangements that are possible for the letters if each arrangement must start and end with an N.

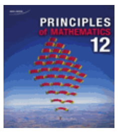


Practice Problem:

Complete “Practising” question 13 on page 127 of your textbook.

Solution:

13. $\frac{11!}{5!\cdot 6!} = 462$ You can take 462 different routes.



Practice Problem:

Complete “Practising” question 14 on page 127 of your textbook.

Solution:

14. e.g., Let’s assign people to the 5-person car first. Let J represent the number of ways to assign the people to this car:

$J = {}_{16}C_5$

$J = \frac{16!}{5!11!}$

$J = 4368$

Now there are 16 – 5 or 11 people left to assign to the remaining two vehicles. Let’s assign people to the 4-person car next. Let K represent the number of ways to assign the people to this car:

$K = {}_{11}C_4$

$K = \frac{11!}{4!7!}$

$K = 330$

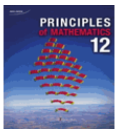
Now there are 11 – 4 or 7 people left to assign to the remaining vehicle. There is only 1 way to assign these people to the 7-passenger van because all of them are going to be assigned to it. Now let T represent the total number of assignments:

$T = J \cdot K \cdot 1$

$T = 4368 \cdot 330$

$T = 1\ 441\ 440$

There are 1 441 440 ways the 16 people can be assigned to the 3 vehicles.



Practice Problem:

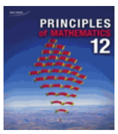
Complete “Practising” question 16 on page 127 of your textbook.

Solution:

16. Case 1: 0 hearts and 5 non-hearts: ${}_{13}C_0 \cdot {}_{39}C_5$
Case 2: 1 heart and 4 non-hearts: ${}_{13}C_1 \cdot {}_{39}C_4$
Case 3: 2 hearts and 3 non-hearts: ${}_{13}C_2 \cdot {}_{39}C_3$
Case 4: 3 hearts and 2 non-hearts: ${}_{13}C_3 \cdot {}_{39}C_2$
Let H represent the number of hands with at most 3 hearts:

$$H = {}_{13}C_0 \cdot {}_{39}C_5 + {}_{13}C_1 \cdot {}_{39}C_4 + {}_{13}C_2 \cdot {}_{39}C_3 + {}_{13}C_3 \cdot {}_{39}C_2$$
$$H = 1 \cdot 575\,757 + 13 \cdot 82\,251 + 78 \cdot 9139 + 286 \cdot 741$$
$$H = 2\,569\,788$$

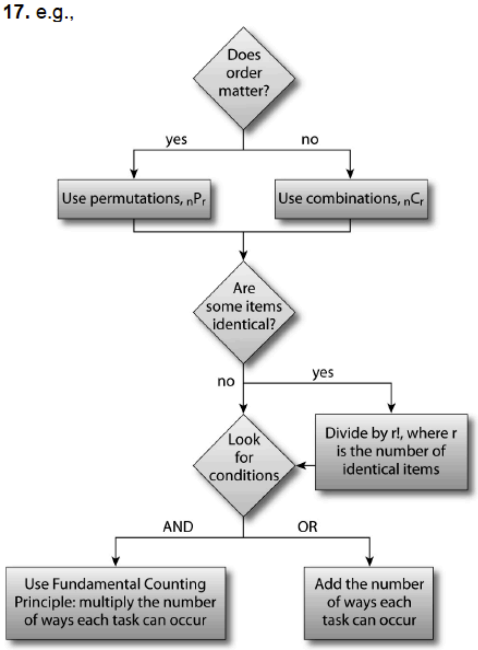
There are 2 569 788 different five-card hands that contain at most three hearts that can be dealt.



Practice Problem:

Complete "Practising" question 17 on page 127 of your textbook.

Solution: 17. e.g.,



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