Math 30-2: U4L5 Teacher Notes

Solving Rational Equations

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Determine the non-permissible values for a rational expression
- Determine, algebraically, the solution to a rational equation, and explain the strategy used to solve the equation.
- Explain why a value obtained in solving a rational equation may not be a solution of the equation.
- Solve a contextual problem that involves a rational equation.

Rational Equations

An equation is a mathematical sentence that has an equal sign. We simplify expressions, but we solve equation. A rational equation is an equation that contains rational expressions.

Examples of Rational Equations		
$\frac{1}{x+2}=\frac{x-1}{4}$	$\frac{5}{x-2}-\frac{2x}{x^2-4}=1$	$\frac{6}{5x^2-125}-7=\frac{x+5}{x-5}$

To solve a rational equation, you must isolate the unknown variable, just as you would to solve any other equation. Below is a list of steps that you can use to solve for rational equations.

Solving Rational Equations	
Step	**Details**
Step 1: Identify the non-permissible values.	• Factor the expressions in each denominator. • Equate each factor in the denominators to 0. • Solve each factor to determine the non-permissible values.
Step 2: Find the LCD and clear the denominators by multiplying each fraction by the LCD.	• Determine the binomial factors required in the LCD that will clear all denominators.
Step 3: Determine the roots of the simplified equation.	• Multiply both sides of the equation by the LCD. • Simplify the equation. • Factor the equation. • Determine the roots of the factored equation.
Step 4: Check for extraneous roots, and check the solution.	• Compare each root to the non-permissible values. • Remove any extraneous roots from the solution. • Check the solution by inserting the value of the root into the original equation.

Example:

$$\frac{3}{3x+2}=\frac{6}{5x}$$

Solution:

The non-permissible values are $x \neq 0, \frac{-2}{3}$.

Determine the lowest common denominator. LCD = $(3x+2)(5x)$

Multiply each term by the LCD.

$$\frac{(3x+2)(5x)}{1}\left(\frac{3}{3x+2}\right)=\frac{(3x+2)(5x)}{1}\left(\frac{6}{5x}\right)$$

Multiply in and simplify both sides of the equation. This will result in an equation with no denominators.

$$\frac{\cancel{(3x+2)}(5x)}{1}\left(\frac{3}{\cancel{3x+2}}\right)=\frac{(3x+2)\cancel{(5x)}}{1}\left(\frac{6}{\cancel{5x}}\right)$$

$$3(5x)=6(3x+2)$$

Simplify and solve the linear equation.

$$15x = 18x + 12$$
$$-3x = 12$$
$$x = -4$$

Check for extraneous solutions. Since -4 is not a non-permissible value it is a solution to the equation.

Verify your answer.

$$\frac{3}{3x+2} = \frac{6}{5x}$$
$$\frac{3}{3(-4)+2} = \frac{6}{5(-4)}$$
$$\frac{3}{-12+2} = \frac{6}{-20}$$
$$\frac{3}{-10} = \frac{6}{-20}$$
$$-0.3 = -0.3$$

Since the LHS = RHS, the solution *x* = -4 is correct.

Example:

Solve the rational equation $\frac{1}{x+2}=\frac{x-1}{4}$

Solution:

$x \neq -2$ Determine the non-permissible value(s).

$4(x+2)$ Determine the lowest common denominator.

Multiply the expression on each side of the equation by the LCD.

$$\frac{1}{x+2}=\frac{x-1}{4},\ x \neq -2$$

- $$4(x+2)\left(\frac{1}{x+2}\right)=4(x+2)\left(\frac{x-1}{4}\right)$$

$$4\cancel{(x+2)}\left(\frac{1}{\cancel{x+2}}\right)=\cancel{4}(x+2)\left(\frac{x-1}{\cancel{4}}\right)$$

$$4=(x+2)(x-1)$$

Solve the resulting equation for *x*. **Hint:** To verify your answer, substitute the value of *x* into the original equation.

$$4 = (x+2)(x-1)$$
$$4 = x^2 + x - 2$$
$$0 = x^2 + x - 6$$
$$0 = (x+3)(x-2),\ x \neq -2$$

So $x + 3 = 0$, which means $x = -3$; or $x - 2 = 0$, which means $x = 2$.

Check to see if your answers are NOT non-permissible values. If they are those solutions would be considered extraneous. In this example the solutions $x = -3$ and $x = 2$ are not the non-permissible value of $x \neq -2$. The solutions are good.

Example:

Solve the rational equation $$\frac{2m}{m-1}+\frac{m-5}{m^2-1}=1$$

Solution:

First step is to factor.

$$\frac{2m}{m-1}+\frac{m-5}{(m+1)(m-1)}=1$$

Determine the non-permissible values.

$m \neq \pm 1$

Find the lowest common denominator (LCD)

LCD = $(m - 1)(m + 1)$

Multiply each term on both sides of the equation by the LCD and cancel common factors.

$$\left(\frac{(m+1)(\cancel{m-1})}{1}\right)\left(\frac{2m}{\cancel{m-1}}\right)+\left(\frac{(\cancel{m+1})(\cancel{m-1})}{1}\right)\left(\frac{m-5}{(\cancel{m+1})(\cancel{m-1})}\right)=\left(\frac{(m+1)(m-1)}{1}\right)\left(\frac{1}{1}\right)$$

Multiply the remaining rational expressions.

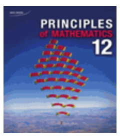
$$2m(m+1)+(m-5)=(m+1)(m-1)$$
$$2m^2+2m+m-5=m^2-1$$

Since we have a quadratic equation, manipulate the equation by bringing all terms to one side and solving by factoring.

$$2m^2 - m^2 + 2m + m - 5 + 1 = 0$$
$$m^2 + 3m - 4 = 0$$
$$(m+4)(m-1) = 0$$

The solutions are m = -4 or m = 1. We must check to see if our solutions are extraneous. Since one of our non-permissible values is, m = 1 is extraneous. Therefore the only solution is m = - 4.

Definition: An **extraneous solution** is a solution that is a non-permissible value of the original rational equation.



Practice Problem:

Complete “Check your Understanding” question 5a and b on page 258 of your textbook.

Solution:

5. a) Denominator:

$x - 1 = 0$

$x = 1$

The non-permissible value for this equation is $x = 1$.

$$\frac{x+3}{x-1} = 0$$

$$\frac{(x+3)(x-1)}{x-1} = 0(x-1)$$

$$x + 3 = 0$$

$$x = -3$$

Verify: Let $x = -3$

$$\frac{(-3)+3}{(-3)-1} = 0$$

$$\frac{0}{-4} = 0$$

$$0 = 0$$

$$\text{LS} = \text{RS}$$

Solution:

b) Denominator:

$x - 1 = 0$

$x = 1$

The non-permissible value for this equation is $x = 1$.

$$\frac{x+3}{x-1} = 2$$

$$\frac{(x+3)(x-1)}{x-1} = 2(x-1)$$

$$x+3 = 2x-2$$

$$x-2x = -2-3$$

$$-x = -5$$

$$x = 5$$

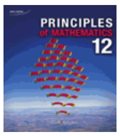
Verify: Let $x = 5$

$$\frac{5+3}{5-1} = 2$$

$$\frac{8}{4} = 2$$

$$2 = 2$$

$$\text{LS} = \text{RS}$$



Practice Problem:

Complete “Check your Understanding” question 7 on page 259 of your textbook.

Solution:

7. Let $l = 15$

Denominator:

$w = 0$ or $15 - w = 0$

$w = 0$ or $w = 15$

The non-permissible values for this equation are $w = 0$ and $w = 15$. In context, w must be greater than 0, because it represents a length.

$$\frac{15}{w} = \frac{w}{15 - w}$$

$$\frac{15w(15 - w)}{w} = \frac{w^2(15 - w)}{15 - w}$$

$$15(15 - w) = w^2$$

$$225 - 15w = w^2$$

$$0 = w^2 + 15w - 225$$

$$w = \frac{-15 \pm \sqrt{15^2 - 4(1)(-225)}}{2}$$

$$w = \frac{-15 \pm \sqrt{225 + 900}}{2}$$

$$w = \frac{-15 \pm \sqrt{1125}}{2}$$

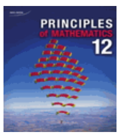
$$w = \frac{-15 \pm 33.541...}{2}$$

$$w = \frac{-15 + 33.541...}{2} \text{ or } w = \frac{-15 - 33.541...}{2}$$

$$w = \frac{18.541...}{2} \text{ or } w = \frac{-48.541}{2}$$

$$w = 9.270... \text{ or } w = -24.270...$$

However, $w = -24.270...$ is an inadmissible value because it is negative. Therefore, the only solution is $w = 9.270...$. The width of the poster will be about 9.27 m.



Practice Problem:

Complete “Check your Understanding” question 10 on page 259 of your textbook.

Solution:

$$\frac{1}{3}+\frac{1}{2}=\frac{1}{t}$$

$$\frac{2}{6}+\frac{3}{6}=\frac{1}{t}$$

$$\frac{5}{6}=\frac{1}{t}$$

$$\frac{5t}{6}=\frac{t}{t}$$

$$\frac{5t}{6}=1$$

$$5t=6$$

$$t=\frac{6}{5}$$

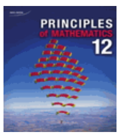
Let t represent the time it would take both Susan and Jacqueline to paint a room if they worked together, in hours.

The non-permissible value for this equation is t = 0. In context, t must be greater than 0, because it represent time.

$$\frac{6}{5}\text{ hours} = \frac{6}{5}\cdot 60\text{ minutes}$$

$$= 72\text{ minutes}$$

It would take Susan and Jacqueline 72 minutes or 1 hour and 12 minutes to paint a room if they worked together.



Practice Problem: (KEY QUESTION)

Complete “Check your Understanding” question 12 on page 259 of your textbook.

Solution:

12. Let t represent the time it would take for Enzo to build an outdoor patio by himself, in hours. Let $t + 9$ represent the time it would tale for Rahj to build an outdoor patio by himself, in hours.
The non-permissible values for this equation are $t = 0$, and $t = -9$. However, in context, t must be greater than 0, because it represents a time.

$$\frac{1}{t}+\frac{1}{t+9}=\frac{1}{20}$$

$$\frac{20t(t+9)}{t}+\frac{20t(t+9)}{t+9}=\frac{20t(t+9)}{20}$$

$$20(t+9)+20t=t(t+9)$$

$$20t+180+20t=t^2+9t$$

$$40t+180=t^2+9t$$

$$0=t^2+9t-40t-180$$

$$0=t^2-31t-180$$

$$0=(t+5)(t-36)$$

$$t=-5 \text{ or } t=36$$

However. $t = -5$ is an inadmissible solution, because it is negative.

Verify: Let t = 36

$$\frac{1}{(36)}+\frac{1}{(36)+9}=\frac{1}{20}$$

$$\frac{1}{36}+\frac{1}{45}=\frac{1}{20}$$

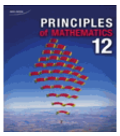
$$\frac{5}{36(5)}+\frac{4}{45(4)}=\frac{1}{20}$$

$$\frac{9}{180}=\frac{1}{20}$$

$$\frac{1}{20}=\frac{1}{20}$$

$$\text{LS}=\text{RS}$$

It would take Enzo 36 h and Rahj 45 h to build outdoor patios by themselves.



Practice Problem:

Complete "Check your Understanding" question 19 on page 260 of your textbook.

Solution:

19. Erin and Dawn are sisters. It takes Erin 20 min longer to wash down their backyard fence than it takes Dawn. Together, it takes the sisters 24 min to finish the fence. How long would it take each sister to complete the task on her own?

Solution: Let t represent the amount of time it would take for Dawn to wash down the backyard fence, in minutes, and let $t + 20$ represent the amount of time it would take for Erin to wash down the backyard fence, in minutes.

The non-permissible values for this equation are $t = 0$ and $t = -20$. In context, t must be greater than 0, because it represents a time.