Math 30-2: U1L3 Teacher Notes
Intersection and Union of Two Sets

Key Math Learnings:

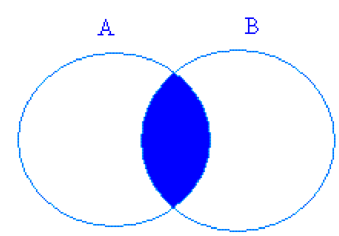
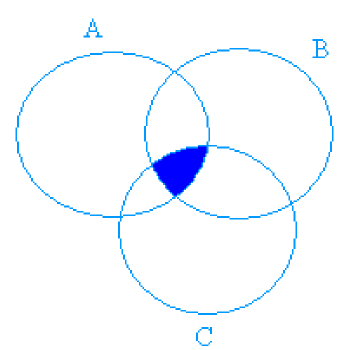
By the end of this lesson, you will learn the following concepts:

- Understand and represent the intersection and union of two sets

Achievement Indicators:

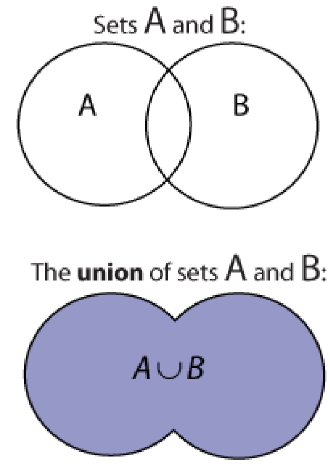
- Explain what a specified region in a Venn diagram represents, using connecting words (and, or) or set notation.
- Determine the elements in the intersection or the union of two sets.
- Identify and correct errors in a solution to a problem that involves sets.
- Solve a contextual problem that involves sets, and record the solution, using set notation.

Intersection of Two or Three Sets

Definition	Notation	Venn Diagram
Given two sets A and B, the **intersection** is the set that contains elements or objects that belong to A and to B at the same time	It is denoted by $A \cup B$ and is read 'A union B' or the 'set of A **and** B	 Shaded region is the intersection of A and B  Shaded region is the intersection of set A, B, and C.

Click here for more notes and examples on Intersection of Sets (Math Goodies)

Union of Two or Three Sets

Definition	Notation	Venn Diagram
The **union** of two sets A and B is the set of elements, which are in A or in B or in both.	It is denoted by $A \cup B$ and is read 'A union B' or the 'set of A **and** B	

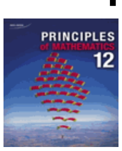
Click here for more notes and examples on Union of Sets (Math Goodies)

Principle of Inclusion and Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

Click here for a video on Principle of Inclusion and Exclusion

Example:



Complete “Practising” question 5 on page 32 of your textbook.

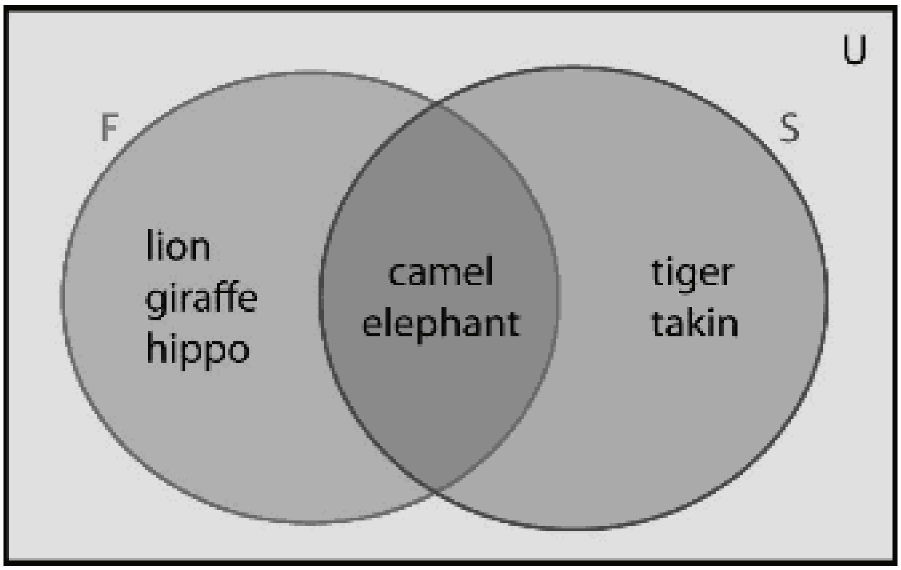
Animals that are native to Africa include the lion, camel, giraffe, hippopotamus, and elephant. Animals that are native to Asia include the elephant, tiger, takin, and camel.

a) Draw a Venn diagram to show these two sets of animals.

b) Determine the union and intersection of these two sets.

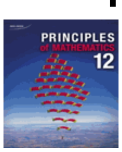
Solution

5. a) Let *U* represent the universal set. Let *F* represent the set of African animals. Let *S* represent the set of Asian animals



b) F = {lion, camel, giraffe, hippo, elephant}
S = {elephant, tiger, takin, camel}
$F \cup S$ = {lion, giraffe, hippo, camel, elephant, tiger, takin}
$F \cap S$ = {camel, elephant}

Example:



Complete "Practising" question 6 on page 33 of your textbook.

Consider the following two sets:

- $A = \{j \mid j = 2x, -3 \leq x \leq 6, x \in \mathrm{I}\}$
- $B = \{k \mid k = 3x, -4 \leq x \leq 5, x \in \mathrm{I}\}$

a) Draw a Venn diagram to show these two sets.

b) Determine $A \cup B$, $n(A \cup B)$, $A \cap B$, and $n(A \cap B)$.

Solution

a. To draw the Venn diagram, you may find it helpful to do the following:

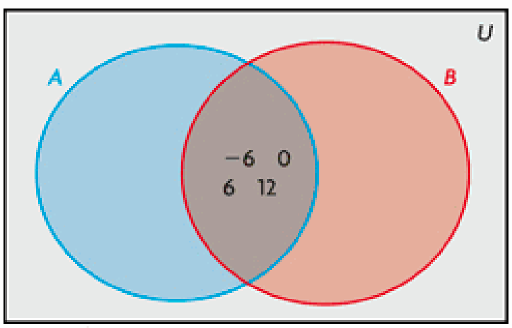
- Start by writing a list of all the elements in *A* and a list of elements in *B*.

  $A = \{-6, -4, -2, 0, 2, 4, 6, 8, 10, 12\}$
  $B = \{-12, -9, -6, -3, 0, 3, 6, 9, 12, 15\}$

- Identify what needs to be in the intersection (i.e., look for the elements common to both sets).

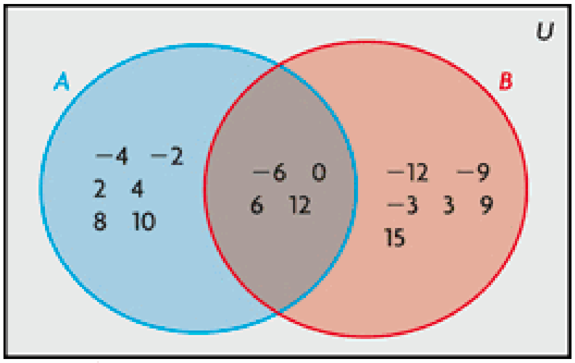
  $A = \{\mathbf{-6}, -4, -2, \mathbf{0}, 2, 4, \mathbf{6}, 8, 10, \mathbf{12}\}$
  $B = \{-12, -9, \mathbf{-6}, -3, \mathbf{0}, 3, \mathbf{6}, 9, \mathbf{12}, 15\}$

- Draw a rectangle to represent the universal set of all numbers in this example and two overlapping circles to represent *A* and *B*. (They overlap because there are elements in the intersection.)



Adapted from *PRINCIPLES OF MATHEMATICS 12* by Canavan-McGrath et al.

- Now put the rest of the elements for *A* and *B* in the non overlapping part of each circle.



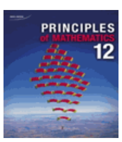
b. $A \cup B$ (the union of *A* and *B*) is all elements in either set.

$\{-12, -9, -6, -4, -3, -2, 0, 2, 3, 4, 6, 8, 9, 10, 12, 15\}$
$n(A \cup B) = 16$

$A \cap B$ (the intersection of *A* and *B*) is all elements in both *A* and *B*.

$\{-6, 0, 6, 12\}$
$n(A \cap B) = 4$

Example:



Complete “Practising” question 7 on page 33 of your textbook.

Rosie asked 25 people at a mystery convention if they liked Sherlock Holmes or Hercule Poirot.

- 4 people did not like either detective.
- 16 people liked Sherlock Holmes.
- 11 people liked Hercule Poirot.

Determine how many people liked both detectives, how many liked only Sherlock Holmes, and how many liked only Hercule Poirot.

Solution

7. Let U represent the universal set. Let H represent the set of people who liked Sherlock Holmes. Let P represent the set of people who liked Hercule Poirot.

$n(H \cup P) = n(U) - n((H \cup P)')$

$n(H \cup P) = 25 - 4$

$n(H \cup P) = 21$

$n(H \cap P) = n(H) + n(P) - n(H \cup P)$

$n(H \cap P) = 16 + 11 - 21$

$n(H \cap P) = 6$

6 people like both detectives.

$n(H \text{ only}) = n(H) - n(H \cup P)$

$n(H \text{ only}) = 16 - 6$

$n(H \text{ only}) = 10$

10 people liked Sherlock Holmes only.

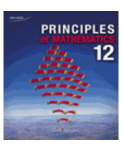
$n(P \text{ only}) = n(P) - n(H \cup P)$

$n(P \text{ only}) = 11 - 6$

$n(P \text{ only}) = 5$

5 people liked Hercule Poirot only.

Example:



Complete "Practising" question 9 on page 33 of your textbook.

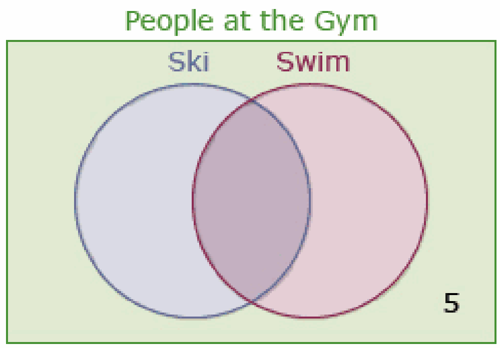
John asked 26 people at a gym if they liked to ski or swim.

- 5 people did not like to do either sport.
- 19 people liked to ski.
- 14 people liked to swim.

Determine how many people liked to ski and swim.

Solution

Create a Venn diagram with a universal set, and then place two intersecting sets inside it. Place the 5 people that do not like to do either sport inside the universal set but not in either circle.



Since 26 people were asked about these sports and 5 liked neither, you have $26 - 5 = 21$ people that like to either swim or ski. Using the principle of inclusion or exclusion, write the equation:

$n(\text{ski} \cup \text{swim}) = n(\text{ski}) + n(\text{swim}) - n(\text{ski} \cap \text{swim})$

Presently you know $n(\text{ski} \cup \text{swim}) = 21$.

$n(\text{ski}) = 19$ (given)
$n(\text{swim}) = 14$ (given)

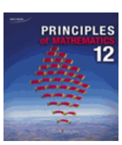
You are looking for people who like to ski and swim, $n(\text{ski} \cap \text{swim})$. Put the numbers into the equation and get $21 = 19 + 14 - n(\text{ski} \cap \text{swim})$. Add $n(\text{ski} \cap \text{swim})$ to both sides and subtract 21 from both sides.

$21 - 21 + n(\text{ski} \cap \text{swim}) = 19 + 14 - n(\text{ski} \cap \text{swim}) + n(\text{ski} \cap \text{swim}) - 21$

Simplify.

$n(\text{ski} \cap \text{swim}) = 19 + 14 - 21$
$n(\text{ski} \cap \text{swim}) = 12$

Example:



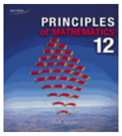
Complete “Practising” question 10 on page 33 of your textbook.

Tiffany volunteers in an elementary classroom. She is helping students understand multiples of 2 and 3 in mathematics. The students are working with the numbers 1 to 30. How can Tiffany use a Venn diagram to show the students how the multiples relate to one another?

Solution

e.g., She could draw a Venn diagram showing the set of multiples of 2 and the set of multiples of 3. The intersection of the sets would be the multiples of 6.

Example: KEY QUESTION



Complete “Practising” question 11 on page 33 of your textbook.

Mark surveyed 100 people at a local doughnut shop.

- 65 people ordered coffee.
- 45 people ordered a doughnut.
- 10 people ordered something else.

Mark wants to determine how many people ordered coffee and a doughnut.

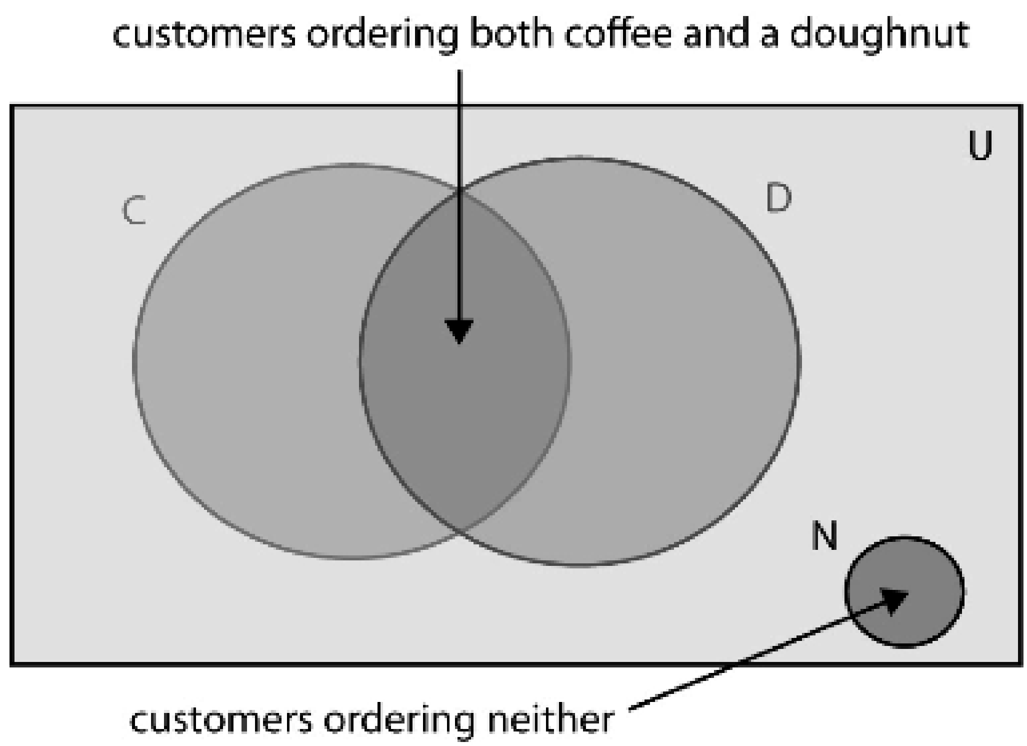
a) Model this situation with sets. Identify the universal set, and explain what subsets you will use.

b) Draw a Venn diagram to model this situation. Explain what each part of your diagram represents.

c) Determine how many people ordered coffee and a doughnut.

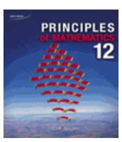
Solution

11. a) U = {all customers surveyed}
C = {customers ordering coffee}
D = {customers ordering donuts}
N = {customers ordering neither coffee nor doughnuts}
b) For the following Venn diagram:
The rectangular area labelled U represents the universal set.
The shaded area labelled D represents the set of people who ordered doughnuts.
The shaded area labelled C represents the set of people who ordered coffee.
The shaded area labelled $D \cap C$ represents the set of people who ordered coffee and doughnuts.
The unshaded area labelled N represents those people did not order coffee or doughnuts.



c) Determine $n(D \cap C)$ using the information available.
$n(U) = 100$, $n(D) = 45$, $n(C) = 65$, $n((D \cup C)') = 10$
$n(D \cup C) = n(U) - n((D \cup C)')$
$n(D \cup C) = 100 - 10$
$n(D \cup C) = 90$
Therefore,
$n(D \cap C) = n(D) + n(C) - n(D \cup C)$
$n(D \cap C) = 45 + 65 - 90$
$n(D \cap C) = 20$
There were 20 people who ordered coffee and a doughnut.

Example:



Complete "Practising" question 12 on page 34 of your textbook.

At a retirement home, 100 seniors were interviewed.
- 16 seniors like to watch television and listen to the radio.
- 67 seniors like to watch television.

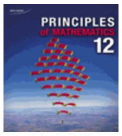
Determine how many seniors prefer to listen to the radio only.

Solution
Let *U* represent the universal set. Let *T* represent the set of seniors who watch television.
Let *R* represent the set of seniors who listen to the radio.

$n(R \text{ only}) = n(U) - n(T)$
$n(R \text{ only}) = 100 - 67$
$n(R \text{ only}) = 33$

33 seniors prefer to listen to the radio only.

Example:



Complete “Practising” question 16 on page 34 of your textbook.

Beyondé solved the following problem:

A total of 48 students were asked how they got to school.

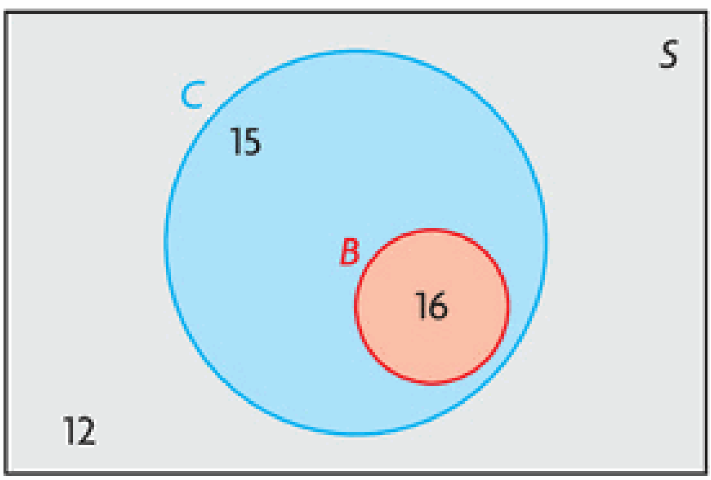
- 31 students drive a car.
- 16 students take a bus.
- 12 students do not drive a car or take a bus.
- Some students drive a car or take a bus.

Determine how many students do not take a bus to school.

Beyondé’s Solution

The total of the three numbers is 59. So, I knew that the region for students who take a bus overlaps the region for students who drive a car.

I drew a Venn diagram with 31 students in the car region and 16 students in the bus region.



15 students drive a car but do not take a bus, 12 students do neither. So, 27 students do not take a bus.

Is Beyondé correct? Justify your answer.

Solution

No. e.g., The three numbers do not add up to 48. There is an overlap between sets B and C, but $B \not\subset C$. The sum of the three values in the problem is 59.

$59 - 48 = 11$

11 students must drive a car and take a bus.

$31 - 11 = 20$

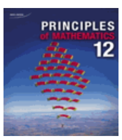
20 students drive a car but do not take a bus.

$16 - 11 = 5$

5 students take a bus but do not drive a car.

There are a total of 15 + 12 = 27 students who do not take a bus.

Example:



Complete "Practising" question 17 on page 35 of your textbook.

Given:
$n(A) + n(B) = n(A \cup B)$ and
$n(A) + n(C) > n(A \cup C)$

a) Which sets do you know are disjoint?

b) Which sets do you know intersect?

c) Are there any sets that could either be disjoint or intersect? If so, which sets? Explain.

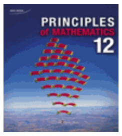
Solution

a) Sets A and B are disjoint sets.

b) Sets A and C intersect.

c) Yes; B and C; e.g., C intersecting A and A and B being disjoint says nothing about the intersection, if any, of B and C.

Example:



Complete “Closing” question 18 on page 35 of your textbook.

Which is more like the addition of two numbers: the union of two sets or the intersection of two sets? Explain.

Solution

e.g., The union of two sets is more like the addition of two numbers because all the elements of each set are counted together, instead of those present in both sets.