# Math 30-2: U6L3 Teacher Notes

## Solving Exponential Equations

## Key Math Learnings:

**By the end of this lesson, you will learn the following concepts:**

- Determine the solution of an exponential equation in which the bases are powers of one another.

## Solving Exponential Equations with a Common Base

Now there are two different types of exponential equations. Equations where:

a) the bases can be changed into the same base - we will cover this type below

b) the bases cannot be changed into the same base - for this type we will have to use logarithms to solve the equation, and will therefore learn how to do this, later in the course.

To solve exponential equations without logarithms, you need to have equations with comparable exponential expressions on either side of the "equals" sign, so you can compare the powers and solve. In other words, you have to have "(some base) to (some power) equals (the same base) to (some other power)", where you set the two powers equal to each other, and solve the resulting equation.

example 3) solve for x in the equation $3^{x+1} = 27$

Solution:

$$3^{x+1} = 27$$

$3^{x+1} = 3^3$ express 27 as a base of 3

$x + 1 = 3$ let the exponents equal each other, and solve the linear equation

$$x = 2$$

**Example 4) Solve for x in the following equation**

**Solution:**

$$2^x = 8^{x+1}$$

**Steps**

1. change the base in each power to the same base

* can be changed to base 2 because $8=2^3$

therefore:

$$2^x = \left(2^3\right)^{x+1}$$

2. multiply out any exponents

make sure the 3 inside the brackets is multiplied by both terms in the exponent outside the brackets

$$2^x = 2^{3x+3}$$

$$x = 3x+3$$

3. let exponents equal each other

$$-2x = 3$$

4. solve for x in the new equation

$$\boxed{x = -\frac{3}{2}}$$

**Example 5) Solve for x in the following equation:**

**Solution:**

$9^{4x+2} = 27^{x-1}$ 1. change bases to same base

both bases can
change to base 3

$(3^2)^{4x+2} = (3^3)^{x-1}$ 2. multiply inside exponents with outside exponents

$3^{8x+4} = 3^{3x-3}$

$8x + 4 = 3x - 3$ 3. let exponents equal each other

$5x = -7$ 4. solve linear equation

$x = -\frac{7}{5}$

**Example 7) Solve the following equation for x**

**Solution:**

$$3(4^{x+1}) = 24$$

$$\frac{3\left(4^{x+1}\right)}{3} = \frac{24}{3}$$ isolate the power with the variable in the exponent, divide both sides by 3

$$\left(4^{x+1}\right) = 8$$

$$2^{2(x+1)} = 2^3$$ change both sides to the same base. A common mistake here would be to think that you could change them both to base 4, but there is no power of 4 that equals 8. However, both 4 and 8 are powers of 2.

$$2x + 2 = 3$$ solve the linear equation

$$2x = 1$$

$$x = \frac{1}{2}$$

**Example 8) Solve the following equation for x**

**Solution:**

$$6 \bullet 2^x = 6$$

$$\frac{6 \bullet 2^x}{6} = \frac{6}{6}$$ isolate the power with the variable in the exponent, divide both sides by 6

we must change both sides to base 2. What exponent can you put on a 2 that will result in a value of 1?

$$2^x = 1$$

Remember that an exponent of zero, will make the power equal 1.
ie) $2^0=1$

$$2^x = 2^0$$ let the exponents equal each other.

$$x = 0$$

**Example:**

The medical isotope iodine-131 is produced at Chalk River Laboratories in Ontario and is used in imaging and diagnosing thyroid problems. A radioactive sample of iodine-131 has a half-life of 8 days. This means that after 8 days, half of the original amount of the sample has decayed.

The equation that can be used to describe the half-life function is

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{8}}$$

where $A$ is the remaining mass of iodine-131, $A_0$ is the original mass of iodine-131, and $t$ is the time in days.

a. Notice that in the function, $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{8}}$ the value of parameter $b$ is $\frac{1}{2}$ . Can you determine if this is exponential growth or exponential decay? Explain your reasoning.

b. How long would it take a 4.0-g sample of iodine-131 to decay to 0.25 g? Determine the time algebraically.

c. Determine the time using a graph. Describe the process you used.

***Solution:***

a. It is a exponential decay since b is between 0 and 1.

b. Write both sides of the equation to have the same base.

$$A = A_o\left(\frac{1}{2}\right)^{\frac{t}{8}}$$

$$0.25 = 4.0\left(\frac{1}{2}\right)^{\frac{t}{8}}$$

$$\frac{0.25}{4.0} = \frac{\cancel{4.0}\left(\frac{1}{2}\right)^{\frac{t}{8}}}{\cancel{4.0}}$$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{t}{8}}$$

$$\frac{1}{2^4} = \left(\frac{1}{2}\right)^{\frac{t}{8}}$$

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\frac{t}{8}}$$

Because the bases are now the same, you can do the following:

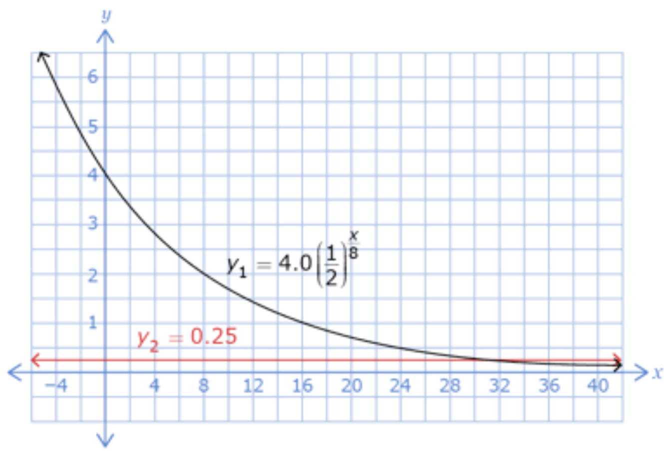
$$4 = \frac{t}{8}$$
$$t = 4(8)$$
$$t = 32$$

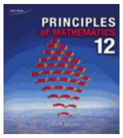
Now substitute the answer into both sides of the equation to verify the solution.

| LS | RS |
|---|---|
| 0.25 | $4.0\left(\frac{1}{2}\right)^{\frac{t}{8}}$ |
| | $4.0\left(\frac{1}{2}\right)^{\frac{32}{8}}$ |
| | $4.0\left(\frac{1}{2}\right)^{4}$ |
| | $4.0\left(\frac{1}{16}\right)$ |
| | $\frac{1}{4}$ |
| | 0.25 |
| LS = RS | |

Click the icon to watch a T4T video on how to solve.

c. This problem could be solved by graphing $y_1 = 4.0\left(\frac{1}{2}\right)^{\frac{x}{8}}$ and $y_2 = 0.25$ and then finding the point of intersection of the two graphs. The solution will be the $x$-value of the point of intersection. From the graph, the point of intersection is (32, 0.25); thus, the solution is 32.






**Practice Problem:**

Complete “Practising” question 4 on page 362 of your textbook.

***Solution:***

**4. a)**

$$\begin{aligned} 10^x &= 1000 \\ 10^x &= 10^3 \\ x &= 3 \end{aligned}$$

**b)**

$$\begin{aligned} 2^{-t} &= 32 \\ 2^{-t} &= 2^5 \\ -t &= -5 \\ t &= -5 \end{aligned}$$

**c)**

$$\begin{aligned} 3^n &= \frac{1}{81} \\ 3^n &= \frac{1}{3^4} \\ 3^n &= 3^{-4} \\ n &= -4 \end{aligned}$$

**d)**

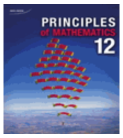
$$\begin{aligned} 3^{3x+1} &= 81 \\ 3^{3x+1} &= 3^4 \\ 3x+1 &= 4 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

**e)**

$$\begin{aligned} 2^{x+2} &= (8)^{\frac{1}{2}} \\ 2^{x+2} &= (2^3)^{\frac{1}{2}} \\ 2^{x+2} &= 2^{\frac{3}{2}} \\ x+2 &= \frac{3}{2} \\ x &= \frac{3}{2} - \frac{4}{2} \\ x &= -\frac{1}{2} \end{aligned}$$

**f)**

$$\begin{aligned} 4^{3x} &= \frac{1}{16} \\ 4^{3x} &= 4^{-2} \\ 3x &= -2 \\ x &= \frac{2}{3} \end{aligned}$$

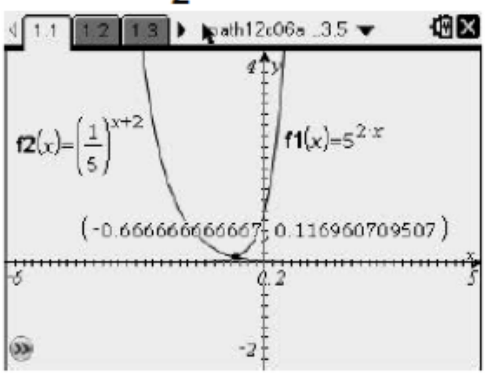



**Practice Problem:**

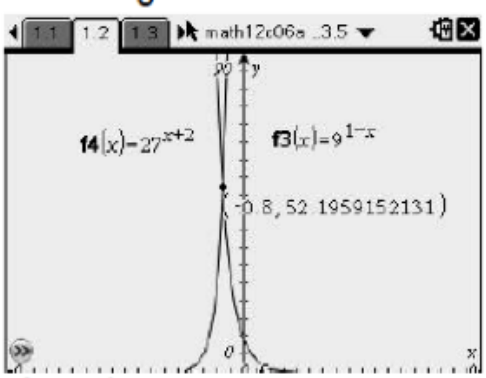
Complete “Practising” question 5 on page 362 of your textbook.

***Solution:***

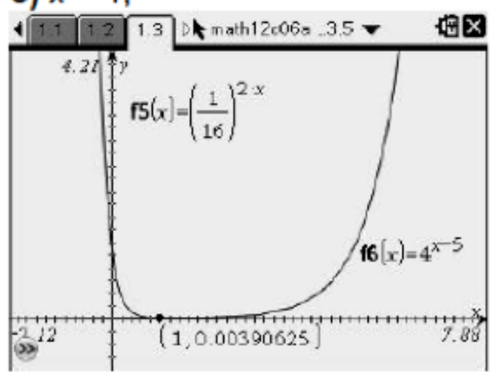
**5. a)** $x = -\frac{3}{2}$

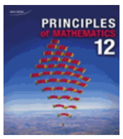



**b)** $x = -\frac{4}{5}$




**c)** $x = 1$,






**Practice Problem:**

Complete "Practising" question 6 on page 362 of your textbook.

***Solution:***

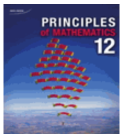
**6. a)** e.g., Using a common base of 8 for both sides:

$$8^{3x-2} = 64^{x+1}$$
$$8^{3x-2} = (8^2)^{x+1}$$
$$8^{3x-2} = 8^{2x+2}$$
$$3x - 2 = 2x + 2$$
$$x = 4$$

**b)** e.g., Using a common base of 2 for both sides:

$$8^{3x-2} = 64^{x+1}$$
$$(2^3)^{3x-2} = (2^6)^{x+1}$$
$$9^{9x-6} = 2^{6x+6}$$
$$9x - 6 = 6x + 6$$
$$3x = 12$$
$$x = 4$$

Yes, you can because the answer is the same one as in part a).




**Practice Problem:**

Complete “Practising” question 7 on page 362 of your textbook.

***Solution:***

**7. a)** $3^{x+1}=9$

$$3^{x+1}=3^2$$
$$x+1=2$$
$$x=1$$

$$8(3)^{\frac{x}{2}}=72$$
$$8(3)^{\frac{x}{2}}=8(9)$$

**b)** $3^{\frac{x}{2}}=3^2$

$$\frac{x}{2}=2$$
$$x=4$$

**c)** $5^{3x+2}=5$

$$3x+2=1$$
$$3x=-1$$
$$x=-\frac{1}{3}$$

**d)** $\sqrt{8}=2^{4x+1}$

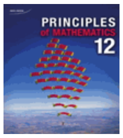
$$8^{\frac{1}{2}}=2^{4x+1}$$
$$(2^3)^{\frac{1}{2}}=2^{4x+1}$$
$$2^{\frac{3}{2}}=2^{4x+1}$$
$$\frac{3}{2}=4x+1$$
$$\frac{3}{2}-\frac{2}{2}=4x$$
$$\frac{1}{2}=4x$$
$$\frac{1}{8}=x$$

**e)** $4^{1-2x}=32$

$$(2^2)^{1-2x}=2^5$$
$$2^{2-4x}=2^5$$
$$2-4x=5$$
$$-4x=3$$
$$x=-\frac{3}{4}$$

**f)** $10^{2(x-3)}=1000$

$$10^{2(x-3)}=10^3$$
$$2x-6=3$$
$$2x=9$$
$$x=\frac{9}{2}$$




**Practice Problem:**

Complete “Practising” question 9 on page 363 of your textbook.

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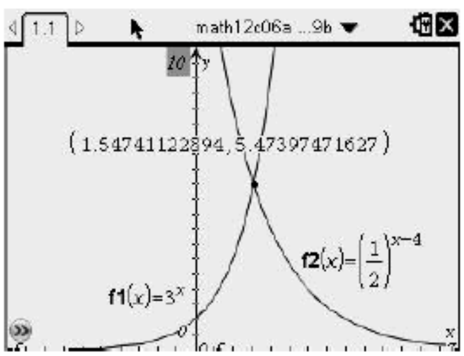
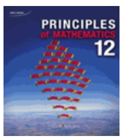
***Solution:***

**9. a)** e.g., Graph each function and identify the intersection point.
**b)** (1.5, 5.5)






**Practice Problem:**

Complete "Practising" question 10 on page 363 of your textbook.

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*Solution:*

**10. a)** $50 = 100\left(\frac{1}{2}\right)^{\frac{t}{5.3}}$

$$\frac{50}{100} = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

$$\frac{1}{2} = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

$$1 = \frac{t}{5.3}$$

$$t = 5.3 \text{ years}$$

It takes 5.3 years.

**b)** $A(t) = 100\left(\frac{1}{2}\right)^{\frac{10}{5.3}}$

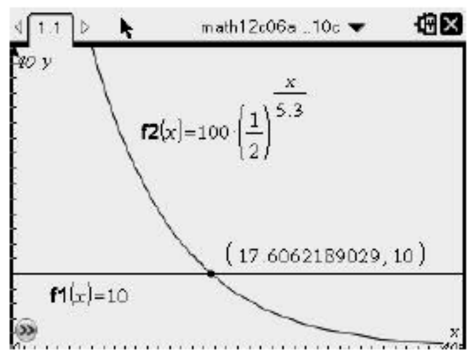
$$A(t) = 100\left(\frac{1}{2}\right)^{1.886\ldots}$$

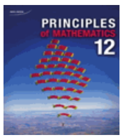
$$A(t) = 100(0.270\ldots)$$

$$A(t) = 27.040\ldots\%$$

27% will remain in a sample after 10 years.

**c)** $t$ = 17.6 years,






**Practice Problem:**

Complete “Practising” question 11 on page 363 of your textbook.

***Solution:***

**11. a)**

$$P(t) = 12\,000(3)^{\frac{t}{4}}$$

$$324\,000 = 12\,000(3)^{\frac{t}{4}}$$

$$12\,000(27) = 12\,000(3)^{\frac{t}{4}}$$

$$12\,000(3)^3 = 12\,000(3)^{\frac{t}{4}}$$

$$3 = \frac{t}{4}$$

$$t = 12 \text{ hours}$$

The value of $t$ is 12 hours.

**b)** The number of bacteria in a Petri dish after 12 h is 324 000.

**c)** Using graphing technology, the answer is $t$ = 0.56 h.

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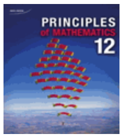



**Practice Problem:** **(KEY QUESTION)**

Complete “Practising” question 15 on page 365 of your textbook.

***Solution:***

15. $307\ 200 = 300(2)^{\frac{h}{12}}$

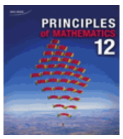
$$1024 = 2^{\frac{h}{12}}$$

$$2^{10} = 2^{\frac{h}{12}}$$

$$10 = \frac{h}{12}$$

$$h = 120\text{ h}$$

It will take 120 h for the bacterial population to grow to 307 200.




**Practice Problem:**

Complete "Practising" question 17 on page 365 of your textbook.

***Solution:***

**17.** e.g., Graphing systems of equations to determine an intersection is like solving exponential equations algebraically because we are trying to locate a point when $x$ and $y$ are equal for both equations. In the graphs, $y = 3$ and $y = 5(2)^x$ are equal at their intersection point.