Math 30-2: U6L2 Teacher Notes

Relating the Characteristics of an Exponential Function to Its Equation

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Describe, orally and in written form, the characteristics of exponential functions by analyzing their graphs.
- Match equations to a given set to their corresponding graphs

How Does the Parameters "a" and "b" Affect the Exponential Function

From the Investigation in the Assessment for Learning Questions you should have discovered the following:

Let's use the information to answer some questions on relating these characteristics to its equation.

| Parameter	Effect on Graph
a	equal to y-intercept
b	If $b > 1$, the function is increasing. If $0 < b < 1$, the function is decreasing.

Example:

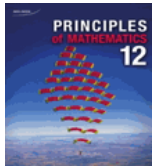Describe the characteristics of the exponential function .

$$y = 2(5)^x$$

Solution:

From the graph, note the following:

- The exponential graph never touches the *x*-axis; therefore, there is no *x*-intercept.

- The *y*-intercept is 2.

- The end behaviour is from quadrant II to quadrant I; the graph is increasing.

- The domain is $\{x \mid x \in R\}$

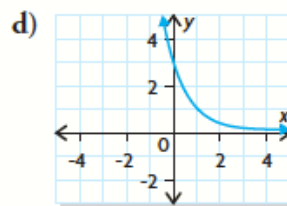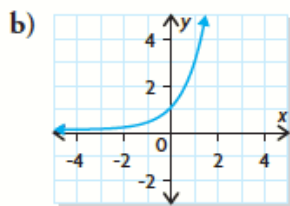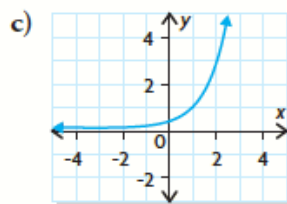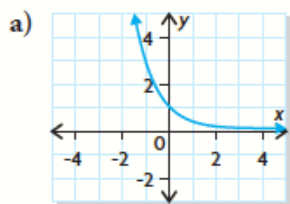- The range is $\{y \mid y > 0, y \in R\}$
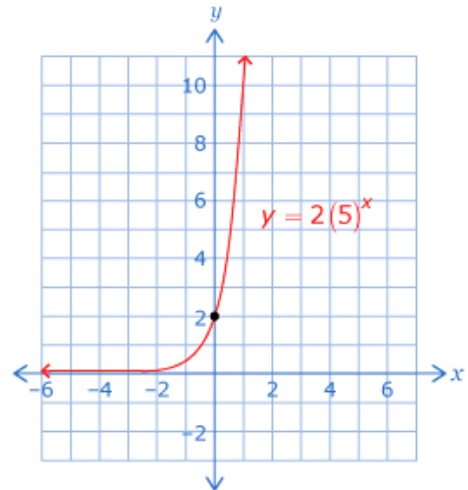
Example:

Turn to Page 340 of your textbook and do Example 1. The symbol *e* or Euler's number will be important in Unit 7.

Example:

Page 345: *Your Turn*

Match each function with the corresponding graph below. Provide your reasoning.

i) $y = (3)^x$ ii) $y = \frac{1}{3}(3)^x$ iii) $y = 3\left(\frac{1}{3}\right)^x$ iv) $y = \left(\frac{1}{3}\right)^x$

a)

c)

b)

d)

Solution:

a) iv -- Since *a* = 1, the y-intercept is 1. Since $0 < b < 1$ it is a decreasing function.

b) i -- Since *a* = 1, the y-intercept is 1. Since $b > 1$ it is an increasing function.

c) ii -- Since *a* = 1/3, the y-intercept is 1/3. Since $b > 1$ it is an increasing function.

d) iii -- Since *a* = 3, the y-intercept is 3. Since $0 < b < 1$ it is a decreasing function.

Example:

Click the icon to take you through an example of Matching Equations to Graphs.

PRINCIPLES of MATHEMATICS 12
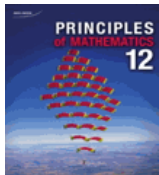
Practice Problem:

Complete "Practising" question 5 on page 347 of your textbook.

Solution:

5. a) i) Yes, an exponential equation can be used to model the function because the rate of change in y-values doubles for each unit increase in x.
ii) y-intercept: $y = 1$, the function is increasing.
b) i) No, y increases by 2 as x increases by 1.
ii) y-intercept: $y = 3$, the function is increasing.
c) i) Yes, an exponential equation can be used to model the function because the rate of change in y-values get divided by 4 for each unit increase in x.
ii) y-intercept: $y = 64$, the function is decreasing.
d) i) No. e.g., y decreases, then increases, then decreases again.
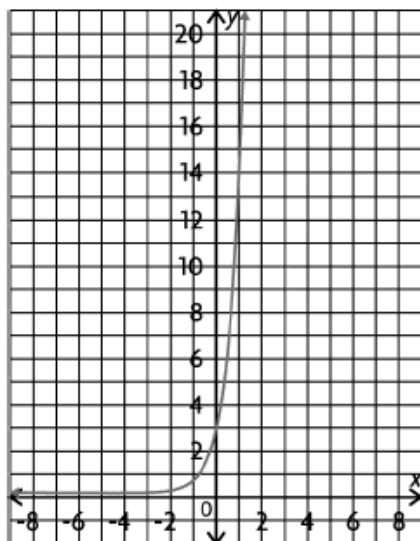ii) y-intercept: $y = 1$, the function is first decreasing than increasing as it reaches the y-intercept.
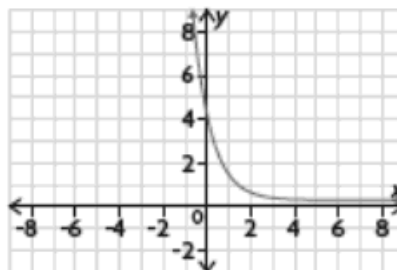
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of MATHEMATICS
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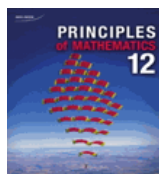
Practice Problem:

Complete "Practising" question 6 on page 347 of your textbook.

Solution:

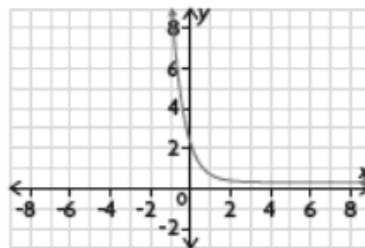**6. a)** Number of *x*-intercepts: 0
y-intercept: $y = 3$; Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI

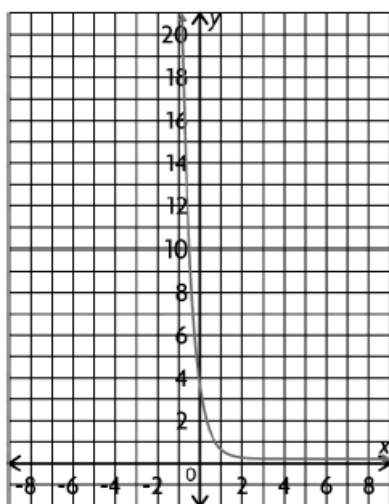**b)** Number of *x*-intercepts: 0; *y*-intercept: $y = 4$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI

c) Number of *x*-intercepts: 0; *y*-intercept: $y = 2$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI

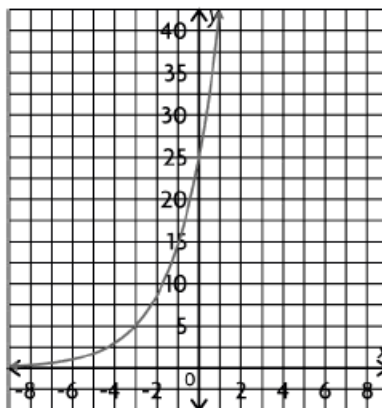**d)** Number of *x*-intercepts: 0; *y*-intercept: $y = 3.5$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI

e) Number of *x*-intercepts: 0; *y*-intercept: $y = 25$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI

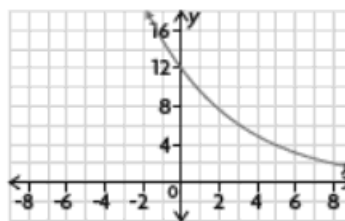**f)** Number of *x*-intercepts: 0; *y*-intercept: $y = 12$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
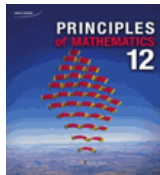
Practice Problem:

Complete "Practising" question 7 on page 348 of your textbook.

Solution:

7. a) The *y*-intercept is positive and since the base is 8, it must mean the function is increasing because the base is larger than 1.
b) The *y*-intercept is positive and since the base is 0.6, it must mean the function is decreasing because the base is less than 1.
c) The *y*-intercept is positive and since the base is *e*, it must mean the function is increasing because *e* is greater than 1.

PRINCIPLES of MATHEMATICS 12

Practice Problem:

Complete "Practising" question 8 on page 348 of your textbook.

Solution:

8. a) Number of *x*-intercepts: 0; *y*-intercept: $y = 4$
Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y > 0, y \in \mathbb{R}\}$
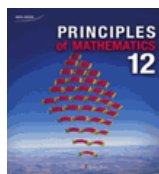End Behaviour: QII to QI
b) Number of *x*-intercepts: 0; *y*-intercept: $y = 8$
Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y > 0, y \in \mathbb{R}\}$
End Behaviour: QII to QI
c) Number of *x*-intercepts: 0; *y*-intercept: $y = 3$
Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y > 0, y \in \mathbb{R}\}$
End Behaviour: QII to QI
d) Number of *x*-intercepts: 0; *y*-intercept: $y = 10$
Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y > 0, y \in \mathbb{R}\}$
End Behaviour: QII to QI

e) Number of *x*-intercepts: 0; *y*-intercept: $y = 30$
Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y > 0, y \in \mathbb{R}\}$
End Behaviour: QII to QI
f) Number of *x*-intercepts: 0; *y*-intercept: $y = 1$
Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y > 0, y \in \mathbb{R}\}$
End Behaviour: QII to QI
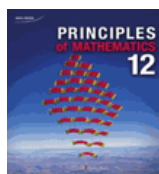**g)** Number of *x*-intercepts: 0; *y*-intercept: $y = 3$
Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y > 0, y \in \mathbb{R}\}$
End Behaviour: QII to QI
h) Number of *x*-intercepts: 0; *y*-intercept: $y = 45$
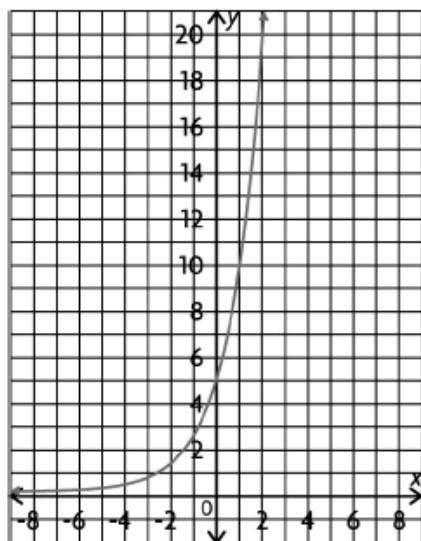Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y > 0, y \in \mathbb{R}\}$
End Behaviour: QII to QI

PRINCIPLES of MATHEMATICS **12**
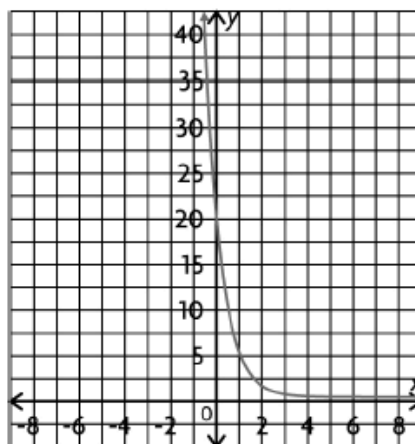
Practice Problem:

Complete "Practising" question 10 on page 348 of your textbook.
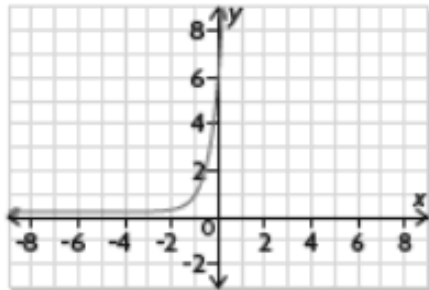
Solution:

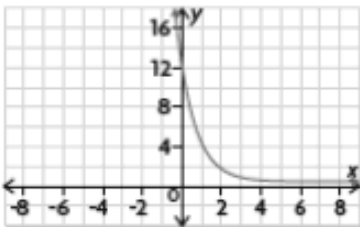**10. a)** It is an increasing function because its base is more than 1.

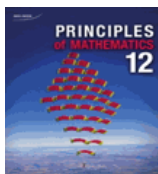**b)** It is a decreasing function because its base is less than 1.

c) It is an increasing function because its base is more than 1.

d) It is a decreasing function because its base is less than 1.

Practice Problem: (KEY QUESTION)

Complete "Practising" question 13 on page 349 of your textbook.

Solution:

13. a) i) Range: $\{y \mid y > 0, y \in R\}$ *a*: 2 *b*: 0.5 This
is a decreasing function.
ii) Range: $\{y \mid y > 0, y \in R\}$ *a*: 1 *b*: 3
This is an increasing function.
iii) Range: $\{y \mid y > 0, y \in R\}$ *a*: 3 *b*: 0.5
This is a decreasing function.
iv) Range: $\{y \mid y > 0, y \in R\}$ *a*: 2 *b*: 4
This is an increasing function.

b) i) This function matches with the graph B.
because it is a decreasing function with a
y-intercept of 2.
ii) This function matches with graph D because it
is an increasing function with a *y*-intercept of 1.
iii) This function matches with the graph A.
because it is a decreasing function with a
y-intercept of 3.
iv) This function matches with the graph C.
because it is an increasing function with a
y-intercept of 2.
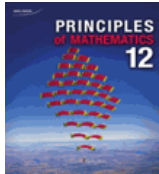
Practice Problem:

Complete "Practising" question 17 on page 350 of your textbook.

Solution:

17. The number of *x*-intercepts, the end behaviour, the domain, and the range are common to all exponential functions. The *y*-intercept and whether the function increases or decreases are unique to the function.