Math 30-2: U6L4 Teacher Notes

Modelling Data Using Exponential Functions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Graph data and determine the exponential function that best approximates the data.
- Interpret the graph of a exponential function that models a situation, and explain the reasoning.
- Solve, using technology, a contextual problem that involves data that is best represent by graphs of exponential functions, and explain the reasoning.

What is Exponential Regression?

Each morning a teacher reads the newspaper. He does this with a nice fresh cup of coffee. Many mornings, however, he becomes so focused on his newspaper that the coffee becomes cold and unpleasant to drink.

The following data shows the temperature of the cup of coffee as it sits cooling.

Time (in min)	Temperature (°C)
0	90
5	71
10	52
15	34
20	30
25	27
30	25

a. Use your graphing calculator to plot the data.

b. Is the temperature increasing or decreasing?

c. What is the shape of your scatter plot? What kind of model do you expect to use with this data?

d. Use your calculator to find a regression equation.

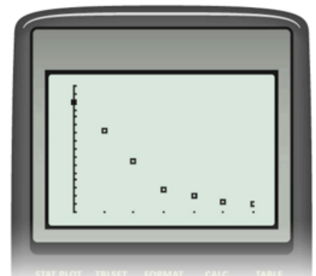
e. Graph the regression equation on the same graph as the scatter plot.

f. What is an appropriate domain and range for this situation?

November 19, 2012

Solution:

a. Use your graphing calculator to plot the data. **Hint:** If you have difficulty creating a scatter plot, refer to your calculator manual or contact your teacher.

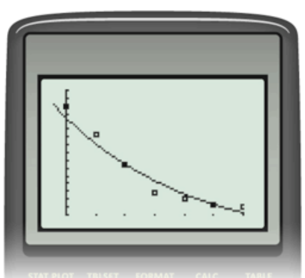


b. The temperature is decreasing.

c. The scatter plot is in the shape of exponential decay. An exponential model will be reasonable for this data.

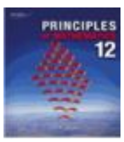
d. The regression equation is an exponential regression: $y = a(b)^x$,
$y = 82.633(0.9558)^x$

e.

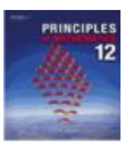


f. An appropriate domain could be $0 \le x \le 30$, where *x* represents time; and an appropriate range could be $25 \le y \le 90$, where *y* represents temperature.

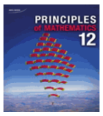
This is the form of exponential equation you saw in Lesson 1 ($y = ab^x$, $b > 0$, $b \neq 1$). The data points lie close to this function; therefore, it is a good model for the data.



Read "Example 1" on pages 371 to 373 of your textbook to see how an exponential regression can be used to make predictions using interpolation and extrapolation. In this problem, time is inputted as 10-year increments instead of using the years listed in the table. Typically, when years (such as 1871 or 1992) are used when performing a regression, the resulting equation is very sensitive to rounding. To avoid this, the number of years past a time is often used for regressions. In this example, 10-year increments are used.



Read and note the section titled "Key Idea" on the bottom of page 376. This provides a summary of exponential regression.

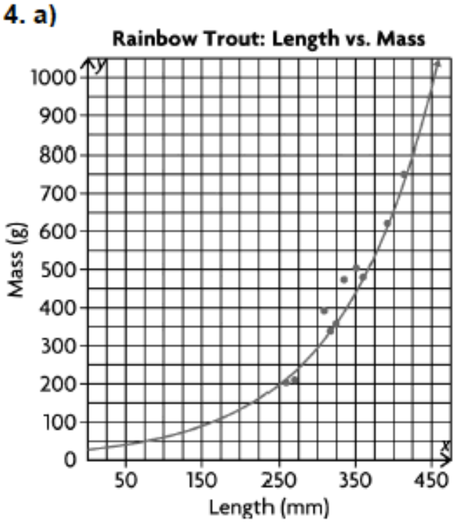


Practice Problem:

Complete “Practising” question 4 on page 378 of your textbook.

Solution:

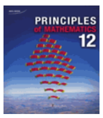
4. a)



b) Using a graphing calculator, the exponential regression function for the data is $y = 26.934\ldots(1.008\ldots)^x$.

c) e.g., 690 g; I identified the point on the curve that had an *x*-value of 400.

d) I would expect this fish to be about 446 mm long. I identified the point on the curve that had an *y*-value of 1000.

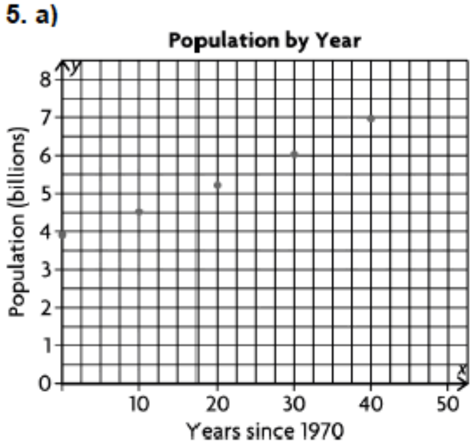


Practice Problem:

Complete “Practising” question 5 on page 378 of your textbook.

Solution:

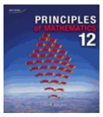
5. a)



b) Using a graphing calculator, the exponential regression function for the data is $y = 3.911\ldots(1.014\ldots)^x$.

c) The population is estimated to be 8.05 billion. To get this number, 50 (for 2020) was substituted for x in the regression equation so that y could be determined.

d) The population is expected to reach 9.50 billion 61.5 years after 1970, that is, during the year 2031. I plotted $y = 9.50$ and the regression function on a graphing calculator. The x-coordinate of the point at which these two functions intersect is the point at which the population is expected to reach 9.50 billion.



Practice Problem:

Complete “Practising” question 6 on page 378 of your textbook.

Solution:

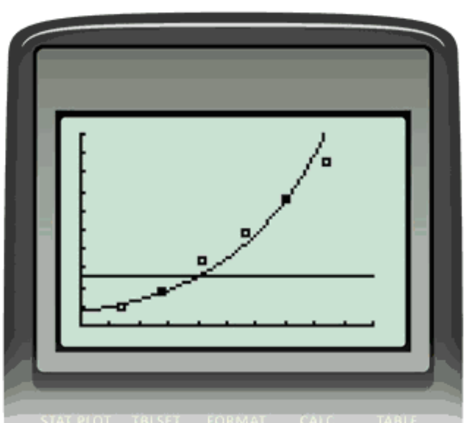
a. $y = 14.4295...(1.0651...)^x$, where x is the day and y is the height

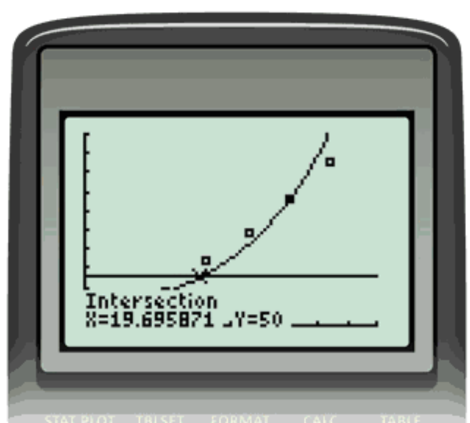
b. $y = 14.4295...(1.0651...)^x$
$= 14.4295...(1.0651...)^{10}$
≈ 27.1 cm

c. $y = 14.4295...(1.0651...)^x$
$= 14.4295...(1.0651...)^{30}$
≈ 95.8 cm

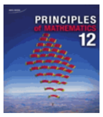
This answer does not make sense because it means the height of the sunflower decreased. (The table of values show the sunflower was 98.0 cm tall after only 28 days.)

d.





From the intersection of the graphs, the sunflower would reach a height of 50 cm on day 19.695..., which means on day 20. On day 19, the height would not be at 50 cm.

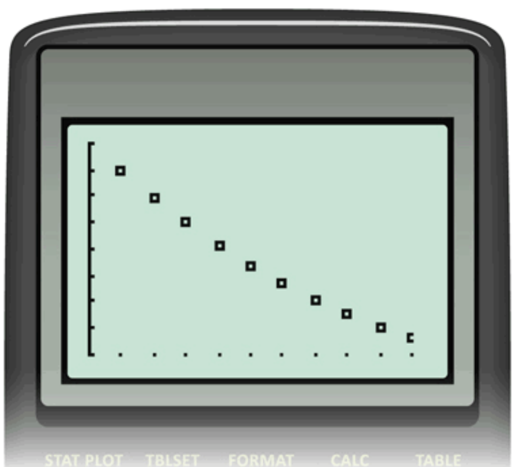


Practice Problem:

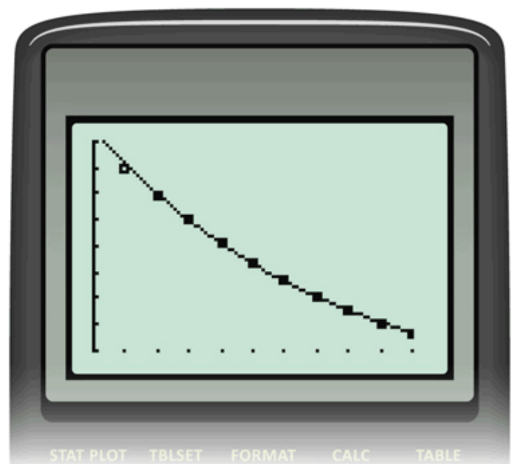
Complete “Practising” question 8 on page 379 of your textbook.

Solution:

a.



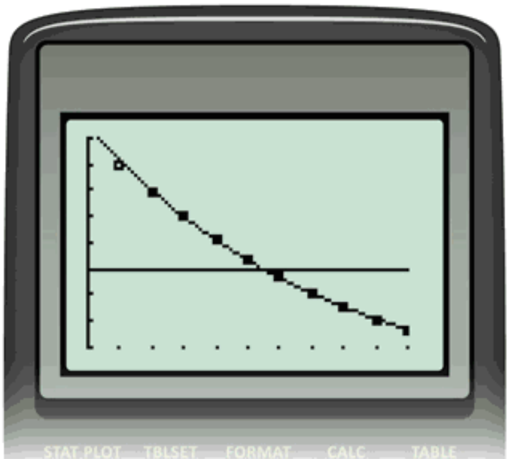
b.

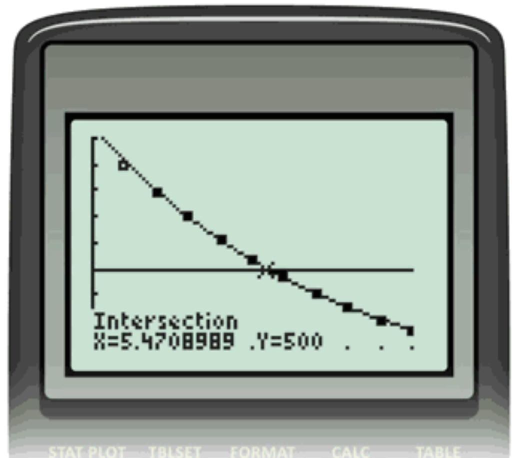


c.

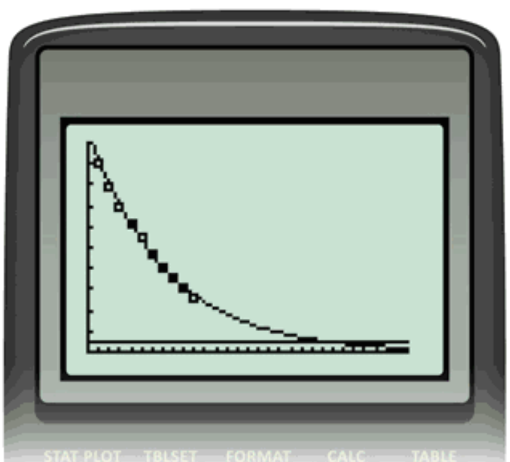
$$y = 1050.311\ldots(0.873\ldots)^x$$
$$= 1050.311\ldots(0.873\ldots)^{15}$$
$$\approx 137.2 \text{ mb}$$

d.

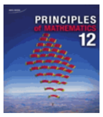




When the pressure is 500 mb, the altitude is approximately 5 km.



When the air pressure is 50 mb, the altitude is approximately 22 km.

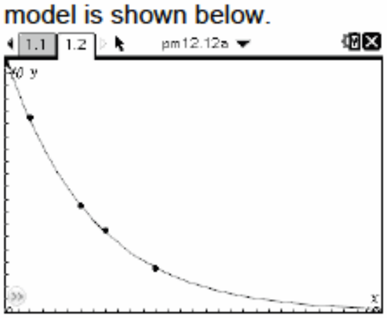


Practice Problem:

Complete “Practising” question 12 on page 381 of your textbook.

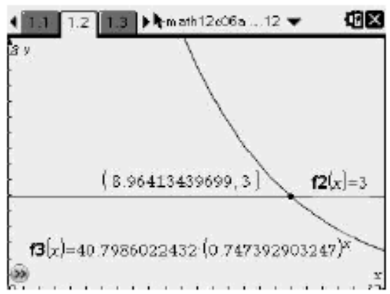
Solution:

12. a) Using graphing technology to determine the regression function, a graphical exponential model is shown below.

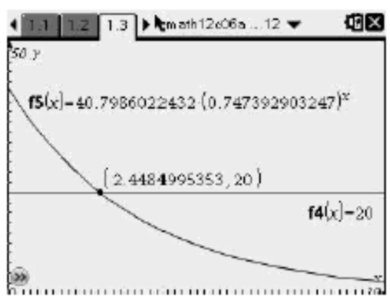


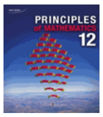
A function that models this situation algebraically is $y = 40.799\ldots(0.747\ldots)^x$.

b) Tristan will have less than 3 mg of caffeine in his body after 9 hours.



c) Half the initial amount of caffeine in Tristan’s body is 20 mg. The half-life of caffeine in Tristan’s body is about 2 h.





Practice Problem:

Complete "Closing" question 7 on page 383 of your textbook.

Solution:

17. a) e.g., Calculate the rate of change in the decrease (*b*) and use 100 for *a*. Use software or a calculator to perform exponential regression to get those values.

b) Using the method from part a) to create an algebraic model, we get our equation as so:

$97 = 100b^1$

$b = 0.97$

$y = 100(0.97)^x$

Now substitute 7 for *x*:

$y = 100(0.97)^7$

$y = 100(0.807\ldots)$

$y = 80.798\ldots\%$

The light intensity would be 80.80%.

c) $100 - 60 = 40$

The light intensity has to go below 40%.

The minimum of gels that would be needed is 31.