Math 30-2: U2L1 Teacher Notes

Counting Principles

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- ★ The Fundamental Counting Principle applies when tasks are related by the word **AND**.
- ★ The Fundamental Counting Principle states that if one task can be performed in *a* ways and another task can be performed in *b* ways, then both tasks can be performed in $a \bullet b$ ways.

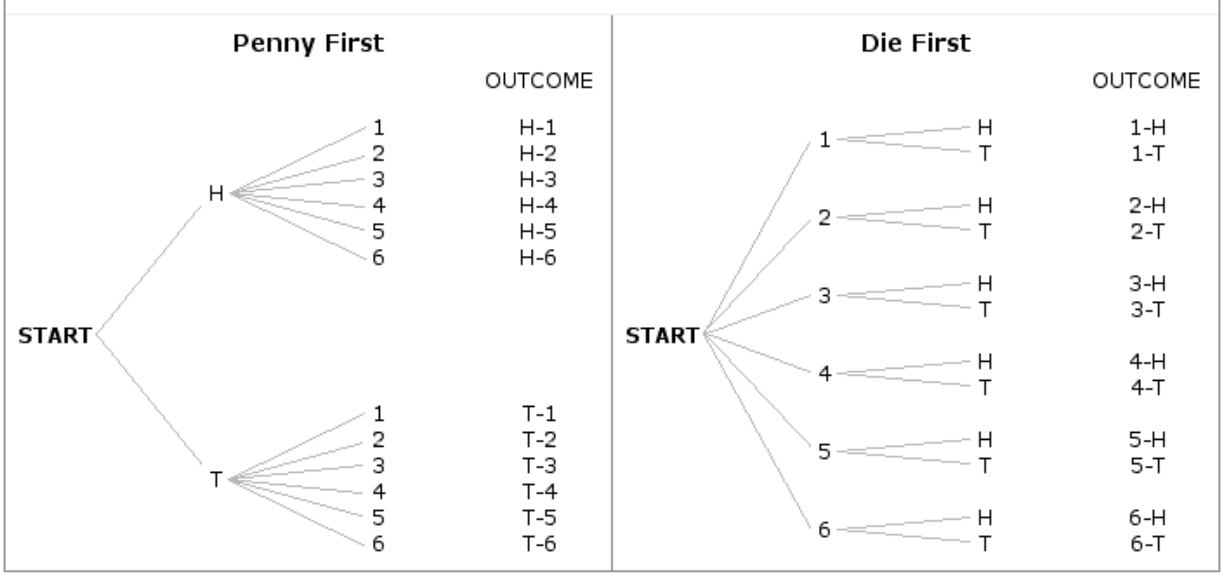
What is a Sample Space and Tree Diagrams?

A **tree diagram** is diagram using branches from locations (like a tree) to show possibilities and relationships including:

all possible outcomes for an experiment (**sample space**)

all possible arrangements of a set of elements

For example, given the experiment of tossing a penny and rolling a six sided die, draw a tree diagram.



TREE DIAGRAMS OF OUTCOMES FOR TOSSING A PENNY AND A DIE

Click the icon to watch a Youtube video on tree diagrams.

What is the Fundamental Counting Principle?

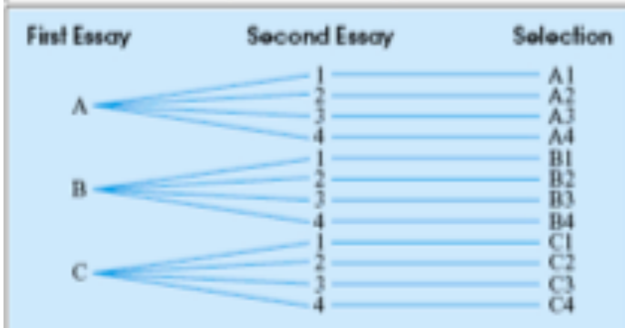
Creating a sample space or a tree diagram for every type of question can be bothersome and time consuming. An easier way to solve problems when tasks are related by the word **AND** is to use the fundamental counting principle.

The **fundamental counting principle** explains that if there are *r* ways to complete one task and *s* ways to complete another task, then there are $r \cdot s$ ways to complete the two tasks. The tasks are related by the word **AND**.

Click the icon to watch a Youtube video on Fundamental Counting Principle

For example, on an English test, a student must write two essays. For the first essay, the student must select from topics A, B, and C. For the second essay, the student must select from topics 1, 2, 3, and 4. How many different ways can the student select the two essay topics?

Method 1: Using a Tree Diagram



The student can select the two essays 12 ways.

Method 2: Using the Fundamental Counting Principle

The task of choosing essay topics has two stages: choosing an essay from topics A, B, and C and choosing an essay from topics 1, 2, 3, and 4.

The total number of ways of selecting the two essays is the product of the choices available at each of the two stages.

$$\therefore \text{Number of ways} = 3 \times 4$$
$$= 12$$

The student can select the two essays 12 ways.

The **fundamental counting principle** can also be extended to more than two tasks: if one task can be performed in *r* ways, another task can be performed in *s* ways, and another task in t ways, and so on, then all these tasks can be performed in *r* • *s* • t • , ... ways.

Let's look at an actual example and try to make sense of this rule. How about a license plate ...

How many different license plates are there altogether? Look at what's used to make a plate:

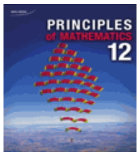
LETTER LETTER LETTER NUMBER NUMBER NUMBER

For each of the letters we have 26 choices. For each of the numbers we have 10 choices.

The number of ways to pick the first letter	The number of ways to pick the second letter	The number of ways to pick the third letter	The number of ways to pick the first number	The number of ways to pick the second number	The number of ways to pick the third number
26	26	26	10	10	10

The Fundamental Counting Principle says that:
The total number of ways to fill the six spaces on a licence plate is
26 *x* ***26*** *x* ***26*** *x* ***10*** *x* ***10*** *x* ***10***
which equals 17,576,000

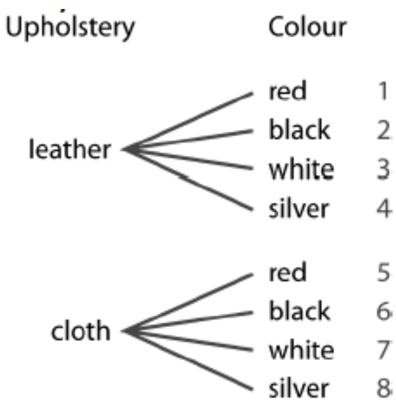
Click the icon to watch a Youtube video on Fundamental Counting Principle on more than two events.



Complete “Check your Understanding” question 2 on page 73 of your textbook.

Solution:

a)



Therefore, there are 8 upholstery-colour choices that are available.

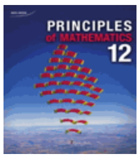
b) The number of upholstery-colour choices, U, is related to the number of colours and the number of kinds of upholstery:

U = (number of colours) · (number of upholstery)

$U = 4 \cdot 2$

$U = 8$

There are 8 upholstery-colour choices that are available. This matches the part a) result.



Complete "Check your Understanding" question 3 on page 73 of your textbook.

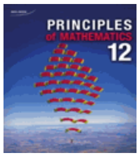
Solution:

3. a) The Fundamental Counting Principle does not apply because tasks in this situation are related by the word OR.

b) The Fundamental Counting Principle does apply because tasks in this situation are related by the word AND.

c) The Fundamental Counting Principle does not apply because tasks in this situation are related by the word OR.

d) The Fundamental Counting Principle does apply because tasks in this situation are related by the word AND.



Complete “Check your Understanding” question 5 on page 73 of your textbook.

Solution:

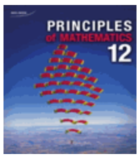
5. The number of colour-size variations, C, is related to the number of colours and the number of sizes:

C = (number of colours) · (number of sizes)

$C = 5 \cdot 4$

$C = 20$

There are 20 colour-size variations that are available.



Complete "Check your Understanding" question 7 on page 74 of your textbook.

Solution:

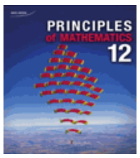
7. The number of possible meals, M, is related to the number of soups (s), the number of sandwiches (sw), the number of drinks (dr), and the number of desserts (d):

$M = (\# \text{ of } s) \cdot (\# \text{ of } sw) \cdot (\# \text{ of } dr) \cdot (\# \text{ of } d)$

$M = 3 \cdot 5 \cdot 4 \cdot 2$

$M = 120$

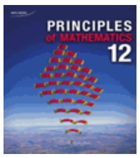
Therefore, there are 120 different meal possibilities.



Complete “Check your Understanding” question 8 on page 74 of your textbook.

Solution:

8. Event *A*: Selecting a rap CD
OR
Event *B*: Selecting a classic rock CD
$n(A \cup B) = n(A) + n(B)$
$n(A \cup B) = 8 + 10$
$n(A \cup B) = 18$
Therefore, Charlene can select from 18 CDs to play in her car stereo that will match Tom’s musical tastes.



Complete “Check your Understanding” question 9 on page 74 of your textbook.

Solution:

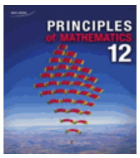
9. a) The number of different PIN combinations, C, is related to the number of digits from which to select for each digit of the PIN, P:

$C = P_1 \cdot P_2 \cdot P_3 \cdot P_4 \cdot P_5$

$C = 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

$C = 59\ 049$

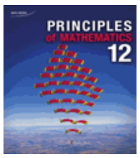
There are 59 049 different five-digit PIN combinations.



Complete “Check your Understanding” question 15 on page 75 of your textbook.

Solution:

15. a) Multiply the number of sizes of the crust, by the number of types of the crust, by the number of types of cheese, by the number of types of tomato sauce.
$2 \cdot 2 \cdot 2 \cdot 2 = 16$
Multiply this number by the number of different toppings.
$16 \cdot 20 = 320$
Therefore, there are 320 different pizzas that can be made with any crust, cheese, tomato sauce, and 1 topping.
b) Multiply the number of types of cheese by the number of types of tomato sauce.
$2 \cdot 2 = 4$
Therefore, there are 4 different pizzas that can be made with a thin whole-wheat crust, tomato sauce, cheese, and no toppings.



Complete "Check your Understanding" question 16a on page 75 of your textbook.

Solution:

16. a) The number of different upper-case letter possibilities, N, is related to the number of upper-case letters from which to choose for each of the first three positions of the Alberta licence plate, P:

$N = P_1 \cdot P_2 \cdot P_3$

$N = 24 \cdot 24 \cdot 24$

$N = 13\ 824$

The number of different digit possibilities, D, is related to the number of digits from which to choose for each of the last three positions of the Alberta licence plate, P:

$D = P_4 \cdot P_5 \cdot P_6$

$D = 10 \cdot 10 \cdot 10$

$D = 1000$

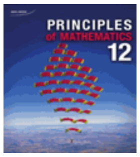
The number of different possible Alberta licence plates, C, is related to the number of upper-case letter possibilities, N, and the number of digit possibilities, D:

$C = N \cdot D$

$C = 13\ 824 \cdot 1000$

$C = 13\ 824\ 000$

So, 13 824 000 Alberta licence plates are possible.



Complete “Check your Understanding” question 16b on page 75 of your textbook.

Solution:

b) The number of different upper-case letter possibilities, N, remains the same since the number of letters in the plates and the number of letters that can be used is the same as in a).

The number of different digit possibilities, D, is related to the number of digits from which to choose for each of the last four positions of the Alberta licence plate, P:

$D = P_4 \cdot P_5 \cdot P_6 \cdot P_7$

$D = 10 \cdot 10 \cdot 10 \cdot 10$

$D = 10\ 000$

The number of different possible Alberta licence plates, C, is related to the number of upper-case letter possibilities, N, and the number of digit possibilities, D:

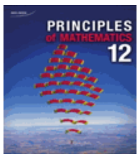
$C = N \cdot D$

$C = 13\ 824 \cdot 10\ 000$

$C = 138\ 240\ 000$

$138\ 240\ 000 - 13\ 824\ 000 = 124\ 416\ 000$

So, 124 416 000 more licence plates are possible.



Complete “Check your Understanding” question 17 on page 75 of your textbook.

Solution:

17. e.g., If multiple tasks are related by AND, it means the Fundamental Counting Principle can be used and the total number of solutions is the product of the solutions to each task. For example: A 4-digit PIN involves choosing the 1st digit AND the 2nd digit AND the 3rd digit AND the 4th digit. So the number of solutions is $10 \cdot 10 \cdot 10 \cdot 10 = 10\,000$. OR means the solution must meet at least one condition so you must add the number of solutions to each condition, and then subtract the number of solutions that meet all conditions. For example: Calculating the number of 4-digit PINs that start with 3 OR end with 3. The solution is the number of PINs that start with 3, plus the number of PINs that end with 3, minus the number of PINs that both start and end with 3:
$1000 + 1000 - 100 = 1900$.