Math 30-2: U4L4 Teacher Notes

Adding and Subtracting Rational Expressions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Compare the strategies for performing a given operation on rational expressions to the strategies for performing the same operation on rational numbers
- Determine the non-permissible values for a rational expression
- Determine, in simplified form, the sum and difference of two rational expressions that have the same denominator
- Determine, in simplified form, the sum and difference of two rational expressions that have different denominators

Adding and Subtracting Rational Numbers

Add the following fractions:

$$\frac{1}{3}+\frac{1}{2}$$

In this example, the denominators are different. First, we have to figure out a common denominator. 6 is the Lowest Common Denominator of 3 and 2.

Now we must write equivalent fractions so that they have a denominator of 6. You must do this separately. Start with 1/3. It becomes 2/6. 1/2 becomes 3/6. Now we have this operation:

$$\frac{2}{6}+\frac{3}{6}$$

Add the numerators to arrive at your final answer:

$$\frac{5}{6}$$

Adding and Subtracting Rational Expressions

To add and subtract rational numbers you often have to find the lowest common multiple (LCM) to create equivalent fractions that have the same denominators. In this section you will explore two different methods of determining the lowest common multiple for a set of rational expressions.

Finding the LCM of Polynomials

Click the icon to watch a Youtube video on Finding LCM of monomials

Click the icon to watch a Youtube video on Finding LCM of polynomials

Now that we know how to find the LCM of polynomials, let's do some examples of adding and subtracting rational expressions.

Example:

$$\frac{4x}{x-3}+\frac{x-2}{x-3}$$

Solution:

$x \neq 3$ ⬅ State the non-permissible values.

Since the denominators are the same, we can proceed to adding the numerators.

$$\frac{4x+x-2}{x-3}$$

⬅ Now each denominator is the same so we can subtract the numerators.

$$\frac{5x-2}{x-3}$$

⬅ Simplify. Ask yourself if you can factor and simplify. In this case you can not.

$$\frac{5x-2}{x-3},\ x \neq 3$$

⬅ Make sure to include the restrictions from the original expressions.

Example: $\frac{7}{15a} - \frac{a-1}{5a^2}$

Solution:

$a \neq 0$ ⬅ State the non-permissible values.

Next, we find the lowest common denominator. Find the prime factorization of each denominator.

$5a^2 = 5 \times a \times a$ and $15a = 3 \times 5 \times a$

Therefore, the LCD is $3 \times 5 \times a \times a = 15a^2$

$\left(\frac{7}{15a} \times \frac{a}{a}\right) - \left(\frac{a-1}{5a^2} \times \frac{3}{3}\right)$ ⬅ Multiply each expression by a fraction that is equal to 1 so each has the denominator $15a^2$.

$\left(\frac{7a}{15a^2}\right) - \left(\frac{3(a-1)}{15a^2}\right)$ ⬅ Now each denominator is the same so we can subtract the numerators.

$$\frac{7a - 3a + 3}{15a^2}$$

Simplify

$$\frac{4a + 3}{15a^2}$$

Ask yourself if you can factor and simplify. In this case you can not.

$$\frac{4a + 3}{15a^2},\ a \neq 0$$

Make sure to include the restrictions from the original expressions.

Example: $\dfrac{5y}{3y^2-15y}+\dfrac{4}{y-5}$

Solution:

$y \neq 0,\ 5$ ⬅ State the non-permissible values.

Find the lowest common denominator. Find the prime factorization of each denominator.

$3y^2-15y=3y(y-5)$ and $y-5$

Therefore, the LCD is $3y(y-5)$

$\left(\dfrac{5y}{3y(y-5)}\right)+\left(\dfrac{4}{y-5}\times\dfrac{3y}{3y}\right)$ ⬅ Multiply each expression by a fraction that is equal to 1 so each has the denominator $3y(y-5)$.

$\left(\dfrac{5y}{3y(y-5)}\right)+\left(\dfrac{4(3y)}{3y(y-5)}\right)$ ⬅ Now each denominator is the same so we can subtract the numerators.

$\dfrac{5y+12y}{3y(y-5)}$ ⬅ Simplify

$$\frac{17y}{3y(y-5)}$$

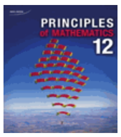
Ask yourself if you can factor and simplify. In this case you can.

$$\frac{y}{y}\times\frac{17}{3(y-5)}$$

Simplify

$$\frac{17}{3(y-5)},\ y\neq 0,\ 5$$

Make sure to include the restrictions from the original expressions.



Practice Problem:

Complete “Check your Understanding” question 4 on page 249 of your textbook.

Solution:

a) $$\frac{-6a+5}{4a^2}+\frac{3}{6a}=\frac{-6a+5}{4a^2}+\frac{1}{2a},\ a\neq 0$$

$$\frac{-6a+5}{4a^2}+\frac{3}{6a}=\frac{-6a+5}{4a^2}+\left(\frac{1}{2a}\cdot\frac{2a}{2a}\right)$$

$$\frac{-6a+5}{4a^2}+\frac{3}{6a}=\frac{-6a+5}{4a^2}+\frac{2a}{4a^2}$$

$$\frac{-6a+5}{4a^2}+\frac{3}{6a}=\frac{-6a+5+2a}{4a^2}$$

$$\frac{-6a+5}{4a^2}+\frac{3}{6a}=\frac{5-4a}{4a^2},\ a\neq 0$$

b) $$\frac{3y-7}{6y}-\frac{1}{12y^2}=\left(\frac{3y-7}{6y}\cdot\frac{2y}{2y}\right)-\frac{1}{12y^2},\ y\neq 0$$

$$\frac{3y-7}{6y}-\frac{1}{12y^2}=\frac{6y^2-14y}{12y^2}-\frac{1}{12y^2}$$

$$\frac{3y-7}{6y}-\frac{1}{12y^2}=\frac{6y^2-14y-1}{12y^2},\ y\neq 0$$

Solution:

c) $$\frac{4}{3x}+5=\frac{4}{3x}+\left(5\cdot\frac{3x}{3x}\right),\ x\neq 0$$

$$\frac{4}{3x}+5=\frac{4}{3x}+\frac{15x}{3x}$$

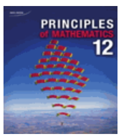
$$\frac{4}{3x}+5=\frac{15x+4}{3x},\ x\neq 0$$

d) $$\frac{3b}{12}-\frac{5}{4b}=\frac{b}{4}-\frac{5}{4b},\ b\neq 0$$

$$\frac{3b}{12}-\frac{5}{4b}=\left(\frac{b}{4}\cdot\frac{b}{b}\right)-\frac{5}{4b}$$

$$\frac{3b}{12}-\frac{5}{4b}=\frac{b^2}{4b}-\frac{5}{4b}$$

$$\frac{3b}{12}-\frac{5}{4b}=\frac{b^2-5}{4b},\ b\neq 0$$



Practice Problem:

Complete “Check your Understanding” question 5 on page 249 of your textbook.

Solution:

a) $\dfrac{5x+12}{x(x+6)}-\dfrac{3}{x}=\dfrac{5x+12}{x(x+6)}-\left(\dfrac{3}{x}\cdot\dfrac{x+6}{x+6}\right),\ x\neq -6,0$

$\dfrac{5x+12}{x(x+6)}-\dfrac{3}{x}=\dfrac{5x+12}{x(x+6)}-\dfrac{3x+18}{x(x+6)}$

$\dfrac{5x+12}{x(x+6)}-\dfrac{3}{x}=\dfrac{5x+12-3x-18}{x(x+6)}$

$\dfrac{5x+12}{x(x+6)}-\dfrac{3}{x}=\dfrac{2x-6}{x(x+6)}$

$\dfrac{5x+12}{x(x+6)}-\dfrac{3}{x}=\dfrac{2(x-3)}{x(x+6)}$

Solution:

b) $\dfrac{-2}{2y-3}+\dfrac{1}{4(y+1)}$

$=\left(\dfrac{-2}{2y-3}\cdot\dfrac{4(y+1)}{4(y+1)}\right)+\left(\dfrac{1}{4(y+1)}\cdot\dfrac{2y-3}{2y-3}\right),\ y\neq -1,\ \dfrac{3}{2}$

$=\dfrac{-8y-8}{4(2y-3)(y+1)}+\dfrac{2y-3}{4(2y-3)(y+1)}$

$=\dfrac{-8y-8+2y-3}{4(2y-3)(y+1)}$

$\dfrac{-6y-11}{4(2y-3)(y+1)},\ y\neq -1,\ \dfrac{3}{2}$

c) $\dfrac{4y-6}{(y-6)(y+6)}-\dfrac{12-3y}{6(y-6)}$

$=\dfrac{4y-6}{(y-6)(y+6)}-\dfrac{4-y}{2(y-6)},\ y\neq -6,\ 6$

$=\left(\dfrac{4y-6}{(y-6)(y+6)}\cdot\dfrac{2}{2}\right)-\left(\dfrac{4-y}{2(y-6)}\cdot\dfrac{y+6}{y+6}\right)$

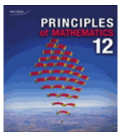
$=\dfrac{8y-12}{2(y-6)(y+6)}-\dfrac{-y^2-2y+24}{2(y-6)(y+6)}$

$=\dfrac{8y-12+y^2+2y-24}{2(y-6)(y+6)}$

$=\dfrac{y^2+10y-36}{2(y-6)(y+6)},\ y\neq -6,\ 6$

Solution:

$$\frac{-2}{a+2}+\frac{5}{a-2}$$
$$=\left(\frac{-2}{a+2}\cdot\frac{a-2}{a-2}\right)+\left(\frac{5}{a-2}\cdot\frac{a+2}{a+2}\right),\ a\neq -2,\ 2$$
$$=\frac{-2a+4}{(a+2)(a-2)}+\frac{5a+10}{(a+2)(a-2)}$$
$$=\frac{-2a+4+5a+10}{(a+2)(a-2)}$$
$$=\frac{3a+14}{(a+2)(a-2)},\ a\neq -2,\ 2$$



Practice Problem:

Complete "Check your Understanding" question 6 on page 249 of your textbook.

Solution:

a)
$$\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{6}{(x+1)(x-1)}+\frac{1}{x+1},\ x\neq -1,\ 1$$
$$\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{6}{(x+1)(x-1)}+\left(\frac{1}{x+1}\cdot\frac{x-1}{x-1}\right)$$
$$\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{6}{(x+1)(x-1)}+\frac{x-1}{(x+1)(x-1)}$$
$$\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{6+x-1}{(x+1)(x-1)}$$
$$\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{x+5}{(x+1)(x-1)},\ x\neq -1,\ 1$$

b)
$$\frac{3a}{4a+2}-\frac{3a^2-6}{4a^2-1}$$
$$=\frac{3a}{2(2a+1)}-\frac{3a^2-6}{(2a+1)(2a-1)},\ a\neq\frac{-1}{2},\ \frac{1}{2}$$
$$=\left(\frac{3a}{2(2a+1)}\cdot\frac{2a-1}{2a-1}\right)-\left(\frac{3a^2-6}{(2a+1)(2a-1)}\cdot\frac{2}{2}\right)$$
$$=\frac{6a^2-3a}{2(2a+1)(2a-1)}-\frac{6a^2-12}{2(2a+1)(2a-1)}$$
$$=\frac{6a^2-3a-6a^2+12}{2(2a+1)(2a-1)}$$
$$=\frac{3(4-a)}{2(2a+1)(2a-1)},\ a\neq\frac{-1}{2},\ \frac{1}{2}$$

Solution:

c) $$\frac{9y}{3y+18}+\frac{y-1}{y+6}=\frac{3(3y)}{3(y+6)}+\frac{y-1}{y+6}, y\neq -6$$

$$\frac{9y}{3y+18}+\frac{y-1}{y+6}=\frac{3y}{y+6}+\frac{y-1}{y+6}$$

$$\frac{9y}{3y+18}+\frac{y-1}{y+6}=\frac{3y+y-1}{y+6}$$

$$\frac{9y}{3y+18}+\frac{y-1}{y+6}=\frac{4y-1}{y+6}, y\neq -6$$

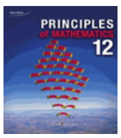
d) $$\frac{a}{2a^2-2}-\frac{a}{2a^2}=\frac{a}{2(a^2-1)}-\frac{1}{2a}, a\neq -1, 0, 1$$

$$\frac{a}{2a^2-2}-\frac{a}{2a^2}=\left(\frac{a}{2(a^2-1)}\cdot\frac{a}{a}\right)-\left(\frac{1}{2a}\cdot\frac{a^2-1}{a^2-1}\right)$$

$$\frac{a}{2a^2-2}-\frac{a}{2a^2}=\frac{a^2}{2a(a^2-1)}-\frac{a^2-1}{2a(a^2-1)}$$

$$\frac{a}{2a^2-2}-\frac{a}{2a^2}=\frac{a^2-a^2+1}{2a(a^2-1)}$$

$$\frac{a}{2a^2-2}-\frac{a}{2a^2}=\frac{1}{2a(a+1)(a-1)}, a\neq -1, 0, 1$$



Practice Problem:

Complete “Check your Understanding” question 7 on page 249 of your textbook.

Solution:

a) $$\frac{7}{2x-6}+\frac{4}{10x-15}=\frac{7}{2(x-3)}+\frac{4}{5(2x-3)},\ x\neq\frac{3}{2},3$$

$$\frac{7}{2x-6}+\frac{4}{10x-15}=\left(\frac{7}{2(x-3)}\cdot\frac{5(2x-3)}{5(2x-3)}\right)+\left(\frac{4}{5(2x+3)}\cdot\frac{2(x-3)}{2(x-3)}\right)$$

$$\frac{7}{2x-6}+\frac{4}{10x-15}=\frac{70x+105}{10(x-3)(2x-3)}+\frac{8x-24}{10(x-3)(2x-3)}$$

$$\frac{7}{2x-6}+\frac{4}{10x-15}=\frac{70x-105+8x-24}{10(x-3)(2x-3)}$$

$$\frac{7}{2x-6}+\frac{4}{10x-15}=\frac{78x-129}{10(x-3)(2x-3)},\ x\neq\frac{3}{2},3$$

Solution:

b) $\dfrac{5}{2b-2}-\dfrac{4+b}{2b^2-2b}=\dfrac{5}{2(b-1)}-\dfrac{4+b}{2b(b-1)},\ b\neq 0,1$

$$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\left(\frac{5}{2(b-1)}\cdot\frac{b}{b}\right)-\frac{4+b}{2b(b-1)}$$

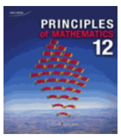
$$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{5b}{2b(b-1)}-\frac{4+b}{2b(b-1)}$$

$$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{5b-b-4}{2b(b-1)}$$

$$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{4b-4}{2b(b-1)}$$

$$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{4(b-1)}{2b(b-1)}$$

$$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{2}{b},\ b\neq 0,1$$



Practice Problem:

Complete “Check your Understanding” question 12 on page 250 of your textbook.

Solution:

12. e.g., First, make sure that the expressions are in simplest terms. Then, determine any common factors in the denominators and the unique factors in each denominator. The LCD is the product of all these factors.

$$\frac{3}{x+1}+\frac{2}{x+2}=\left(\frac{3}{x+1}\cdot\frac{x+2}{x+2}\right)+\left(\frac{2}{x+2}\cdot\frac{x+1}{x+1}\right),\ x\neq -2, -1$$

$$\frac{3}{x+1}+\frac{2}{x+2}=\frac{3x+6}{(x+1)(x+2)}+\frac{2x+2}{(x+1)(x+2)}$$

$$\frac{3}{x+1}+\frac{2}{x+2}=\frac{3x+6+2x+2}{(x+1)(x+2)}$$

$$\frac{3}{x+1}+\frac{2}{x+2}=\frac{5x+8}{(x+1)(x+2)}$$