Math 30-2: U4L2 Teacher Notes

Simplifying Rational Expressions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers
- determine the non-permissible values for a rational expression and explain why such values are non-permissible determine equivalent rational expressions to given rational expressions and explain why the non-permissible values of both are the same
- simplify a rational expression and identify, correct, and explain errors in the simplification of a rational expression

How Do We Simplify Rational Expressions?

To simplify a rational expression means to write an equivalent expression where the greatest common factor between the numerator and the denominator is 1. Such an expression is described as being in simplified form or reduced to lowest terms.

To simplify a rational expression, we first factor both the numerator and denominator completely then reduce the expression by cancelling common factors.

Example:

Simplify $\frac{2x-3}{2x}$

Solution:

This particular rational expression cannot be simplified.

Why, you ask?? First of all it cannot be factored, and secondly there are no common factors.

Some people are tempted to cross out the (2x) that is both in the top and the bottom, but since the (2x) in the top is part of a binomial, this cannot be done.

<u>Example 1:</u> Simplify the rational expression

$$\frac{4x - 2}{2x - 1}$$

<u>Detailed Solution to Example 1</u>

- Factor both the numerator and denominator completely.

$$\frac{2(2x - 1)}{2x - 1}$$

- Cancel common factors to reduce and simplify the given expression.

$$= \frac{2(2x - 1)}{2x - 1} = 2 \text{ , with x not equal to 1}$$

Note: The given expression is equal to 2 on the condition that x is not equal to 1. x = 1 is not in the domain of the expression.

Example 2: Simplify the rational expression

$$\frac{4x + 16}{x^2 - 16}$$

Detailed Solution to Example 2

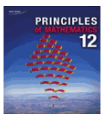
- Factor both the numerator and denominator completely.

$$\frac{4x + 16}{x^2 - 16} = \frac{4(x + 4)}{(x + 4)(x - 4)}$$

- Cancel common factors to simplify the expression.

$$= \frac{4(x + 4)}{(x - 4)(x + 4)}$$

$$= \frac{4}{(x - 4)} \quad \text{, with x not equal to -4.}$$



Practice Problem:

Complete "Check your Understanding" question 3 on page 229 of your textbook.

Solution:

3. a) $\frac{6y}{4y^3} = \frac{2y(3)}{2y(2y^2)}, y \neq 0$

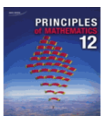
$\frac{6y}{4y^3} = \frac{3}{2y^2}, y \neq 0$

b) $\frac{-17y^5}{7y^6} = \frac{y^5(-17)}{y^5(7y)}, y \neq 0$

$\frac{-17y^5}{7y^6} = \frac{-17}{7y}, y \neq 0$

c) $\frac{240y^2}{168y} = \frac{24y(10y)}{24y(7)}, y \neq 0$

$\frac{240y^2}{168y} = \frac{10y}{7}, y \neq 0$



Practice Problem:

Complete "Check your Understanding" question 4 on page 229 of your textbook.

Solution:

4. a)

$$\frac{20a^2+30a^3}{25a^2}=\frac{5a^2(4+6a)}{5a^2(5)},\ a\neq 0$$

$$\frac{20a^2+30a^3}{25a^2}=\frac{4+6a}{5},\ a\neq 0$$

b)

$$\frac{6a+2}{6a}=\frac{2(3a+1)}{2(3a)},\ a\neq 0$$

$$\frac{6a+2}{6a}=\frac{3a+1}{3a},a\neq 0$$

$$\frac{6a+2}{6a}=\frac{3a}{3a}+\frac{1}{3a},a\neq 0$$

$$\frac{6a+2}{6a}=1+\frac{1}{3a},a\neq 0$$

c)

$$\frac{18a^2-24a^3}{36a^2}=\frac{6a^2(3-4a)}{6a^2(6)},\ a\neq 0$$

$$\frac{18a^2-24a^3}{36a^2}=\frac{3-4a}{6},\ a\neq 0$$

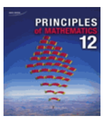
d)

$$\frac{-15x^2-10x}{15x^2}=\frac{5x(-3x-2)}{5x(3x)},\ x\neq 0$$

$$\frac{-15x^2-10x}{15x^2}=\frac{-3x-2}{3x},\ x\neq 0$$

$$\frac{-15x^2-10x}{15x^2}=\frac{-3x}{3x}-\frac{2}{3x},\ x\neq 0$$

$$\frac{-15x^2-10x}{15x^2}=-1-\frac{2}{3x},\ x\neq 0$$



Practice Problem:

Complete “Check your Understanding” question 6 on page 230 of your textbook.

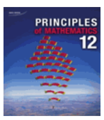
Solution:

6. a) $\dfrac{-10x^2-15x}{4x^3-9x}=\dfrac{-5x(2x+3)}{x(4x^2-9)},\ x\neq\dfrac{-3}{2},0,\ \dfrac{3}{2}$

$$\frac{-10x^2-15x}{4x^3-9x}=\frac{-5x(2x+3)}{x(2x+3)(2x-3)}$$

$$\frac{-10x^2-15x}{4x^3-9x}=\frac{-5}{2x-3},\ x\neq\frac{-3}{2},\ 0,\frac{3}{2}$$

b) e.g., Simplifying may remove a factor from the denominator that creates a non-permissible value.



Practice Problem:

Complete "Check your Understanding" question 8 on page 230 of your textbook.

Solution:

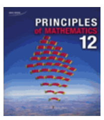
8. a) Sharon did not begin her solution by determining the non-permissible values, so her solution is missing the non-permissible value of $x = 2$.

$$\frac{12x-6x^2}{10x^3-20x^2}=\frac{3(2x)(2-x)}{5x(2x)(x-2)},\ x\neq 0,\ 2$$

$$\frac{12x-6x^2}{10x^3-20x^2}=\frac{-3(2x)(x-2)}{5x(2x)(x-2)}$$

$$\frac{12x-6x^2}{10x^3-20x^2}=\frac{-3}{5x},\ x\neq 0,\ 2$$

b) Sharon's error may be a common error because you might forget to determine the non-permissible values before factoring and simplifying.



Practice Problem:

Complete “Check your Understanding” question 9 on page 230 of your textbook.

Solution:

9. a) i) When $t = 0$

$$\frac{4t}{4+2t} = \frac{4(0)}{4+2(0)}$$

$$\frac{4t}{4+2t} = 0$$

At 0 hours, the concentration of chlorine would be 0 mg/L.

ii) When $t = 2$

$$\frac{4t}{4+2t} = \frac{4(2)}{4+2(2)}$$

$$\frac{4t}{4+2t} = \frac{8}{4+4}$$

$$\frac{4t}{4+2t} = \frac{8}{8}$$

$$\frac{4t}{4+2t} = 1$$

At 2 h, the concentration of chlorine would be 1 mg/L.

iii) When $t = 6$

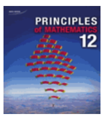
$$\frac{4t}{4+2t} = \frac{4(6)}{4+2(6)}$$

$$\frac{4t}{4+2t} = \frac{24}{4+12}$$

$$\frac{4t}{4+2t} = \frac{24}{16}$$

$$\frac{4t}{4+2t} = 1.5$$

At 6 h, the concentration of chlorine would be 1.5 mg/L.



Practice Problem:

Complete “Check your Understanding” question 14 on page 231 of your textbook.

Solution:

14. e.g., To simplify a rational expression, you need to divide the numerator and denominator by all possible common factors. This is similar to dividing the numerator and denominator of a rational number by the largest possible common (numerical) factor.
$\frac{6x^2}{14x^2-2x}$, $x \neq 0, \frac{1}{7}$ simplifies to $\frac{3x}{7x-1}$, $x \neq 0, \frac{1}{7}$ after factoring $2x$ from the numerator and denominator.

Solution:

b) $\frac{4t}{4+2t} = \frac{2(2t)}{2(2+t)}, t \neq -2$

$\frac{4t}{4+2t} = \frac{2t}{2+t}, t \neq -2$

c) Yes. e.g., Time cannot be negative.