Math 30-2: U5L2 Teacher Notes

Characteristics of the Equations of Polynomial Functions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Describe, orally and in written form, the characteristics of polynomial functions by analyzing their graphs.
- **Match equations in a given set to their corresponding graphs.**

What is Standard Form of a Polynomial?

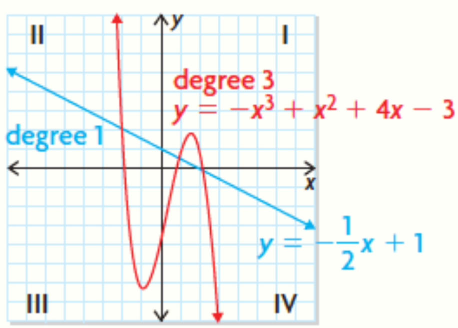
Polynomials can be written in different forms. Standard form is when the polynomial is expanded and simplified as much as possible. In this course you will need to work with linear, quadratic and cubic functions in standard form.

Polynomial	Standard Form	Examples
Linear	$f(x) = ax + b,\ a \neq 0$	$f(x) = 2x - 1$ $f(x) = -3x$
Quadratic	$f(x) = ax^2 + bx + c,\ a \neq 0$	$f(x) = 2x^2 - 3x + 1$ $f(x) = -3x^2 - 2$
Cubic	$f(x) = ax^3 + bx^2 + cx + d,\ a \neq 0$	$f(x) = 2x^3 - 4x^2 + 3x + 1$ $f(x) = -3x^3$

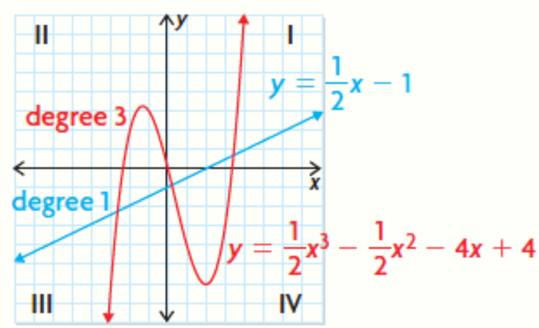
How Does the Leading Coefficent (*a*) Affect the Graph?

Linear and Cubic Functions that have positive or negative leading coefficients have similar end behaviors.

If the leading coefficient is negative, then the graph of the function extends from quadrant II to quadrant IV.

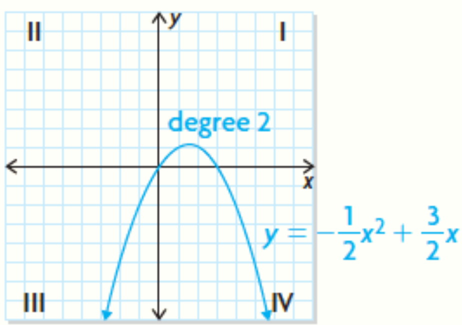


If the leading coefficient is positive, then the graph of the function extends from quadrant III to quadrant I.

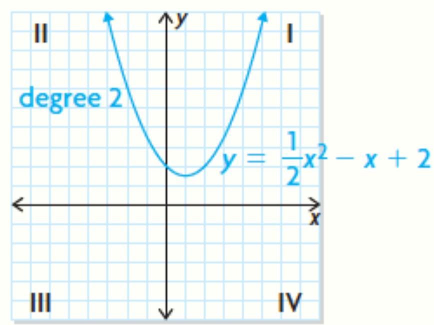


Quadratic polynomial functions have unique end behavior.

If the leading coefficient is negative, then the graph of the function extends from quadrant III to quadrant IV.

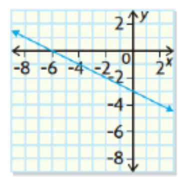
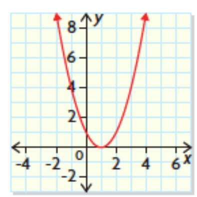
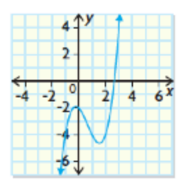


If the leading coefficient is positive, then the graph of the function extends from quadrant II to quadrant I.



How Does the Constant Term Affect the Graph?

The constant term in the equation of the polynomial function is the y-intercept of its graph.

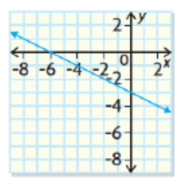
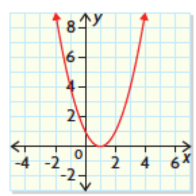
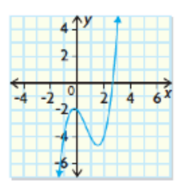
Function	Constant Term	Graph	Y-intercept
$h(x) = -\frac{1}{2}x - 3$	-3		(0,-3)
$k(x) = x^2 - 2x + 1$	+1		(0, 1)
$p(x) = x^3 - 2x^2 - x - 2$	-2		(0, -2)

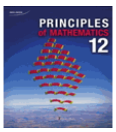
What is the Maximum Number of X-Intercepts and Turning Points for the Graph?

The maximum number of x-intercepts the graph may have is equal to the degree of the function.

Maximum Number of X-Intercepts		
Linear Functions	**Quadratic Functions**	**Cubic Functions**
degree = 1 maximum number of x-intercepts is 1	degree = 2 maximum number of x-intercepts is 2 A quadratic function can have 0, 1, or 2 x-intercepts.	degree = 3 maximum number of x-intecepts is 3 A cubic function can have 1, 2 or 3 x-intercepts.

The maximum number of turning points a graph may have is equal to one less than the degree of the function. For example, a cubic function has a degree of 3, then at most it has 3 - 1 = 2 turning points.

Maximum Number of Turning Points			
Function	**Degree**	**Graph**	**Maximum Number of Turning Points**
$h(x) = -\frac{1}{2}x - 3$	1		0
$k(x) = x^2 - 2x + 1$	2		1
$p(x) = x^3 - 2x^2 - x - 2$	3		2

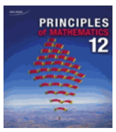


Practice Problem:

Complete “Practising” question 4 on page 287 of your textbook.

Solution:

4. a) e.g., $y = 5$
b) e.g., $y = x + 5$
c) e.g., $y = x^2 + x + 5$
d) e.g., $y = x^3 + x^2 + x + 5$

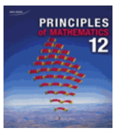


Practice Problem:

Complete “Practising” question 5 on page 288 of your textbook.

Solution:

5. a) The graph extends from quadrant III to quadrant I.
b) The graph extends from quadrant III to quadrant IV.
c) The graph extends from quadrant II to quadrant I.
d) The graph extends from quadrant II to quadrant IV.
e) The graph extends from quadrant II to quadrant IV.
f) The graph extends from quadrant III to quadrant I.



Practice Problem:

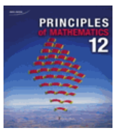
Complete “Practising” question 6 on page 288 of your textbook.

Solution:

6. a) The correct function for this graph is v. Only v and vi are possible choices, because they are the only ones that are linear. The graph extends from quadrant II to quadrant IV, and the slope is –1. This means that the leading coefficient is –1. Therefore, v is the proper choice.
b) The correct function for this graph is i. Only i and iv are possible choices, because they are the only ones that are cubic. The graph extends from quadrant II to quadrant IV, which means that the leading coefficient is negative. Therefore, i is the proper choice.
c) The correct function for this graph is ii. Only ii and iii are possible choices, because they are the only ones that are quadratic. The graph has a *y*-intercept of 4, which means that the constant term is 4. Therefore, ii is the proper choice.

Solution:

d) The correct function for this graph is vi. Only v and vi are possible choices, because they are the only ones that are linear. The graph extends from quadrant II to quadrant IV, and the slope is –2. This means that the leading coefficient is –2. Therefore, vi is the proper choice.
e) The correct function for this graph is iii. Only ii and iii are possible choices, because they are the only ones that are quadratic. The graph has a *y*-intercept of 2, which means that the constant term is 2. Therefore, iii is the proper choice.
f) The correct function for this graph is iv. Only i and iv are possible choices, because they are the only ones that are cubic. The graph extends from quadrant III to quadrant I, which means that the leading coefficient is positive. Therefore, iv is the proper choice.



Practice Problem:

Complete “Practising” question 7 on page 288 of your textbook.

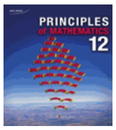
Solution:

7. a) Possible number of *x*-intercepts: 1
y-intercept: 5
End behaviour: graph extends from quadrant II to quadrant IV
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Possible number of turning points: 0

b) Possible number of *x*-intercepts: 0, 1, or 2
y-intercept: –6
End behaviour: graph extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \geq \text{minimum}, y \in R\}$
Possible number of turning points: 1

c) Possible number of *x*-intercepts: 1, 2, or 3
y-intercept: –1
End behaviour: graph extends from quadrant III to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Possible number of turning points: 0 or 2

d) Possible number of *x*-intercepts: 1, 2, or 3
y-intercept: 0
End behaviour: graph extends from quadrant II to quadrant IV
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Possible number of turning points: 0 or 2

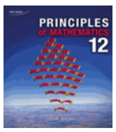


Practice Problem:

Complete “Practising” question 8 on page 288 of your textbook.

Solution:

8. a) e.g., $y = -x^2 + 2$
b) e.g., $y = x^3 - 3x^2 - x + 3$
c) e.g., $y = x - 3$
d) e.g., $y = x^3 - 5x^2 - x + 5$
e) e.g., $y = -x^2 + 2$

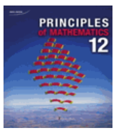


Practice Problem:

Complete “Practising” question 14 on page 290 of your textbook.

Solution:

14. a) The degree of this function is 3, so it is a cubic function. The leading coefficient is positive so the function is increasing from left to right. The curve extends from quadrant III to quadrant I. It has a *y*-intercept of 25.720 and may have 1, 2, or 3 *x*-intercepts. Also, it may have 0 or 2 turning points.
b) In this case, the constant term equals the retail price of gas in 1979.



Practice Problem:

Complete “Practising” question 16 on page 291 of your textbook.

Solution:

16. e.g., I would ask for the degree of the function, its leading coefficient, and the *x*-intercepts. Knowing the degree of the function is essential to describing it, and knowing the leading coefficient as well will allow you to know the end behaviour of the function. With the *x*-intercepts, as well as the other 2 aspects of the function, you can create the formula, which will tell you the *y*-intercept as well. You may also be able to deduce how many turning points a function has from the number of *x*-intercepts.