Math 30-2: U1L1 Teacher Notes
Types of Sets and Set Notation

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

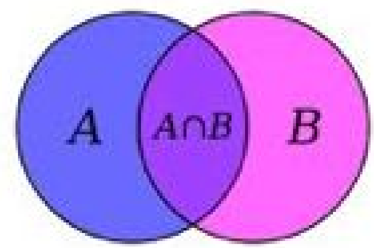
- Understand sets and set notation

Achievement Indicators:

- Provide examples of the empty set, disjoint sets, subsets and universal sets in context, and explain the reasoning.
- Organize information such as collected data and number properties, using graphic organizers, and explain the reasoning.

What is Set Theory?

Set theory is the study of groups of unique objects and the relationships between the objects within the group.

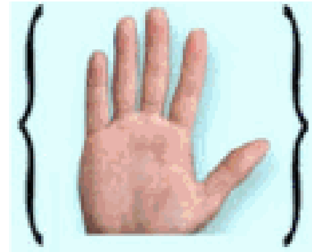


What is a Set?

A **set** is a group or collection of objects or numbers, considered as an entity unto itself.

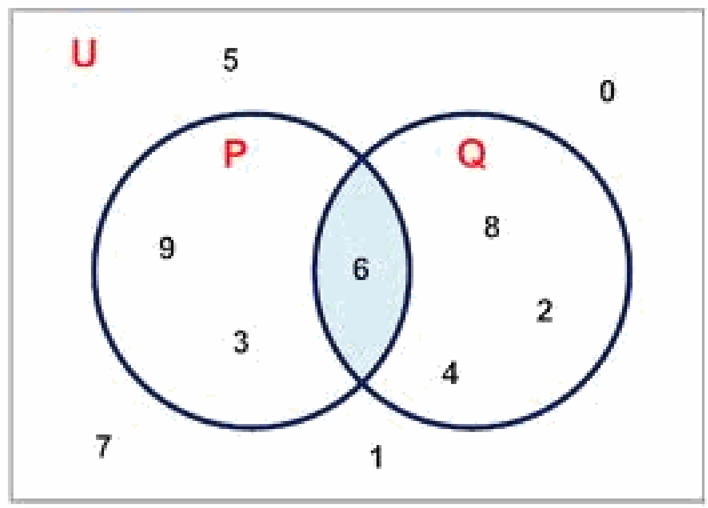
Every object in a set is unique: The same object cannot be included in the set more than once.

For Example: What is the set of all fingers?



Solution:
P = {thumb, index, middle, ring, little}

Elements of a Set



Each object or number in a set is called a **member or element** of the set. Lowercase letters denote the elements in a set.

For Example: the elements in Set ***P*** are 9, 3 and 6

When you are calculating the number of elements in a set we use the notation *n*(*A*): means the number of elements in set *A*

For Example: n(*Q*) = 4 since there are 4 elements in set *Q*.

The Universal Set

The **Universal Set:** A set of all the elements in the group. The universal set as the symbol *U*.

For Example: all the elements in the rectangle are considered the universal set.

Representing a Set

You can represent a set of elements by:

1. **Listing the elements:** A list of the elements in a set, separated by commas and surrounded by {French curly braces}.

2. **Using words or a sentence**

3. **Using set notation:** Mathematical shorthand for precisely stating allnumbers of a specific set that possesses a specific property.

For Example:

Listing the Elements	Words	Set Notation
{2, 3, 4, 5, 6}	the natural numbers between and including 2 and 6	$\{x \mid 2 \leq x \leq 6, x \in N\}$

Subsets

Subset: A set whose elements all belong to another set. The symbol for subset is $\subset$. The symbol for not a subset is $\not\subset$.

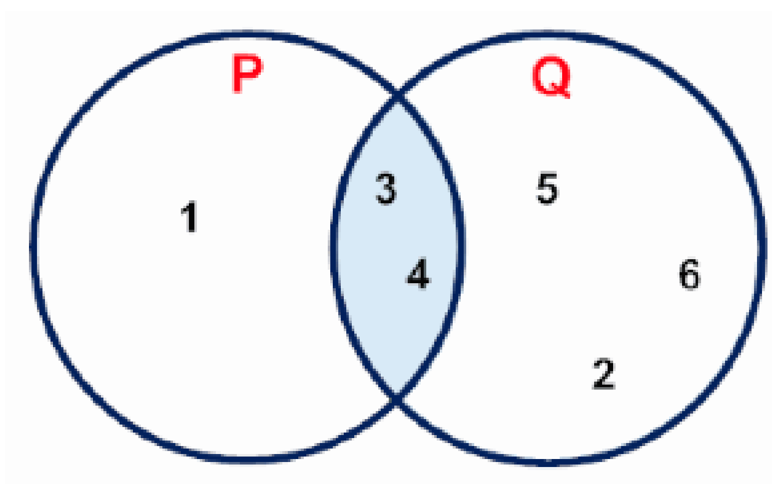
For Example: Given $A = \{1, 3, 5\}$ and $B = \{1, 2, 3, 5, 6\}$. Describe the relationship between sets A and B.

Solution:
Insert a venn diagram here
In order to see the relationship we can draw a picture (we will learn more about Venn Diagrams later) to help us with the relationship.

Since Set A is completely inside Set B we can say that Set A is a subset of Set B and can write it using the symbols $A \subset B$.

For Example: Given P = {1, 3, 4} and Q = { 2, 3, 4, 5, and 6}, what is the relationship between these sets?

Solution:
We say that P is not a subset of Q since not every element of P is not contained in Q. For example, we can that that 1 is not a member of set Q.



Therefore, the statement "P is not a subset Q" is denoted by:

$$P \not\subset Q$$

Note that these sets do have some elements in common. The intersection of these sets is shown in the Venn diagram to the right.

Complements

Complement: All the elements of a universal set that do not belong to a subset of it. The symbol for complement is A'. The Set Builder notation of complement is $A' = \{ x \mid x \in U \text{ and } x \notin A \}$

There are several ways to represent the complement of a set. You may see A^c **or** $\bar{A}$
All of these notations have the same meaning.

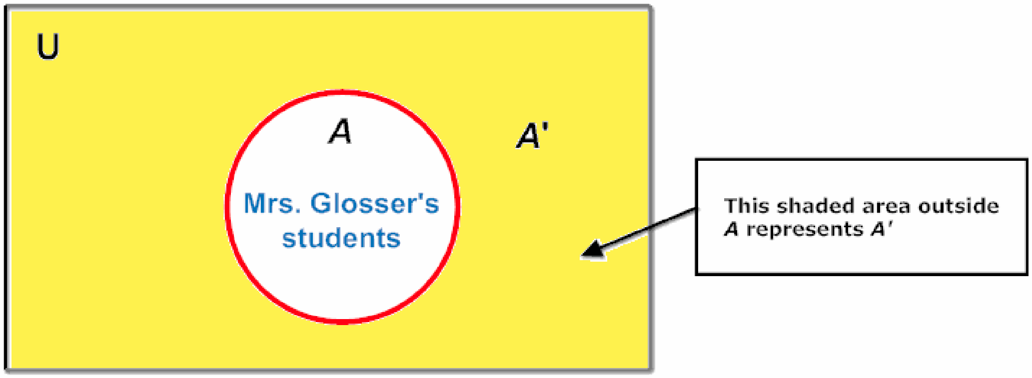
However, for the purpose of this instructional unit, we have chosen to use A', and it is read as A prime.

$A \cup A' = U$ **The union of a set and its complement is the universal set.**

$A \cap A' = \emptyset$ **The intersection between a set and its complement is the null set or empty set.**

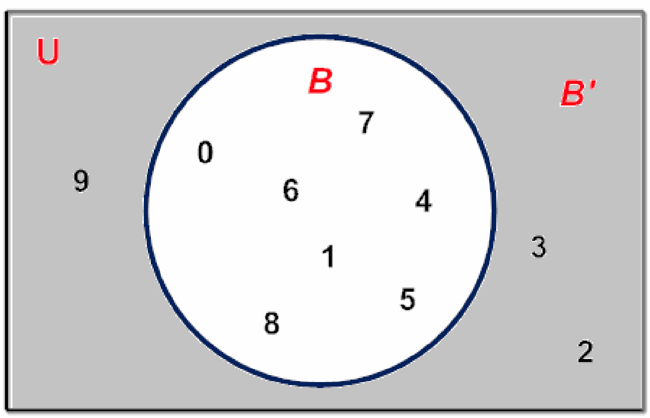
Example 1: Given U = {students who attend The Kewl School} *and* A = {students in Mrs. Glosser's class}. What is the set of all students who attend The Kewl School that are not in Mrs. Glosser's class?

Analysis: The relationship between these sets is illustrated in the Venn diagram below.



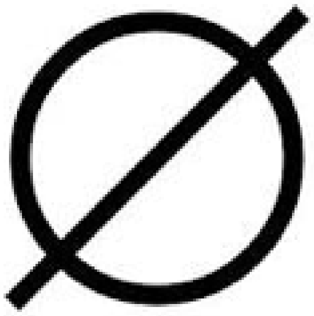
Answer: The shaded area outside *A* represents *A'*, which is all students who attend The Kewl School that are not in Mrs. Glosser's class.

Example 2: Given U = {single digits} and B = {0, 1, 4, 5, 6, 7, 8}, find the complement of *B*.



Answer: B' = {2, 3, 9}

Empty Set

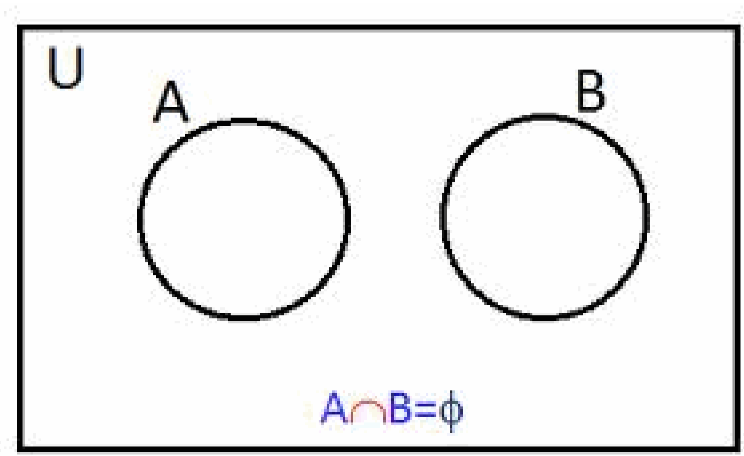


Empty Set: A set with no elements. The symbol for empty set is $\emptyset$

Disjoint Sets

Disjoint: Two or more sets having no elements in common.

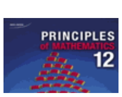
For Example: the sets A = {a,b,c} and B = {d,e,f} are disjoint.



Mutually exclusive sets: Two or more events that cannot occur at the same time. Two sets are that disjoint are said to be Mutually exclusive sets.

Infinite Set: A set with an infinite number of elements

Practice Problem:

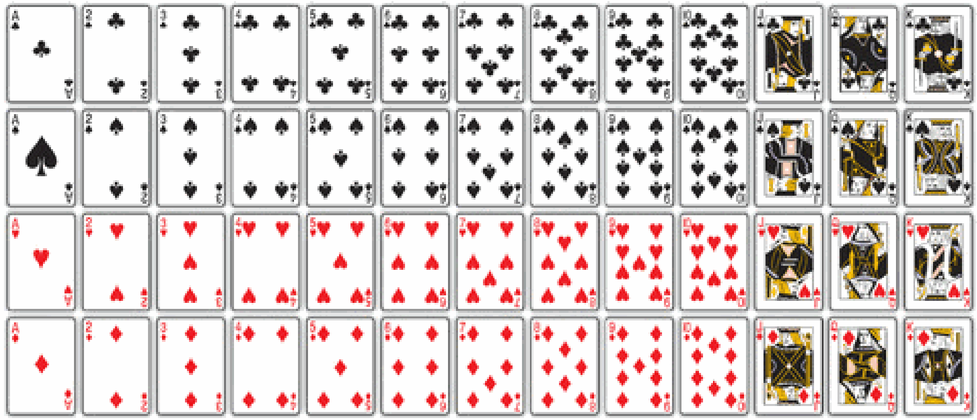


Complete “Practising” question 4 on page 15 of your textbook.

For this question, the universal set, U, is a standard deck of 52 cards as shown.

a) Represent the following sets and subsets using a Venn diagram:

- B = {black cards}
- R = {red cards}
- S = {spades ♠}
- H = {hearts ♥}
- C = {clubs ♣}
- D = {diamonds ♦}



b) List all defined sets that are subsets of B.

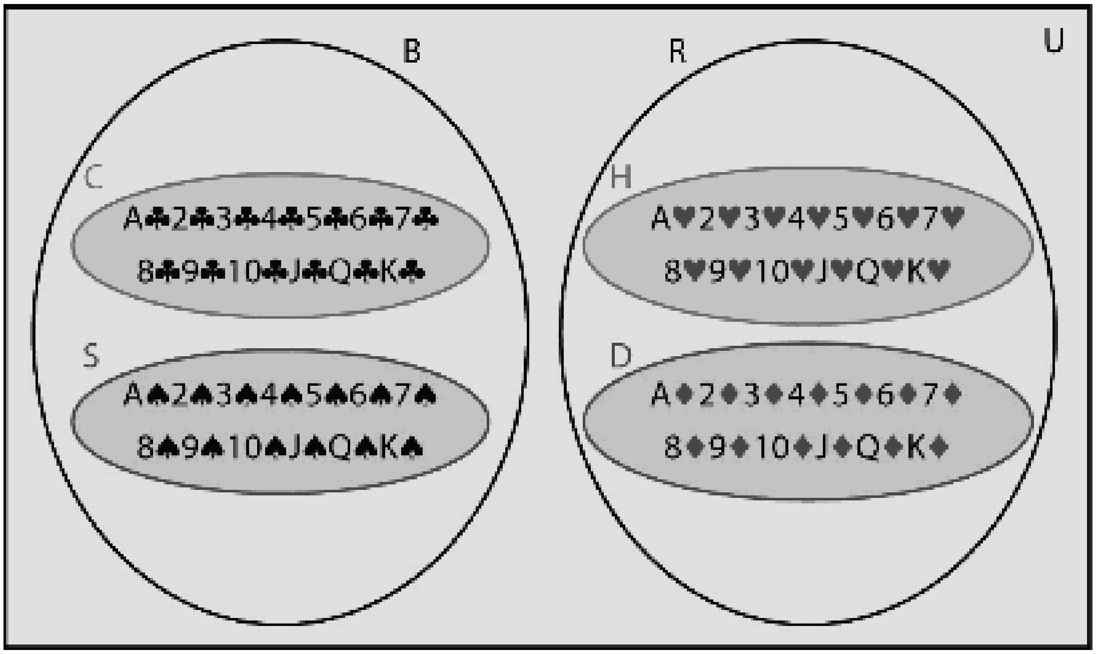
c) List all defined sets that are subsets of R.

d) Are sets S and C disjoint? Explain.

e) Suppose you draw one card from the deck. Are the events drawing a heart and drawing a diamond mutually exclusive? Explain.

f) Is the following statement correct?
$n(S \text{ or } D) = n(S) + n(D)$
Provide your reasoning. Determine the value of $n(S \text{ or } D)$.

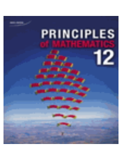
Solution:



b) Subsets of set B: $C \subset B$ and $S \subset B$
c) Subsets of set R: $H \subset R$ and $D \subset R$
d) Yes, the sets S and C are disjoint. e.g., A card cannot be both a spade and a club.
e) Yes, the events in sets H and D are mutually exclusive. e.g., You cannot draw a card that is a heart and a diamond at the same time.
f) Yes, that statement is correct. e.g., Because these sets are disjoint, they contain no common elements. Therefore, when the numbers of elements in each set are added, no element will be counted twice.

$n(S \text{ or } D) = n(S) + n(D)$
$n(S \text{ or } D) = 13 + 13$
$n(S \text{ or } D) = 26$

Practice Problem:



Complete “Practising” question 6 on page 16 of your textbook.

Consider the following information:

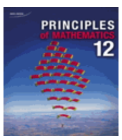
- the universal set $U =$ {natural numbers from 1 to 100 000}
- $X \subset U$
- $n(X) = 12$

Determine $n(X')$, if possible. If it is not possible, explain why.

Solution:

$n(X') = n(U) - n(X)$
$n(X') = 100\,000 - 12$
$n(X') = 99\,988$

Practice Problem:



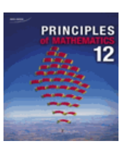
Complete "Practising" question 8 on page 16 of your textbook.

Determine $n(U)$, the universal set, given $n(X) = 34$ and $n(X') = 42$.

Solution:

$n(U) = n(X) + n(X')$
$n(U) = 34 + 42$
$n(U) = 76$

Practice Problem:



Complete "Practising" question 9 on page 16 of your textbook.

Consider this universal set:

A = {A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z}

a) List the following subsets:

- S = {letters drawn with straight lines only}
- C = {letters drawn with curves only}

b) Is this statement true or false?

$C = S'$

Provide your reasoning.

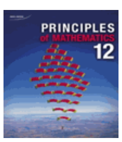
Solution:

a) S = {A, E, F, H, I, K, L, M, N, T, V, W, X, Y, Z}

C = {C, O, S}

b) False. e.g., B is not in S or C.

Practice Problem:



Complete “Practising” question 11 on page 17 of your textbook.

a) Organize the following sets of numbers in a Venn diagram:

- $U = \{\text{integers from } -10 \text{ to } 10\}$
- $A = \{\text{positive integers from 1 to 10 inclusive}\}$
- $B = \{\text{negative integers from } -10 \text{ to } -1 \text{ inclusive}\}$

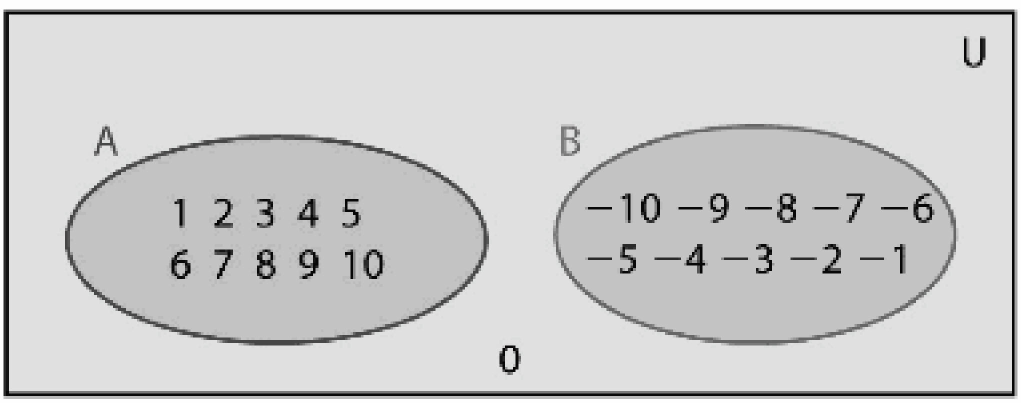
b) List the disjoint subsets, if there are any.

c) Is each statement true or false? Explain.

i) $A \subset B$
ii) $B \subset A$
iii) $A' = B$
iv) $n(A) = n(B)$
v) For set U, the set of integers from -20 to -15 is $\{\ \}$.

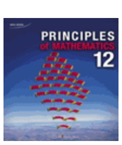
Solution:

11. a)



b) Sets *A* and *B* are disjoint sets.
c) i) False. e.g., 1 is not in *B*.
ii) False. e.g., –1 is not in *A*.
iii) False. e.g., 0 is in *A′* but not in *B*.
iv) True. e.g., $n(A) = 10$, $n(B) = 10$.
v) True. e.g., No integer from –20 to –15 is in *U*.

Practice Problem:



Complete “Practising” question 12 on page 17 of your textbook.

Semiprime numbers are numbers that are the product of two prime numbers (1 is not considered to be a prime number). For example, 10 is a semiprime number because it is the product of 2 and 5.

a) Use the set of natural numbers from 1 to 50, inclusive, as the universal set. Organize these numbers into the following sets:
- S = {semiprimes less than 50}
- W = {other numbers}

b) Define one subset of S.

c) Determine $n(W)$ without counting.

d) Consider the set A = {all semiprimes}. Can you determine $n(A)$? Explain why or why not.

Solution:

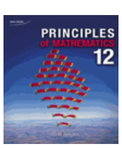
12. a) $S = \{4, 6, 9, 10, 14, 15, 21, 22, 25, 26, 33, 34, 35, 38, 39, 46, 49\}$
$W = \{1, 2, 3, 5, 7, 8, 11, 12, 13, 16, 17, 18, 19, 20, 23, 24, 27, 28, 29, 30, 31, 36, 37, 40, 41, 42, 43, 44, 45, 47, 48, 50\}$

b) e.g., E = {even semiprime numbers}
$E = \{4, 6, 10, 14, 22, 26, 34, 38, 46\}$

c)
$n(W) = n(U) - n(S)$
$n(W) = 50 - 17$
$n(W) = 33$

d) No, it is not possible to determine $n(A)$. e.g., There is an infinite number of prime numbers, so there is an infinite number of semiprime numbers.

Practice Problem:



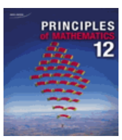
Complete “Practising” question 14 on page 17 of your textbook.

Cynthia claims that the $\subset$ sign for sets is similar to the $\leq$ sign for numbers. Explain whether you agree or disagree.

Solution:

14. Agree; e.g., $A \subset B$ means that set A is a part of set B, and it could be that set A and set B are equal. If $A \subset B$, then set A will have the same number or fewer elements than set B. With numbers, $x \leq y$ means that x is less than or equal to y. Or, or if $A \subset B$, then $n(A) \leq n(B)$. The number of elements in a subset must be equal to or less than the number of elements in the set.

Practice Problem:



Complete “Practising” question 15 on page 17 of your textbook.

a) Indicate the multiples of 25 and 50, from -1000 to 1000, using set notation. List any subsets.

b) Represent the sets and subsets in a Venn diagram.

Solution:

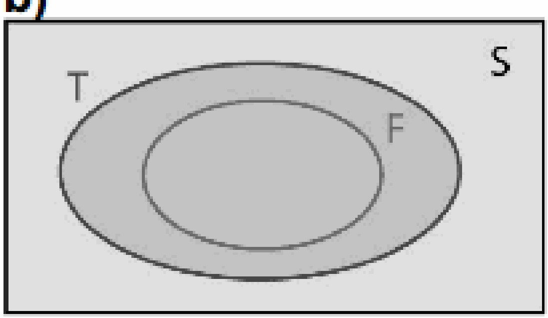
15. a) $S = \{x \mid -1000 \leq x \leq 1000, x \in I\}$

$T = \{t \mid t = 25x, -40 \leq x \leq 40, x \in I\}$

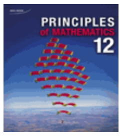
$F = \{f \mid f = 50x, -20 \leq x \leq 20, x \in I\}$

$F \subset T \subset S$

b)



Practice Problem: **(KEY QUESTION)**



Complete “Practising” question 16 on page 17 of your textbook.

Carol tosses a nickel, a dime, and a quarter. Each coin can turn up heads (H) or tails (T).

a) List the elements of the universal set, U, for this situation.

b) $E = \{\text{second coin turns up tails}\}$ List the elements of E.

c) Determine $n(U)$ and $n(E)$.

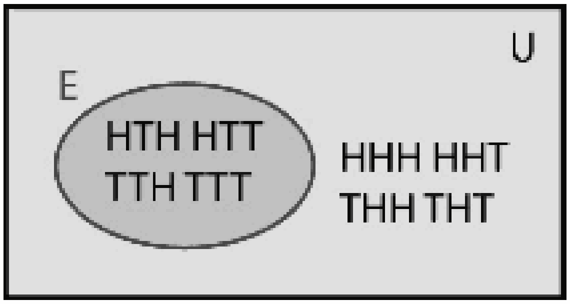
d) Is $E \subset U$?

e) Describe E' in words. Determine $n(E')$ using $n(U)$ and $n(E)$. Confirm your answer by listing the elements of E'.

f) Are E and E' disjoint sets? Explain.

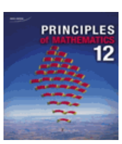
Solution:

16. a) U = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
b) E = {HTH, HTT, TTH, TTT}
c) $n(U) = 8$, $n(E) = 4$
d) Yes, e.g., because each element of E is also an element of U, and there are some elements of U that are not elements of E.



e) For example, E' is the set of elements of U where the second coin turns up heads.
$n(E') = n(U) - n(E)$
$n(E') = 8 - 4$
$n(E') = 4$
E' = {HHH, HHT, THH, THT} and $n(E') = 4$
f) Yes. e.g., A coin cannot show both heads and tails at the same time.

Practice Problem:



Complete "Closing" question 19 on page 18 of your textbook.

Explain how to determine each of the following, and give an example.
a) Whether one set is a subset of another
b) Whether one set is a complement of another

Solution:

19. a) e.g., $A \subset B$ if all elements of A are also in B. For example, all weekdays are also days of the week, so weekdays is a subset of days of the week.
b) e.g., A' consists of all the elements in the universal set but not in A. For example, all days of the week that are not weekdays are weekend days. So weekend days is the complement of weekdays.