Math 30-2: U1L4 Teacher Notes
Application of Set Theory

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Use sets to model and solve problems

Achievement Indicators

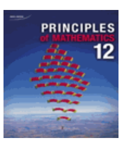
- Explain how set theory is used in applications such as Internet searches, database queries, data analysis, games and puzzles.
- Identify and correct errors in a solution to a problem that involves sets.
- Solve a contextual problem that involves sets, and record the solution, using set notation.

The best way to practice application type of questions by example.

Click here for more examples on Practice with Sets (Math Goodies)

Click here for more examples on Challenge Problems with Sets (Math Goodies)

Example:



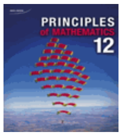
Complete "Practising" question 3 on page 51 of your textbook.

Someone left a backpack full of school books on a transit bus. The only identification is the name "David Smith," so the bus driver takes the backpack to the public school board office. The staff search their database and learn that 56 students have this name. How can the staff narrow their search using search tools and other items in the backpack?

Solution

e.g., Staff could look at how any David Smiths were on that bus route or they could look at the books in the bag and see how many David Smiths are taking courses that use those books.

Example:



Complete “Practising” question 4 on page 51 of your textbook.

Jennifer is an optician. She is trying to decide whether she should offer a package deal to customers who buy glasses and contact lenses. She hires a survey company to research consumer preferences. A survey of 641 people provides the following information:

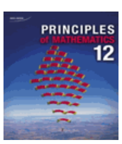
- 83 wear contact lenses.
- 442 wear glasses.
- 167 do not use corrective lenses.

What percent of Jennifer’s customers might use a package deal? Use set notation in your answer.

Solution

4. P = {population surveyed}
$n(P) = 641$
L = {people wearing corrective lenses}
L' = {people not wearing corrective lenses}
$n(L') = 167$
G = {people wearing glasses}
C = {people wearing contact lenses}
$n(L) = n(P) - n(L')$
$n(L) = 641 - 167$
$n(L) = 474$
$n(G \cup C) = n(L)$
$n(G \cup C) = n(G) + n(C) - n(G \cap C)$
$474 = 442 + 83 - n(G \cap C)$
$51 = n(G \cap C)$
51 people might make use of a package deal. This is
$\frac{51}{574}$ = 10.759...% or about 10.8% of all potential
customers.

Example:



Complete “Practising” question 6 on page 52 of your textbook.

A total of 58 teens attended a sports camp to train in at least one of three sports: swimming, cycling, and running.

- 35 trained in swimming, 32 trained in cycling, and 38 trained in running.
- 9 trained in swimming and cycling, but not in running.
- 11 trained in cycling and running, but not in swimming.
- 13 trained in swimming and running, but not in cycling.

A triathlon consists of swimming, cycling, and running. How many teens might be training for the upcoming triathlon?

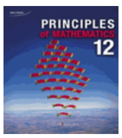
Solution

6. Using the principle of inclusion and exclusion for three sets:

$$\begin{aligned} 32 + 35 + 38 - (9 + x) - (11 + x) - (13 + x) + x &= 58 \\ 105 - 9 - x - 11 - x - 13 - x + x &= 58 \\ 72 - 2x &= 58 \\ -2x &= 58 - 72 \\ -2x &= -14 \\ x &= 7 \end{aligned}$$

7 teens are training for the upcoming triathlon.

Example:



Complete “Practising” question 7 on page 52 of your textbook.

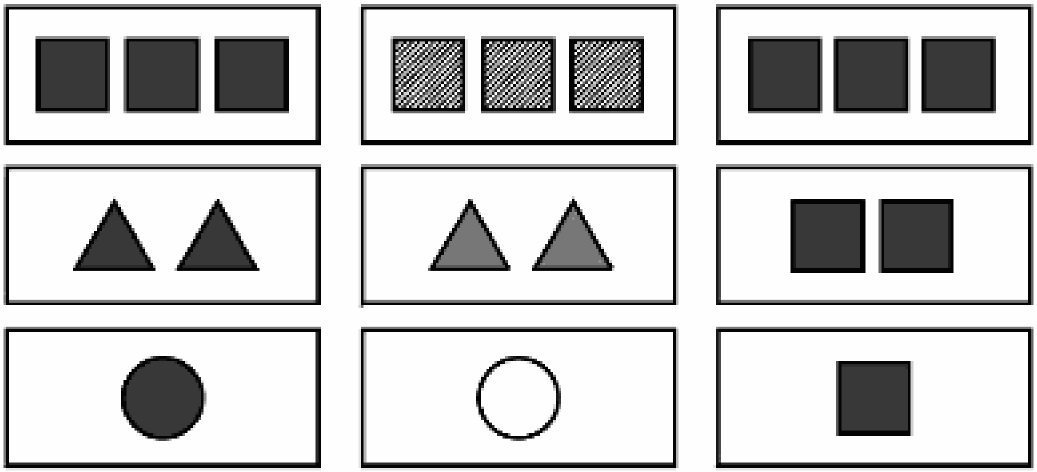
These nine attribute cards have three different shapes, numbers, and shadings (clear, striped, or solid).
Determine three sets, with three cards in each set. Each set of three cards must have

- the same number or three different numbers, and
- the same shape or three different shapes, and
- the same shading or three different shadings.

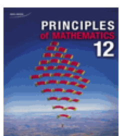
All the cards can be used more than once.

Solution

7. • same numbers, same shapes, different shadings
• same numbers, different shape, different shading
• same numbers, different shape, different shading



Example:



Complete “Practising” question 9 on page 52 of your textbook.

John was asked to solve the following problem:

240 students were surveyed to determine which restaurants they like.

- 90 like Chicken and More.
- 90 like Fast Pizza.
- 90 like Gigantic Burger.
- 37 like Chicken and More and Fast Pizza, but not Gigantic Burger.
- 19 like Chicken and More and Gigantic Burger, but not Fast Pizza.
- 11 like Fast Pizza and Gigantic Burger, but not Chicken and More.
- 13 like all three restaurants.

How many students do not like any of these restaurants?

John solved the problem as follows:

John's Solution:

I added up the first six results of the survey and subtracted the number of students who ate at all three restaurants. Then I subtracted this value from the total number of students surveyed.

$$90 + 90 + 90 + 37 + 19 + 11 - 13 = 324$$
$$240 - 324 = -84$$

This answer is not possible, so I knew that I made an error.

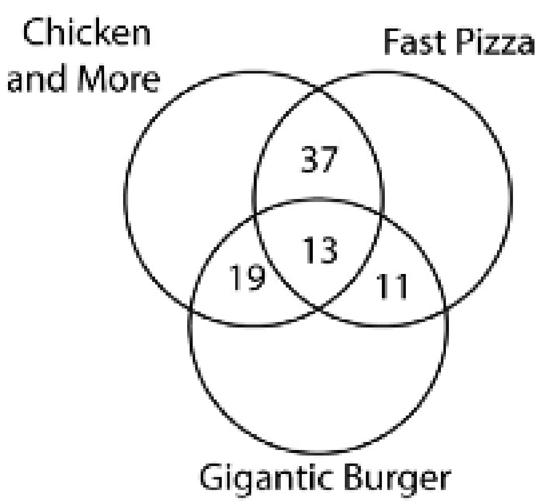
What error did John make? What is the correct answer?

Solution

9. e.g., John assumed that 90 people ate at only one restaurant for each of the 3 restaurants. He did not calculate the correct number of people eating at only one of each of the 3 restaurants.
I defined these sets.

C = {students who like only Chicken and More
F = {students who like only Fast Pizza}
G = {students who like only Gigantic Burger}

I listed the values I knew and entered them in a Venn diagram.
$n(C \cap P \setminus B) = 37$; $n(C \cap B \setminus P) = 19$;
$n(P \cap B \setminus C) = 11$
$n(C \cap B \cap P) = 13$

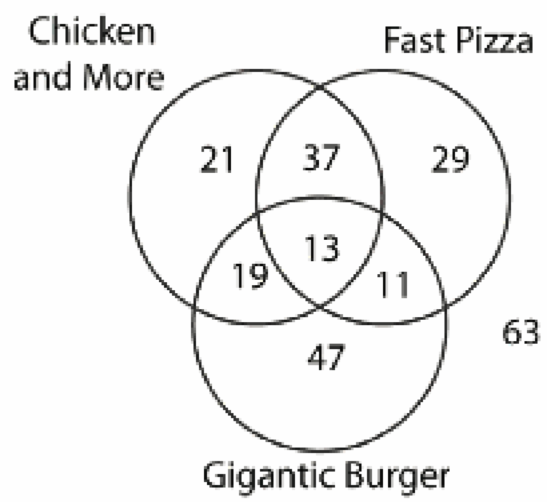


I used these figures and diagram to determine the unknown values.

$$\begin{aligned} n(C \setminus B \setminus P) &= 90 - n(C \cap P \setminus B) - n(C \cap B \setminus P) \\ &\quad - n(C \cap B \cap P) \\ &= 90 - 37 - 19 - 13 \\ &= 21 \end{aligned}$$

$$\begin{aligned} n(B \setminus P \setminus C) &= 90 - n(C \cap B \setminus P) - n(P \cap B \setminus C) \\ &\quad - n(C \cap B \cap P) \\ &= 90 - 19 - 11 - 13 \\ &= 47 \end{aligned}$$

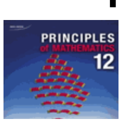
$$\begin{aligned} n(P \setminus B \setminus C) &= 90 - n(P \cap B \setminus C) - n(C \cap P \setminus B) \\ &\quad - n(C \cap B \cap P) \\ &= 90 - 11 - 37 - 13 \\ &= 29 \end{aligned}$$



$n(R) = 21 + 29 + 47 + 37 + 19 + 11 + 13$
$n(R) = 177$

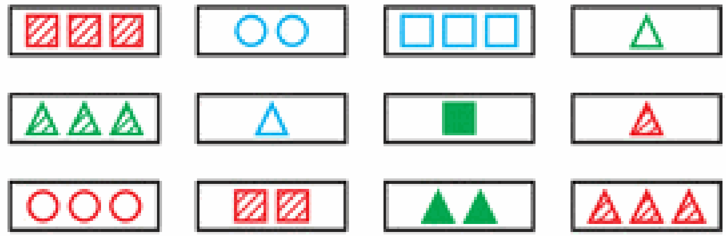
177 students like at least one of these restaurants.
$240 - 177 = 63$
So, 63 students do not like any of the restaurants.

Example:



Complete “Practising” question 11 on page 53 of your textbook.

These 12 cards have three different colours, shapes, numbers, and shadings.



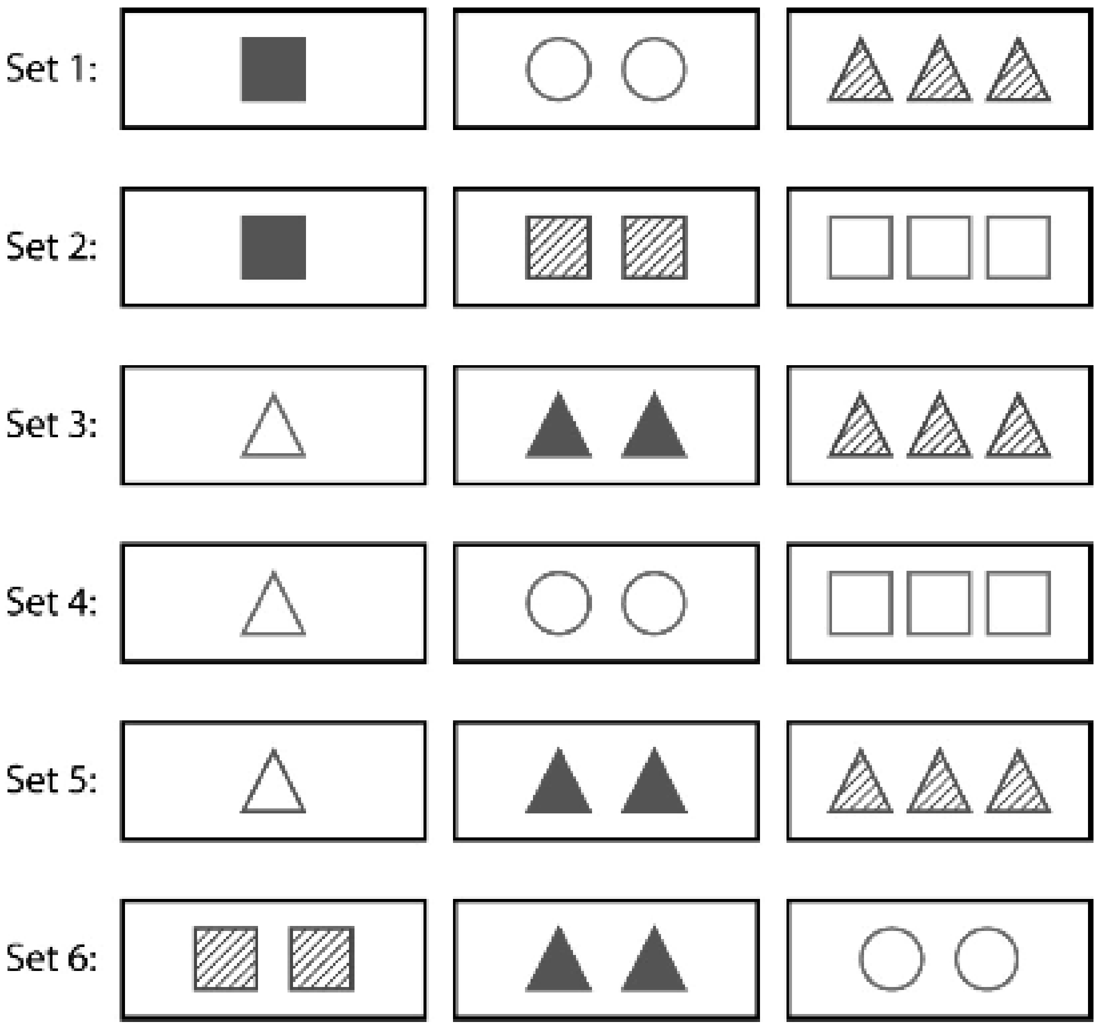
Determine six sets of cards, with three cards in each set. Each set of three cards must have

- the same number or three different numbers, and
- the same shape or three different shapes, and
- the same colour or three different colours, and
- the same shading or three different shadings.

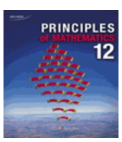
All the cards can be used more than once.

Solution

Set 1: different numbers, different colours, different shading, different shape
Set 2: different numbers, different colours, different shading, same shape
Set 3: different numbers, different colours, different shading, same shape Set 4: different numbers, same colour, same shading, different shape
Set 5: different numbers, same colour, different shading, same shape Set 6: same number, different colours, different shading, different shape



Example: **(KEY QUESTION)**



Complete "Practising" question 12 on page 53 of your textbook.

The cards in question 11 are part of a complete deck of cards. Determine the following amounts:

a) $n(D)$, the total number of cards in the deck

b) $n(T)$, the total number of triangle cards in the deck

c) $n(G)$, the total number of green cards in the deck

d) $n(S)$, the total number of cards with shading

e) $n(T \cup G)$

f) $n(G \cap S)$

Solution

12. a) $n(D)$, the total number of cards in the deck: there are 3 shapes, 3 colours, 3 numbers, and 3 shadings, so in total there are $3 \cdot 3 \cdot 3 \cdot 3$ or 81 cards.

b) $n(T)$, the total number of triangle cards in the deck: there are 3 colours, 3 numbers, and 3 shadings, so in total there are $3 \cdot 3 \cdot 3$ or 27 triangle cards.

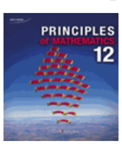
c) $n(G)$, the total number of green cards in the deck: 3 shapes, 3 numbers, and 3 shadings, so in total, there are $3 \cdot 3 \cdot 3$ or 27 green cards.

d) $n(S)$, the total number of cards with shading: there are 27 cards with striped shading and 27 cards with solid shading, so there are $27 + 27$ or 54 cards with shading.

e) $n(T \cup G)$: there are 27 triangle cards and 27 green cards, but 9 triangle cards are also green, so there are $54 - 9$ or 45 cards that have triangles or are green.

f) $n(G \cap S)$: there are 27 green cards. Since 2/3 of the cards have either striped shading or solid shading, 18 cards are both green and have shading.

Example:



Complete “Practising” question 13 on page 53 of your textbook.

A small web-hosting service specializes in websites involving outdoor activities.

- 35 sites involve boats: 20 of these sites deal with fishing boats and 25 deal with power boats.
- 21 sites involve fishing: these sites include all the sites that deal with fishing boats; 3 sites deal with fly fishing.

a) How many sites from this service would appear in a search for *fishing boats*? Explain.

b) Why might a search for *fishing* and *boats* turn up a different result than a search for "*fishing boats*"?

c) If the only search word was *fishing*, how many results would not involve boats?

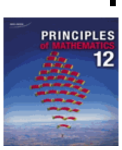
Solution

a) 36 sites would appear in a search for fishing boats. There are 35 sites that involve boats, 20 of which deal with fishing boats. 21 sites involve fishing, but these sites include the 20 sites that deal with fishing boats.

b) e.g., Because *fishing* and *boats* will turn up sites that deal with boats and fishing, but not just fishing boats.

c) 20 of the 21 fishing sites deal with fishing boats, so 1 site would not have boats.

Example:



Complete "Closing" question 14 on page 54 of your textbook.

James searched for "*string bean*" on the Internet with quotation marks. Elinor searched for *string bean* without quotation marks. Did they get the same results? Explain, using set theory and a Venn diagram.

Solution

e.g., No, they did not get the same results. Elinor got all of Jamesí results, plus others dealing with either string or bean, but not both.