Math 30-2: U5L1 Teacher Notes

Exploring the Graphs of Polynomial Functions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Describe, orally and in written form, the characteristics of polynomial functions by analyzing their graphs.

Polynomial Functions

Polynomial Function
A polynomial function contains only the operations of multiplication and addition with real numbers and variables.
$f(x) = 5x^3 + 6x^2 - 3x + 7$
The function can also be written as
$f(x) + 5(x)(x)(x) + 6(x)(x) + (-3)(x) + 7$

Degree

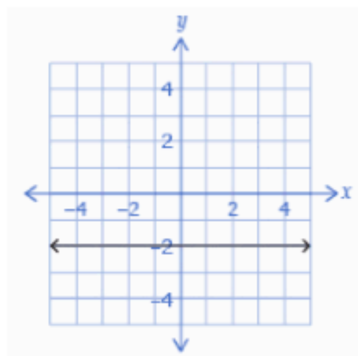
The highest exponent (on a variable) is called the degree of the polynomial. In Math 30-2, you will work with degree 0, 1, 2, and 3 polynomial functions.

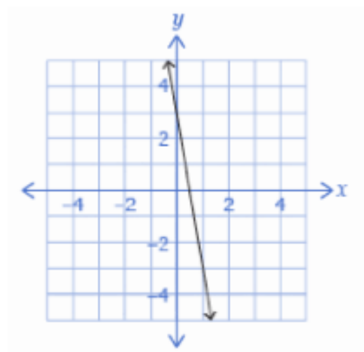
Degree	Name	Example
0	constant	$f(x) = 3.5$
1	linear	$f(x) = 4x - 1$
2	quadratic	$f(x) = 3x^2 + 6x - 1$
3	cubic	$f(x) = 5x^3 - 4x^2 + 7x + 3$

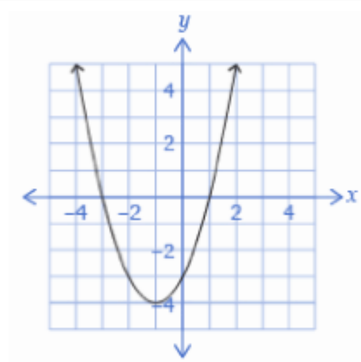
Graphs of Constant, Linear, Quadratic and Cubic Functions

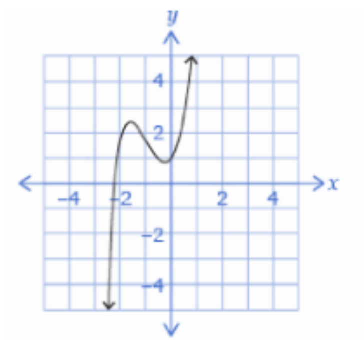
In order to do this lesson we need to review a few concepts of graphs of polynomial functions.

- **x - intercept** is the point or points at which the graph crosses the x-axis.
- **y - intercept** is the point at which the graph crosses the y - axis.
- The **domain** of a function is the set of all possible input values (usually x), which allows the function formula to work.
- The **range** is the set of all possible output values (usually y), which result from using the function formula.

Type of Function	Example Equation	Example Graph	Characteristics of Example Graphs
Constant	$f(x) = -2$		The graphs of constant functions are horizontal lines. There is no x-intercept (does not cross the x-axis) y-intercept = -2 (crosses the y-axis at -2) Domain: x is a member of real numbers Range is y = -2 or the constant term

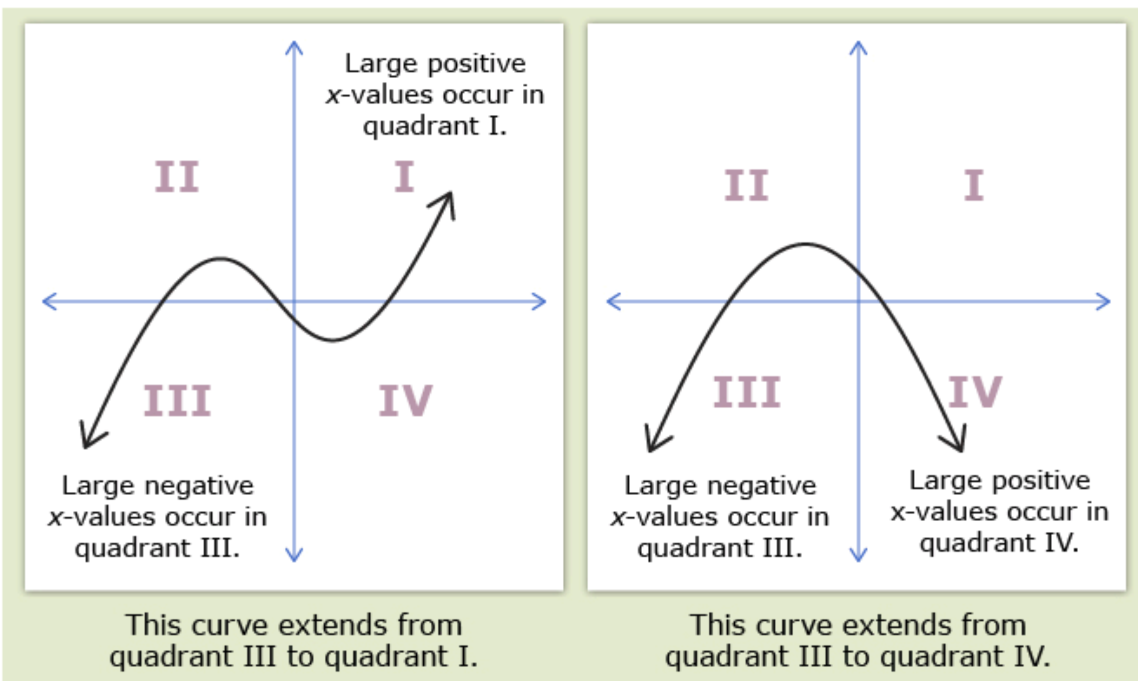
Linear	$f(x) = -5x + 3$		The graphs of linear functions are diagonal lines. x-intercept = 0.5 (crosses the x-axis at 0.5) y-intercept = 3 (crosses the y-axis at 3) Domain: x is a member of real numbers Range: y is a member of real numbers

Quadratic	$f(x) = x^2 + 2x - 3$		The graphs of quadratic functions are parabolas. x-intercepts = -3 and 1 y-intercept = -3 Domain: x is a member of real numbers Range: y is greater than or equal to -4

Cubic	$f(x) = 2x^3 + 5x^2 + 2x + 1$		The graphs of cubic functions are curvy. x-intercept = -2.2 y-intercept = 1 Domain: x is a member of real numbers Range: y is a member of real numbers

End Behavior

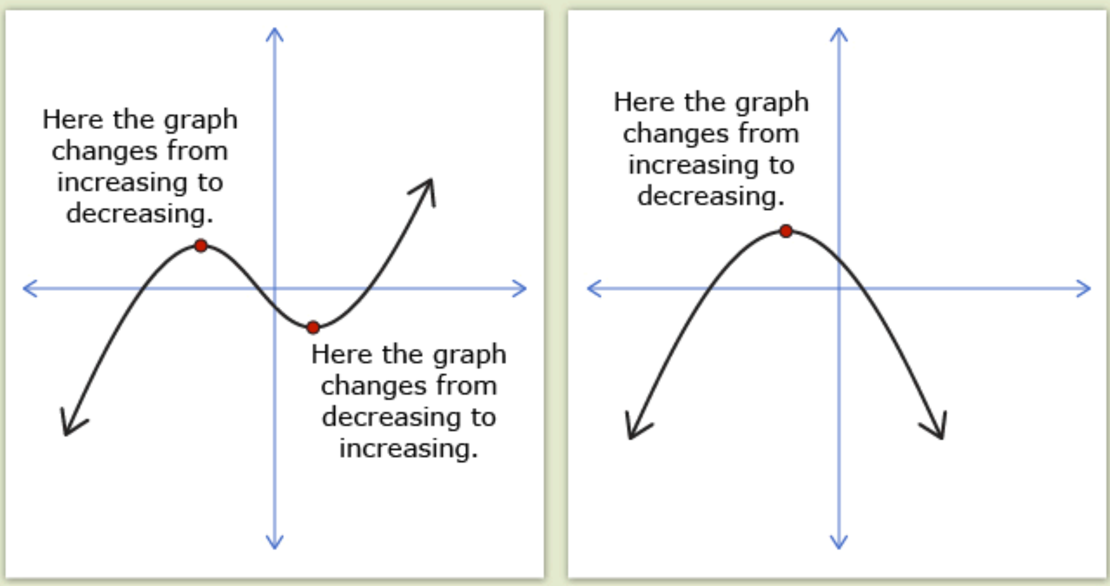
The **end behaviour** of a graph describes what happens as the *x*-values become very large positive or very large negative numbers. In this course, you will typically describe end behaviour by stating the quadrant the graph is in for large negative *x*-values and large positive *x*-values.



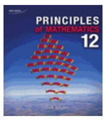
Turning Points

A **turning point** occurs when a graph changes from increasing to decreasing or from decreasing to increasing.

In the definition for turning point, describing a function that is decreasing means the curve is falling from left to right. Describing a function that is increasing means the curve is rising from left to right.



This graph contains two turning points. This graph contains one turning point.

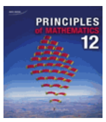


Practice Problem:

Complete “Further your Understanding” question 1 on page 277 of your textbook.

Solution:

1. a) This is not a polynomial function since the graph has infinitely many turning points.
b) This is a polynomial function since the graph extends from quadrant II to quadrant I, it has 1 *y*-intercept, 1 turning point and 2 *x*-intercepts. It is a quadratic function.
c) This is a polynomial function since the graph extends from quadrant III to quadrant I, it has 1 *y*-intercept, 2 turning points and 3 *x*-intercepts. It is a cubic function.
d) This is a polynomial function since the graph extends from quadrant II to quadrant IV, it has 1 *y*-intercept, no turning points and 1 *x*-intercept. It is a linear function.
e) This is not a polynomial function because it has no *x*-intercepts and it is not a constant function.
f) This is not a polynomial function since the domain of this graph is not $\{x \mid x \in R\}$.



Practice Problem:

Complete “Further your Understanding” question 2 b, c and d on page 277 of your textbook.

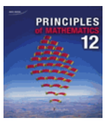
Solution:

2. b) *x*-intercepts: –5, –1
y-intercept: 2
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \geq -2.5, y \in R\}$
Number of turning points: 1

Solution:

c) *x*-intercepts: –2, –1, 1
y-intercept: –5
End behaviour: curve extends from quadrant III to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Number of turning points: 2

d) *x*-intercept: 0.5
y-intercept: 2
End behaviour: line extends from quadrant II to quadrant IV
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Number of turning points: 0

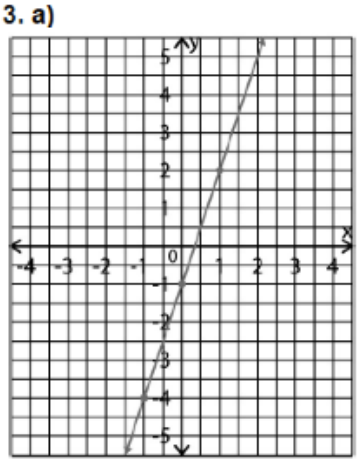


Practice Problem:

Complete “Further your Understanding” question 3 on page 277 of your textbook.

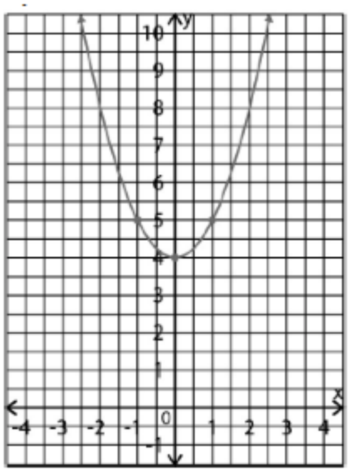
Solution:

3. a)



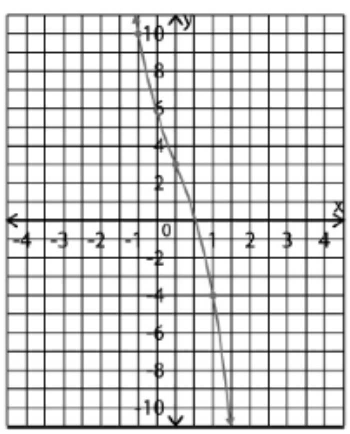
Number of x-intercepts: 1
y-intercept: –1
End behaviour: line extends from quadrant III to quadrant I
Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$
Number of turning points: 0

b)



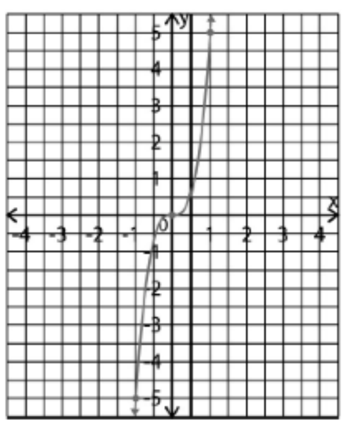
Number of x-intercepts: 0
y-intercept: 4
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \geq 4, y \in R\}$
Number of turning points: 1

c)



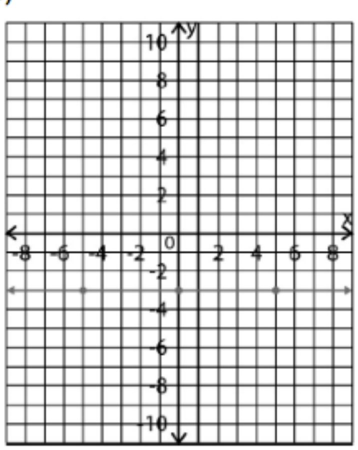
Number of x-intercepts: 1
y-intercept: 3
End behaviour: curve extends from quadrant II to quadrant IV
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Number of turning points: 0

d)



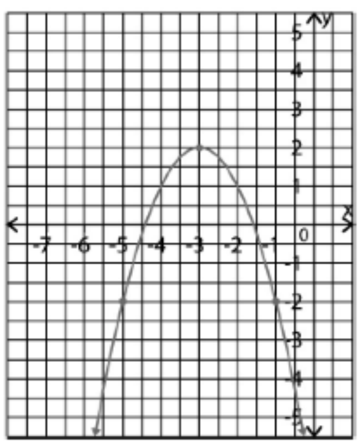
Number of *x*-intercepts: 1
y-intercept: 0
End behaviour: curve extends from quadrant III to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Number of turning points: 0

e)

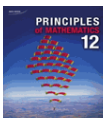


Number of *x*-intercepts: 0
y-intercept: –3
End behaviour: line extends from quadrant III to quadrant IV
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y = -3\}$
Number of turning points: 0

f)



Number of *x*-intercepts: 2
y-intercept: –7
End behaviour: curve extends from quadrant III to quadrant IV
Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \leq 2, y \in \mathrm{R}\}$
Number of turning points: 1

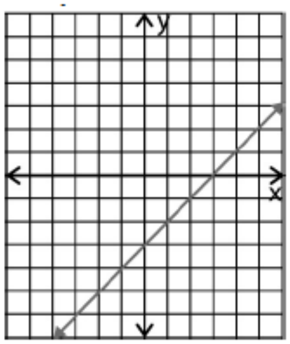


Practice Problem:

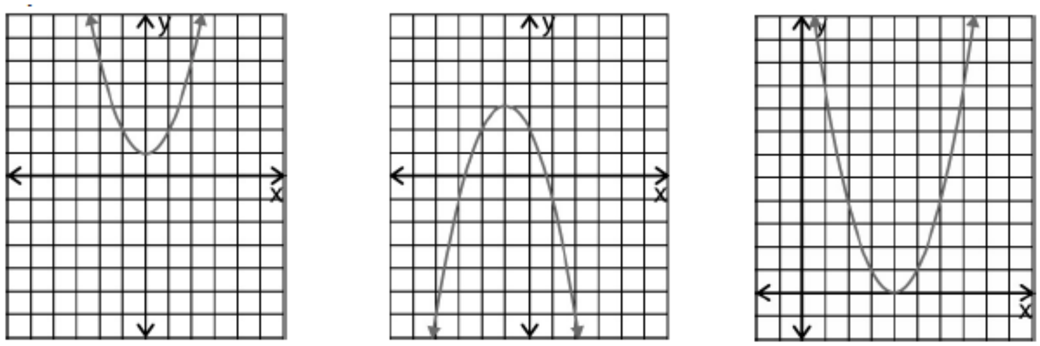
Complete “Further your Understanding” question 4 on page 277 of your textbook.

Solution:

a)



b)



c)