Math 30-2: U5L4 Teacher Notes

Modelling Data with a Curve of Best Fit

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Graph data and determine the polynomial function that best approximates the data.
- Interpret the graph of a polynomial function that models a situation, and explain the reasoning.
- Solve, using technology, a contextual problem that involves data that is best represent by graphs of polynomial functions, and explain the reasoning.

Quadratic and Cubic Regression

If the points on a scatterplot follow a quadratic or cubic trend, technology can be used to determine and graph the equation of the **curve of best fit**. The **curve of best fit** is drawn so that the points are evenly distributed on either side of the curve. It is considered to be the line or curve (next lesson) that best represents the data plotted.

We follow all the similar steps as in linear regression but if data follows a quadratic trend then we use **quadratic regression** and if it follows a cubic trend then we use a **cubic regression**.

Follow along in the next two Practice Problems to help you solve problems with Quadratic and Cubic Regression.

Practice Problem: Question 3 on page 314

Josie hit a golf ball from the top of a hill. The height of the ball above the green is given in the table below.

Time (s)	1	2	3	4	5
Height (m)	52.5	73.2	74.6	55.8	16.1

a) Describe the characteristics of the data.
b) Determine the equation of the quadratic regression function that models the data.
c) Use your equation to determine the height of the ball at
i) 0 s **ii)** 2.5 s **iii)** 4.5 s
d) When did the ball hit the ground?

Solution:

a) The ball rises, peaks at about 3 s, and then falls.

b) The regression equation is $h(t) = -10.071t^2 + 51.408t + 11$

c)

i) The height of the ball at 0 seconds is 11 m.

$$h(t) = -10.071t^2 + 51.408t + 11$$
$$h(0) = -10.071(0)^2 + 51.408(0) + 11$$
$$h(0) = 11$$

ii) The height of the ball at 2.5 seconds is 76.6 m.

$$h(t) = -10.071t^2 + 51.408t + 11$$
$$h(2.5) = -10.071(2.5)^2 + 51.408(2.5) + 11$$
$$h(2.5) = 76.575$$

iii) The height of the ball at 4.5 seconds is 38.4 m.

$$h(t) = -10.071t^2 + 51.408t + 11$$
$$h(4.5) = -10.071(4.5)^2 + 51.408(4.5) + 11$$
$$h(4.5) = 38.392$$

d) The ball hit the ground when its height was zero. Determine the solutions to the equation.

$$0 = -10.071t^2 + 51.408t + 11$$

Graph each side of the equation as a separate function.

$$y_1 = 0 \text{ and } y_2 = -10.071t^2 + 51.408t + 11 .$$

The t-coordinators of the intersection points are solutions to the equation. **To solve for the point of intersection, press 2nd TRACE 5:Intersect**

$$t = 5.310 \text{ or } t = -0.205$$

Since time can't be negative, the ball hit the ground at about 5.3 seconds.

Practice Problem: Question 4 on page 314

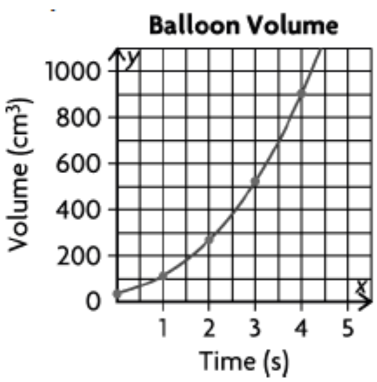
A spherical balloon is being inflated. The volume, V, in cubic centimetres is related to the time, t, in seconds.

Volume, V (cm^3)	33.51	113.10	268.08	523.60	904.78
Time, t (s)	0	1	2	3	4

a) Use technology to plot the data as a scatter plot. Describe the trend you see.
b) Use cubic regression to create a curve of best fit.
c) Determine the volume of the balloon at 10.5 s.

Solution:

a)



As the time increases, the volume is increasing at an increasing rate, but it does not appear to be quadratic.

b) The cubic regression equation is

$$y = 4.189x^3 + 25.130x^2 + 50.267x + 33.510$$

c) Substitute 10.5 in for *x* and solve for *y*.

$$y = 4.189(10.5)^3 + 25.130(10.5)^2 + 50.267(10.5) + 33.510$$
$$y = 8181.469$$

At 10.5 s, the volume of the balloon will be about 8181.5cm^3 .

Practice Problem: Question 7 on page 315

The fertility rate for Canadian women over 40, for several years after 1977, is shown in the table at the right. The rate is the number of babies born, on average, to a group of 1000 women.

a) Use quadratic regression to interpolate two missing data values.

b) Compare your interpolated values to the actual values in the table below.

Years after 1977	1	2	7	8	12	13	18	19	20	24	25
Rate per 1000 Females	3.9	3.5	3.1	3.1	3.8	3.9	5.0	5.3	5.4	6.3	6.4

Solution:

7. a) e.g., The quadratic regression equation is
$y = 0.007\ldots x^2 - 0.065\ldots x + 3.521\ldots.$
Interpolations:
When $x = 1$, $y = 3.5$
When $x = 2$, $y = 3.4$
When $x = 7$, $y = 3.4$
When $x = 8$, $y = 3.5$
When $x = 12$, $y = 3.8$
When $x = 13$, $y = 3.9$
When $x = 18$, $y = 4.8$
When $x = 19$, $y = 5.0$
When $x = 20$, $y = 5.2$
When $x = 24$, $y = 6.3$
When $x = 25$, $y = 6.6$

b) e.g., The values found for when x is equal to 12, 13, and 24 are exactly correct, while the estimate for when x is equal to 2 is very close. The interpolated values found when x is equal to 1, 2, 18,19, and 20 are too small, while the values found when x is equal to 7, 8, and 25 are too large.

Practice Problem: **(KEY QUESTION)** Question 8 on page 315

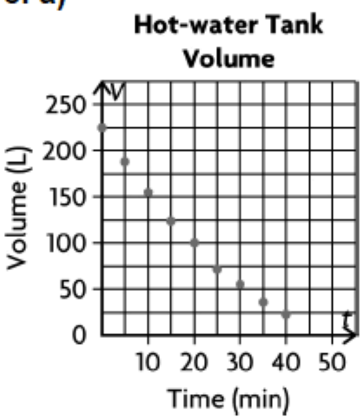
A 225 L hot-water tank sprung a leak at $t = 0$ min. The remaining volume was measured every 5 min for the first 40 min.

Volume, V (L)	225	188	155	124	100	72	55	36	23
Time, t (min)	0	5	10	15	20	25	30	35	40

a) Plot the data as a scatter plot. Describe the trend.
b) Determine the equation of the quadratic regression function that models the data.
c) Determine when the tank was half full.
d) Determine when the tank was empty.

Solution:

8. a)



e.g., As the time increases, the volume of the tank decreases.

b) The quadratic regression equation is
$V = 0.064\ldots t^2 - 7.650\ldots t + 224.885\ldots$

c) The tank is half full when its volume is $\frac{225}{2}$ or 112.5 L. Use technology to determine where $V = 112.5$ intersects with the quadratic regression function. The t-coordinates of the intersection points are solutions to the equation
$0.064\ldots t^2 - 7.650\ldots t + 224.885\ldots = 112.5$

The t-coordinates of the intersection points are 17.2 min and 100.6 min.
$t = 100.6$ is not a possible value in this scenario since the tank is draining and the volume is already lower than half at 20 min.
After 17.2 min, the tank will be half full.

d) The tank is empty when its volume is 0 L. Use technology to determine where $V = 0$ intersects with the quadratic regression function. The t-coordinates of the intersection points are solutions to the equation $0.064\ldots t^2 - 7.650\ldots t + 224.85\ldots = 0$

The t-coordinates of the intersection points are 56.2 and 61.6.
Since the tank is draining, once it is the empty the first time, it will always be empty. Therefore, the tank was empty at about 56.2 min.

Practice Problem: Question 10 on page 316

x	y	x	y
0	1	12	69
5	49	15	85
10	75	18	65
15	80	21	82
20	78	24	80
25	83	27	100
30	100	30	105
35	150	33	110
40	200	36	150
50	400	39	190
3	15	42	220
6	55	48	400
9	70	51	450

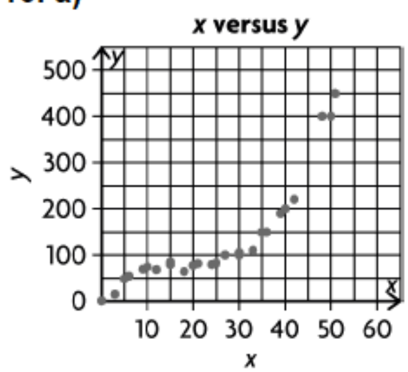
a) Create a scatter plot for the data in the table at the left.

b) Determine the equation of the cubic regression function that models the data.

c) Use your equation to determine the y-value when x is 25. How close is your calculated value to the y-value in the table?

Solution:

10. a)



b) The cubic regression equation is
$y = 0.010\ldots x^3 - 0.557\ldots x^2 + 11.078\ldots x + 1.409\ldots$

c) When $x = 25$:
$y = 0.010\ldots(25)^3 - 0.557\ldots(25)^2 + 11.078\ldots(25) + 1.409\ldots$
$y = 87.688\ldots$

Using the cubic regression equation, when $x = 25$, $y = 87.7$. This value is very close to the y-value in the chart. It is off by approximately 4.7.

November 09, 2012