Math 30-2: U4L1 Teacher Notes

Equivalent Rational Expressions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers
- determine the non-permissible values for a rational expression and explain why such values are non-permissible determine equivalent rational expressions to given rational expressions and explain why the non-permissible values of both are the same
- simplify a rational expression and identify, correct, and explain errors in the simplification of a rational expression

What is Rational Expression

Recall that the definition of a **rational number** is a real number that can be expressed in the form $\frac{a}{b}$ where *a* and *b* are integers and $b \neq 0$.

Rational expression
A **rational expression** has a similar definition. A rational expression can be expressed in the form of $\frac{a}{b}$, where *a* and *b* **are polynomials** and $b \neq 0$.

Click the icon to watch a LearnAlberta video Identifying Rational Expressions

Now that you have the definition of a rational expression, work through Identifying Rational Expressions to help you identify a rational expression.

Example:

Which of the following expressions are rational? Explain.

1. $\frac{1}{1-x^2}$
2. $\frac{2-\sqrt{x}}{4-x}$
3. $\frac{1-x}{1+\frac{1}{x}}$
4. $3x$

Solution:

The textbook defines a rational expression as a fraction that has a polynomial in both the numerator and denominator. To determine whether an expression is in fact a rational expression, examine both the numerator and denominator, checking to see if both are polynomials.

1. This is a rational expression because the numerator and denominator are polynomials. (Remember that a constant can also be classified as a polynomial.)
2. This is not a rational expression because the numerator is not a polynomial (due to the square root).
3. This is not a rational expression because the denominator is not a polynomial (due to the $\frac{1}{x}$).
4. This is a rational expression because it can be written as $\frac{3x}{1}$, and the numerator and denominator are polynomials.

Why are Rational Expressions Undefined for Certain Values?

A rational expression is not defined when its denominator is 0. Therefore, any value of the variable that would result in a denominator of 0 is called a **non-permissible value**. These non-permissible values are also called **restrictions**.

Click the icon to watch a YouTube video Non-permissible Values

Example:

Determine the non-permissible values.

a. $\frac{x-3}{23}$

b. $\frac{23}{x-3}$

c. $\frac{2x+1}{x^2+1}$

d. $\frac{2x+1}{x^2-4}$

Solution:

Non-permissible values are found by determining the values that will make the denominator equal to zero.

a. $\frac{x-3}{23}$

In this case, the denominator will never be 0, so there are no non-permissible values.

b. $\frac{23}{x-3}$

The denominator will equal 0 when $x = 3$.

c. $\frac{2x+1}{x^2+1}$

In this case, solving for a 0 denominator results in the following equation:

$$\begin{aligned} x^2+1 &= 0 \\ x^2 &= -1 \end{aligned}$$

A squared number can never be negative, so this equation has no solutions. Therefore, there are no non-permissible values.

d. $\frac{2x+1}{x^2-4}$

To find the non-permissible values, first factor the denominator; then determine what will make each factor equal to 0. Solving for a 0 denominator results in the following equation:

$$\begin{aligned} x^2-4 &= 0 \\ (x-2)(x+2) &= 0 \\ x &= -2,\ 2 \end{aligned}$$

Therefore, the non-permissible values are -2 and 2.

How Do We Create Equivalent Rational Expressions?

What happens to a number if you multiply it by 1? If you said that the number remains unchanged, you are absolutely correct. **This property of 1 is critical in determining equivalent expressions in mathematics.**

The fractions 2/3, 4/6, 6/9 are all equivalent since:

$$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6} \quad \text{and} \quad \frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$$

We can create a equivalent Rational Expression by multiplying the expression by using the same Property of 1. Look at the example below to help you with this concept.

Example:

Find two equivalent expression to $\dfrac{2x}{x-2}$

Solution:

Use the Property of 1

$$\frac{2x}{x-2} \times \frac{10}{10} = \frac{20x}{10(x-2)} = \frac{20x}{10x-20}$$

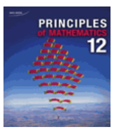
Remember you can chose whatever 1 you want, as long as you follow the proper rules of multiplying two fractions together. Lets find another equivalent fraction using the Property of 1 with a variable.

$$\frac{2x}{x-2} \times \frac{x}{x} = \frac{(x)(2x)}{x(x-2)} = \frac{2x^2}{x^2-2x}$$

You can even chose a binomial for your 1.

$$\frac{2x}{x-2} \times \frac{x+1}{x+1} = \frac{2x(x+1)}{(x-2)(x+1)}$$

Click the icon to link to another lesson from PurpleMath on Equivalent Rational expressions



Practice Problem:

Complete “Check your Understanding” question 1 on page 222 of your textbook.

Solution:

1. a) i) $\dfrac{16x^3}{10x} = \dfrac{2x(8x^2)}{2x(5)}$

$\dfrac{16x^3}{10x} = \dfrac{8x^2}{5},\ x \neq 0$

ii) $\dfrac{2x+6}{4} = \dfrac{2(x+3)}{2(2)}$

$\dfrac{2x+6}{4} = \dfrac{x+3}{2}$

iii) $\dfrac{x-2}{4x^2} = \dfrac{x-2}{4x^2} \cdot \dfrac{x}{x}$

$\dfrac{x-2}{4x^2} = \dfrac{x^2-2x}{4x^3},\ x \neq 0$

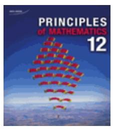
iv) $\dfrac{x-3}{x-1} = \dfrac{x-3}{x-1} \cdot \dfrac{2}{2}$

$\dfrac{x-3}{x-1} = \dfrac{2x-6}{2x-2},\ x \neq 1$

b) i) Denominator:
$10x = 0$
$x = 0$
The non-permissible value for this pair of equivalent expressions is $x = 0$.
ii) Denominator: $4 = 0$
This pair of expressions has no non-permissible values.
iii) Denominator:
$4x^2 = 0$
$x = 0$
The non-permissible value for this pair of equivalent expressions is $x = 0$.
iv) Denominator:
$x - 1 = 0$
$x = 1$
The non-permissible value for this pair of equivalent expressions is $x = 1$.
c) e.g., ii) substitute $x = 2$:

$$\frac{2(2)+6}{4}=\frac{5}{2}, \frac{(2)+3}{2}=\frac{5}{2}$$

Both expressions produce the same result for this value of x.



Practice Problem:

Complete “Check your Understanding” question 2 on page 222 of your textbook.

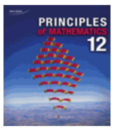
Solution:

2. a) Let t represent the time of the journey, in hours.

$$\text{Speed} = \frac{450}{t} \text{ km/h}, \ t > 0$$

b) Let V represent the total volume melted, in cubic metres.

$$\text{Volume} = \frac{V}{15} \text{ m}^3\text{/yr}, \ V \geq 0$$



Practice Problem:

Complete “Check your Understanding” question 3 on page 223 of your textbook.

Solution:

3. a) Denominator:

$-5 - x = 0$

$x = -5$

The non-permissible value for this pair of equivalent expressions is $x = -5$.

$$\frac{2-7x}{-5-x} = \frac{2-7x}{-5-x} \cdot \frac{-1}{-1}$$

$$\frac{2-7x}{-5-x} = \frac{7x-2}{x+5},\ x \neq -5$$

$$\frac{2-7x}{-5-x} = \frac{2-7x}{-5-x} \cdot \frac{2}{2}$$

$$\frac{2-7x}{-5-x} = \frac{4-14x}{-10-2x},\ x \neq -5$$

b) Denominator:

$a^2 + a = 0$

$a(a + 1) = 0$

$a = 0$ or $a + 1 = 0$

$a = 0$ or $a = -1$

The non-permissible values for this pair of equivalent expressions are $a = 0$ and $a = -1$.

$$\frac{4a^2}{a^2+a} = \frac{a(4a)}{a(a+1)}$$

$$\frac{4a^2}{a^2+a} = \frac{4a}{a+1}, a \neq 0, -1$$

$$\frac{4a^2}{a^2+a} = \frac{4a^2}{a^2+a} \cdot \frac{3}{3}$$

$$\frac{4a^2}{a^2+a} = \frac{12a^2}{3a^2+3a}, a \neq 0, -1$$

c) Denominator:

$3t^2 - t = 0$

$t(3t - 1) = 0$

$t = 0$ or $3t - 1 = 0$

$t = 0$ or $\quad t = \frac{1}{3}$

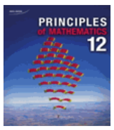
The non-permissible values for this pair of equivalent expressions are $t = 0$ and $t = \frac{1}{3}$.

$$\frac{4t^3}{3t^2 - t} = \frac{t(4t^2)}{t(3t - 1)}$$

$$\frac{4t^3}{3t^2 - t} = \frac{4t^2}{3t - 1},\ t \neq 0,\ \frac{1}{3}$$

$$\frac{4t^3}{3t^2 - t} = \frac{4t^3}{3t^2 - t} \cdot \frac{-t}{-t}$$

$$\frac{4t^3}{3t^2 - t} = \frac{-4t^4}{t^2 - 3t^3},\ t \neq 0,\ \frac{1}{3}$$



Practice Problem:

Complete "Check your Understanding" question 4 on page 223 of your textbook.

Solution:

4. A. Non-permissible value:
$16a^2 = 0$
$a = 0$
D. Non-permissible value:
$64a^3 = 0$
$a = 0$
Both expressions have the same non-permissible value.

$$\text{Expression A} = \frac{4a-4}{16a^2},\ a \neq 0$$

$$\text{Expression A} = \frac{4a-4}{16a^2} \cdot \frac{4a}{4a}$$

$$\text{Expression A} = \frac{16a^2-16a}{64a^3},\ a \neq 0$$

Expression A = Expression D
Therefore, expressions A and D are equivalent.

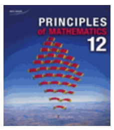
B. Non-permissible value:
$32a^2 = 0$
$a = 0$
D. Non-permissible value:
$-128a^2 = 0$
$a = 0$
Both expressions have the same non-permissible value.

$$\text{Expression B} = \frac{2-2a}{32a^2},\ a \neq 0$$

$$\text{Expression B} = \frac{2-2a}{32a^2} \cdot \frac{-4}{-4}$$

$$\text{Expression B} = \frac{8a-8}{-128a^2},\ a \neq 0$$

Expression B = Expression C
Therefore, expressions B and C are equivalent.

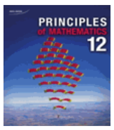


Practice Problem:

Complete "Check your Understanding" question 9 on page 223 of your textbook.

Solution:

9. a) i) Denominator:
$x + 5 = 0$
$x = -5$
The non-permissible value for this expression is $x = -5$.
ii) Denominator:
$-3x - 15 = 0$
$-3x = 15$
$x = -5$
The non-permissible value for this expression is $x = -5$.
iii) Denominator:
$2x^3 + 10x^2 = 0$
$2x^2(x + 5) = 0$
$2x^2 = 0$ or $x + 5 = 0$
$x = 0$ or $x = -5$
The non-permissible values for this expression are $x = 0$ and $x = -5$.
b) These expressions are not equivalent, because expression iii) has an additional non-permissible value.



Practice Problem:

Complete “Check your Understanding” question 17 on page 224 of your textbook.

Solution:

17. a) $\dfrac{13}{2x-5x^2},\ x \neq 0,\ \dfrac{2}{5}$

b) $\dfrac{3x}{12x-6x^2} = \dfrac{3(x)}{3(4x-2x^2)},\ x \neq 0,\ 2$

$\dfrac{3x}{12x-6x^2} = \dfrac{x}{4x-2x^2},\ x \neq 0,\ 2$

$\dfrac{3x}{12x-6x^2} = \dfrac{x(3)}{x(12-6x)},\ x \neq 0,\ 2$

$\dfrac{3x}{12x-6x^2} = \dfrac{3}{12-6x},\ x \neq 0,\ 2$