Math 30-2: U4L3 Teacher Notes

Multiplying and Dividing Rational Expressions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Compare the strategies for performing a given operation on rational expressions to the strategies for performing the same operation on rational numbers
- Determine the non-permissible values for a rational expression
- Determine, in simplified form, the product or quotient of two rational expressions

How do We Multiply Rational Numbers (Fractions)

There are 3 steps to mulitplying fractions:

1. Multiply the top numbers (numerators)
2. Multiply the bottom numbers (denominators)
2. Simplify the fraction if needed (reduce to lowest terms)

Multiplying Rational Expressions with Monomials

Multiplying Rational expressions is very similar to multiplying rational numbers. We do have to to take into consideration our non-permissible values.

Example:

Multiply. $\frac{x^2}{7} \times \frac{14}{x^3}$

Solution:

$x \neq 0$ State the restrictions on the original expression. We must look at both denominators.

$\frac{14x^2}{7x^3}$ ⬅ Multiply the numerators together and multiply the denominators together.

$\frac{14x^2}{7x^3} = \frac{2 \cdot 7 \cdot x \cdot x}{7 \cdot x \cdot x \cdot x}$ ⬅ Simplify using prime factorization.

$\frac{14x^2}{7x^3} = \frac{7}{7} \times \frac{x}{x} \times \frac{x}{x} \times \frac{2}{x}$ ⬅ Rearranging the factors and using the Property of 1

$\frac{14x^2}{7x^3} = 1 \times 1 \times 1 \times \frac{2}{x}$ ⬅ Simplify

$\frac{2}{x},\ x \neq 0$

Example:

Aki was presented a similar problem, $\left(\frac{12a^3}{8a}\right)\left(\frac{a^2}{4a}\right)$, and had a different idea on how to solve.

Solution:

$a \neq 0$ First, Aki stated her non-permissible values by looking at the denominators of each fraction.

$$\left(\frac{4 \cdot 3 \cdot a \cdot a \cdot a}{4 \cdot 2 \cdot a}\right)\left(\frac{a \cdot a}{4 \cdot a}\right)$$

Her next step involved simplifying each individual fraction using prime factorization.

$$\left(\frac{4}{4} \times \frac{a}{a} \times \frac{3 \cdot a \cdot a}{2}\right)\left(\frac{a}{a} \times \frac{a}{4}\right)$$

She used the Property of 1 to simplify each fraction

$$\left(\frac{3a^2}{2}\right)\left(\frac{a}{4}\right)$$

$$\frac{3a^3}{8}$$

Then Aki multiplied the two numerators and denominators.

$$\frac{3a^3}{8},\ a \neq 0$$

She then looked for further simplification.

When you multiply rational expressions together you can either:

- Multiply first and then simplify.
- Simplify each individual expression, then multiply. Check the product for further simplication.

Multiply the rational expressions as you would rational numbers. Reduce to lowest terms where possible. Include the non-permissible values with your final answer.

Multiplying Rational Expressions with Binomials

When we multiply binomials together we follow a similar procedure.

1. State the non-permissible values

2. Factor all numerators and denominators completely.

3. Divide numerators and denominators by common factors by looking for properties of 1.

4. Multiply the remaining factor

Example:

Multiply: $\frac{x - 3}{x + 5} \cdot \frac{10x + 50}{7x - 21}$.

Solution:

$x \neq -5, \text{ and } 3$ — State the non-permissible values

$\frac{x - 3}{x + 5} \cdot \frac{10(x + 5)}{7(x - 3)}$ — Factor as much as possible.

$\frac{x-3}{x-3} \cdot \frac{x+5}{x+5} \cdot \frac{10}{7}$ — Divide numerators and denominators by common factors by looking for properties of 1.

$\frac{10}{7}, \ x \neq -5, \text{ and } 3$

Example:

Multiply: $\frac{x-7}{x-1} \cdot \frac{x^2-1}{3x-21}$.

Solution:

$x \neq 1, \text{ and } 7$ — State the non-permissible values

$\frac{x-7}{x-1} \cdot \frac{(x+1)(x-1)}{3(x-7)}$ — Factor as much as possible.

$\frac{x-1}{x-1} \cdot \frac{x-7}{x-7} \cdot \frac{(x+1)}{3}$ — Divide numerators and denominators by common factors by looking for properties of 1.

$\frac{(x+1)}{3}, \; x \neq 1, \text{ and } 7$

How do I Divide Rational Numbers (Fractions)?

Step 1: Change the division symbol to multiplication. Change the right-hand fraction (the divisor) to its reciprocal. ("Flip it.")

Step 2: Multiply the numerators and denominators separately; then simplify (reduce) wherever possible.

Dividing Rational Expressions

Now that you have explored multiplication of rational expressions, you will focus on division. Recall that dividing rational numbers is a process that involves multiplying the first number by the reciprocal of the second number.

One concept we have to be very careful of is stating the non-permissible values of the expression. When finding the non-permissible values of the rational expression when we are dividing is a bit more complex since we have two divisions to deal with.

For division, remember to consider both the numerator and the denominator of the divisor (the second fraction).

Example:

Divide: $(x + 5) \div \frac{x - 2}{x + 9}$.

Solution:

$x \neq 2,\ \text{and -9}$ — State the non-permissible values

$\frac{x+5}{1} \div \frac{x-2}{x+9}$ — Factor as much as possible.

$\frac{x+5}{1} \times \frac{x+9}{x-2}$ — Change the division symbol to multiplication. Change the right-hand fraction (the divisor) to its reciprocal.

$\frac{(x+5)(x+9)}{(x-2)},\ x \neq 2,\ \text{and -9}$

Example:

Divide: $\frac{x+1}{3} \div \frac{3x+3}{7}$

Solution:

$x \neq -1$ — State the non-permissible values

$\frac{x+1}{3} \div \frac{3(x+1)}{7}$ — Factor as much as possible.

$\frac{x+1}{3} \times \frac{7}{3(x+1)}$ — Change the division symbol to multiplication. Change the right-hand fraction (the divisor) to its reciprocal.

$\frac{x+1}{x+1} \times \frac{7}{(3)(3)}$ — Simplify and multiply.

$\frac{7}{9},\ x \neq 2,\ -2$

Example:

Divide: $\frac{x^2 - 4}{x - 2} \div \frac{x + 2}{4x - 8}$

Solution:

$x \neq 2, \ -2$ ⬅ State the non-permissible values

$\frac{(x+2)(x-2)}{x-2} \div \frac{x+2}{4(x-2)}$ ⬅ Factor as much as possible.

$\frac{(x+2)(x-2)}{x-2} \times \frac{4(x-2)}{x+2}$ ⬅ Change the division symbol to multiplication. Change the right-hand fraction (the divisor) to its reciprocal.

$\frac{x+2}{x+2} \times \frac{x-2}{x-2} \times \frac{4(x-2)}{1}$ ⬅ Simplify and multiply.

- $4(x-2), \ x \neq 2,$ and -2

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Practice Problem:

Complete “Check your Understanding” question 4 on page 238 of your textbook.

Solution:

4. a)

$$\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right)=\left(\frac{(27x^3)x}{14x}\right)\left(-\frac{3x^3(7x)}{3x^3(2)}\right),\ x\neq 0$$

$$\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right)=\left(\frac{27x^3}{14}\right)\left(\frac{-7x}{2}\right)$$

$$\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right)=\frac{-189x^4}{28}$$

$$\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right)=\frac{7(-27x^4)}{7(4)}$$

$$\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right)=\frac{-27x^4}{4},\ x\neq 0$$

Solution:

b) $\frac{6a}{9} \div \frac{4a^2}{3} = \frac{3(2a)}{3(3)} \div \frac{4a^2}{3}, a \neq 0$

$\frac{6a}{9} \div \frac{4a^2}{3} = \frac{2a}{3} \div \frac{4a^2}{3}$

$\frac{6a}{9} \div \frac{4a^2}{3} = \frac{2a}{3} \cdot \frac{3}{4a^2}$

$\frac{6a}{9} \div \frac{4a^2}{3} = \frac{6a}{12a^2}$

$\frac{6a}{9} \div \frac{4a^2}{3} = \frac{6a}{6a(2a)}$

$\frac{6a}{9} \div \frac{4a^2}{3} = \frac{1}{2a}, a \neq 0$

c) $\frac{(3y^2)^2}{4y^3} \cdot \frac{20}{y} = \frac{9y^4}{4y^3} \cdot \frac{20}{y}, y \neq 0$

$\frac{(3y^2)^2}{4y^3} \cdot \frac{20}{y} = \frac{(9y)y^3}{4y^3} \cdot \frac{20}{y}$

$\frac{(3y^2)^2}{4y^3} \cdot \frac{20}{y} = \frac{9y}{4} \cdot \frac{20}{y}$

$\frac{(3y^2)^2}{4y^3} \cdot \frac{20}{y} = \frac{180y}{4y}$

$\frac{(3y^2)^2}{4y^3} \cdot \frac{20}{y} = \frac{4y(45)}{4y}$

$\frac{(3y^2)^2}{4y^3} \cdot \frac{20}{y} = 45, y \neq 0$

Solution:

d) $-\frac{15m}{20m^2} \div \frac{3}{14m} = -\frac{5m(3)}{5m(4m)} \div \frac{3}{14m}, \; m \neq 0$

$-\frac{15m}{20m^2} \div \frac{3}{14m} = \frac{-3}{4m} \div \frac{3}{14m}$

$-\frac{15m}{20m^2} \div \frac{3}{14m} = \frac{-3}{4m} \cdot \frac{14m}{3}$

$-\frac{15m}{20m^2} \div \frac{3}{14m} = \frac{-42m}{12m}$

$-\frac{15m}{20m^2} \div \frac{3}{14m} = \frac{6m(-7)}{6m(2)}$

$-\frac{15m}{20m^2} \div \frac{3}{14m} = \frac{-7}{2}, \; m \neq 0$

Practice Problem:

Complete "Check your Understanding" question 5 on page 238 of your textbook.

Solution:

5. a)
$$\frac{3x(2x-1)}{7x^2(x-6)}\cdot\frac{14(6-x)}{8x^3}$$
$$=\frac{x(3)(2x-1)}{x(7x)(x-6)}\cdot\frac{2(7)(6-x)}{2(4x^3)},\ x\neq 0,\ 6$$
$$=\frac{3(2x-1)}{7x(x-6)}\cdot\frac{7(6-x)}{4x^3}$$
$$=\frac{3(2x-1)}{7x(x-6)}\cdot\frac{7(-1)(x-6)}{4x^3}$$
$$=\frac{-21(2x-1)(x-6)}{28x^4(x-6)}$$
$$=\frac{7(x-6)(-3)(2x-1)}{7(x-6)(4x^4)}$$
$$=\frac{-3(2x-1)}{4x^4},\ x\neq 0,\ 6$$

b)
$$\frac{4b^2(2b+1)}{2b+3}\div\frac{10(1+2b)}{b+3}$$
$$=\frac{4b^2(2b+1)}{2b+3}\cdot\frac{b+3}{10(1+2b)},\ b\neq -3,\ \frac{-3}{2},\ \frac{-1}{2}$$
$$=\frac{4b^2(2b+1)(b+3)}{10(2b+3)(1+2b)}$$
$$=\frac{2(2b+1)(2b^2)(b+3)}{2(2b+1)(5)(2b+3)}$$
$$=\frac{2b^2(b+3)}{5(2b+3)},\ b\neq -3,\ \frac{-3}{2},\ \frac{-1}{2}$$

Practice Problem:

Complete "Check your Understanding" question 6 on page 239 of your textbook.

Solution:

6. a) $\frac{a^2+3a}{25}\cdot\frac{15a^3}{9a+3a^2}$

$$=\frac{a(a+3)}{25}\cdot\frac{3a(5a^2)}{3a(3+a)},\ a\neq -3,0$$

$$=\frac{a(a+3)}{25}\cdot\frac{5a^2}{a+3}$$

$$=\frac{5a^3(a+3)}{25(a+3)}$$

$$=\frac{5a^3(a+3)}{5(5)(a+3)}$$

$$=\frac{a^3}{5},\ a\neq -3,0$$

b) $\frac{y^4}{5y^2-2y^3}\cdot\frac{2y^3-5y^2}{y^3}$

$$=\frac{y^2(y^2)}{y^2(5-2y)}\cdot\frac{y^2(2y-5)}{y^2(y)},\ y\neq 0,\ \frac{5}{2}$$

$$=\frac{y^2}{5-2y}\cdot\frac{2y-5}{y}$$

$$=\frac{y^2(2y-5)}{y(5-2y)}$$

$$=\frac{y(-y)(5-2y)}{y(5-2y)}$$

$$=-y,\ y\neq 0,\ \frac{5}{2}$$

Solution:

c) $\dfrac{x+2x^2}{1-x}\cdot\dfrac{x^2-x}{2x^4+x^3}$

$=\dfrac{x(1+2x)}{1-x}\cdot\dfrac{x(x-1)}{x(x^2)(2x+1)},\ x\neq\dfrac{-1}{2},\ 0$

$=\dfrac{x(1+2x)}{1-x}\cdot\dfrac{x-1}{x^2(2x+1)}$

$=\dfrac{x(1+2x)(x-1)}{x^2(1-x)(2x+1)}$

$=\dfrac{x(2x+1)(-1)(1-x)}{x(x)(1-x)(2x+1)}$

$=\dfrac{-1}{x},\ x\neq\dfrac{-1}{2},\ 0$

$\dfrac{a^2-9}{3a+9}\cdot\dfrac{15a^3}{12a^2-36a}$

$=\dfrac{(a+3)(a-3)}{3(a+3)}\cdot\dfrac{3a(5a^2)}{3a(4)(a-3)},\ a\neq-3,0,3$

d) $=\dfrac{a-3}{3}\cdot\dfrac{5a^2}{4(a-3)}$

$=\dfrac{5a^2(a-3)}{12(a-3)}$

$=\dfrac{5a^2}{12},\ a\neq-3,0,3$

Practice Problem: (KEY QUESTION)

Complete "Check your Understanding" question 9 on page 239 of your textbook.

Solution:

9. $$\frac{5a^2}{10a^3-5a^2}\cdot\frac{3a-6}{a+2}=\frac{5a^2}{5a^2(2a-1)}\cdot\frac{3(a-2)}{a+2},\ a\neq -2,0,\frac{1}{2}$$

$$\frac{5a^2}{10a^3-5a^2}\cdot\frac{3a-6}{a+2}=\frac{1}{2a-1}\cdot\frac{3(a-2)}{a+2}$$

$$\frac{5a^2}{10a^3-5a^2}\cdot\frac{3a-6}{a+2}=\frac{3(a-2)}{(2a-1)(a+2)},\ a\neq -2,0,\frac{1}{2}$$

Practice Problem:

Complete "Check your Understanding" question 14 on page 239 of your textbook.

Solution:

14. e.g., Two expressions with this product are

$\frac{y+2}{4y}$ and $\frac{y^2}{3y^2+6y}$

$$\frac{y+2}{4y}\cdot\frac{y^2}{3y^2+6y}=\frac{y+2}{4y}\cdot\frac{y(y)}{y(3)(y+2)}, y\neq-2, 0$$

$$\frac{y+2}{4y}\cdot\frac{y^2}{3y^2+6y}=\frac{y+2}{4y}\cdot\frac{y}{3(y+2)}$$

$$\frac{y+2}{4y}\cdot\frac{y^2}{3y^2+6y}=\frac{y(y+2)}{12y(y+2)}$$

$$\frac{y+2}{4y}\cdot\frac{y^2}{3y^2+6y}=\frac{1}{12}, y\neq-2, 0$$

Either or both of the expressions are not defined at the non-permissible values, so their product cannot be defined at those values either.