Math 30-2: U6L5 Teacher Notes

Financial Applications Involving Exponential Functions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Determine the solution of an exponential equation in which the bases are powers of one another.
- Determine the solution of an exponential equation in which the bases are NOT powers of one another.
- Solve problems that involve the application of exponential equations to loans, mortgages and investments.

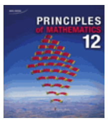
Compound Interest

One application of exponential functions is compound interest. **Compound Interest** is the interest earned on both the original amount that was invested and any interest that has accumulated over time.

Compound Interest
$A(n) = P(1+i)^n$
$A(n)$ = **Future value** (the amount that an investment will be worth after a specified period of time)
P = **Principal** (the original amount of money that is invested or borrowed)
i = **interest rate per compounding period**
n = **number of compounding periods** (the time over which interest is calculated and paid on a investment or loan)

Compounding periods are usually daily, weekly, monthly, quarterly, semi-annually, or annually. The chart below will help you find the "***n"*** and the "***i"*** for the compound interest.

Compounding Period	Number of Compounding Periods "n"	Interest Rate per Compounding Periods, "i"
Daily -- 365 times a year	365 x number of years = *n*	$i = \frac{\text{annual rate}}{365}$
Weekly -- 52 times a year	52 x number of years = *n*	$i = \frac{\text{annual rate}}{52}$
Semi-monthly -- 24 times a year	24 x number of years = *n*	$i = \frac{\text{annual rate}}{24}$
Monthly -- 12 times a year	12 x number of years = *n*	$i = \frac{\text{annual rate}}{12}$
Quarterly -- 4 times a year	4 x number of years =*n*	$i = \frac{\text{annual rate}}{4}$
Semi-annually - 2 times a year	2 x number of years = *n*	$i = \frac{\text{annual rate}}{2}$
Annually - 1 time a year	number of years = *n*	i = annual rate



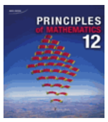
Practice Problem:

Complete “Practising” question 4 on page 395 of your textbook.

Solution:

4. Use the equation $y = 150(1.04)^x$, substitute 1, 2, 3, 4, and 5 for *x*, and solve for *y*. By doing this, I get the following costs for the shoes:

After 1 year: \$156
After 2 years: \$162.24
After 3 years: \$168.73
After 4 years: \$175.48
After 5 years: \$182.50



Practice Problem:

Complete “Practising” question 5 on page 396 of your textbook.

Solution:

5. a)

End of Year	Amount Owed ($)	Increase in Amount Owed (%)
1	2157.84	
2	2330.47	8.000…
3	2516.90	7.999…
4	2718.26	8.000…

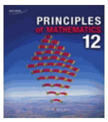
From the table above, the annual interest rate is about 8%, which is approximately the ratio between successful amounts owed.

b) To solve for the cost of the dirt bike, use the amount owed at the first year divided by 1.08.

$$2157.84 = P(1.08)^1$$

$$P = \$1998$$

The dirt bike cost $1998.



Practice Problem:

Complete “Practising” question 7 on page 396 of your textbook.

Solution:

7. a) e.g., First divide amounts in consecutive rows to determine the base, then use the initial amount and the base to write an exponential formula.

b)

Time (years)	Amount ($)	Increase in Amount (%)
0	1500.00	
1	1560.00	4
2	1622.40	4
3	1687.30	4.000…
4	1754.79	3.999…

The annual interest rate is 4%.

c) The function, which we must use, is

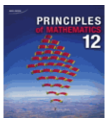
$y = 1500(1.04)^x$.

$y = 1500(1.04)^{10}$

$y = 1500(1.480\ldots)$

$y = \$2220.366\ldots$

Paula will have $2220.37 in her account after 10 years.



Practice Problem:

Complete “Practising” question 8 on page 396 of your textbook.

Solution:

8. a) The equation that represents Carl’s investment is $A(x) = 10\,000(1.05)^x$.
To determine the value of Carl's investment as the end of each of the next 4 years, substitute 1, 2, 3, and 4 for x into the above equation, and solve for y.

$$y = 10000(1.05)^1$$
$$y = \$10500$$
$$y = 10000(1.05)^2$$
$$y = \$11025$$
$$y = 10000(1.05)^3$$
$$y = \$11576.25$$
$$y = 10000(1.05)^4$$
$$y = \$12155.062...$$

Carl’s investment will be worth \$10 500 at the end of year 1, \$11 025 at the end of year 2, \$11 576.25 at the end of year 3, and \$12 155.06 at the end of year 4.

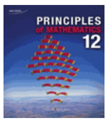
Solution:

b) $y = 10\,000(1.05)^x$

c) $y = 10\,000\left(1.05\right)^8$

$y = \$14\,774.554...$

The value of Carl's investment after 8 years will be $14 774.55.

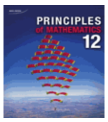


Practice Problem:

Complete “Practising” question 9 on page 396 of your textbook.

Solution:

9. e.g., If interest is compounded k times per year, then replace i with $\frac{i}{k}$. For example, if the interest rate is 12% per year, compounded monthly (that is, 12 times per year), then the formula is $A(n) = P\left(1 + \frac{0.12}{12}\right)^{12n}$ or $P(1.01)^{12n}$ where n is the number of years.

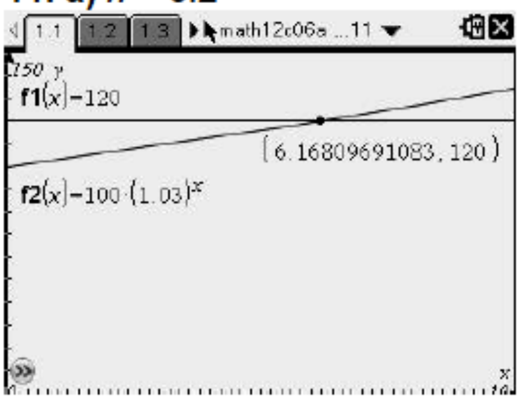


Practice Problem:

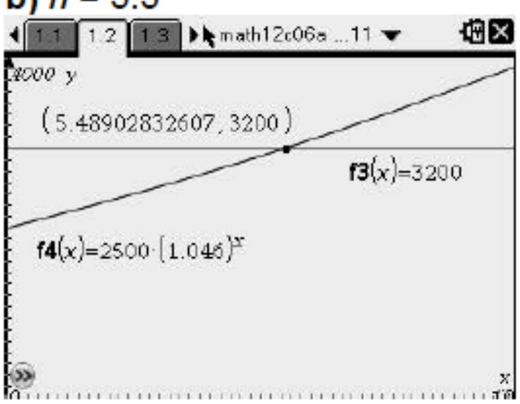
Complete “Practising” question 11a and b on page 397 of your textbook.

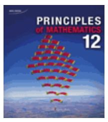
Solution:

11. a) $n = 6.2$



b) $n = 5.5$





Practice Problem:

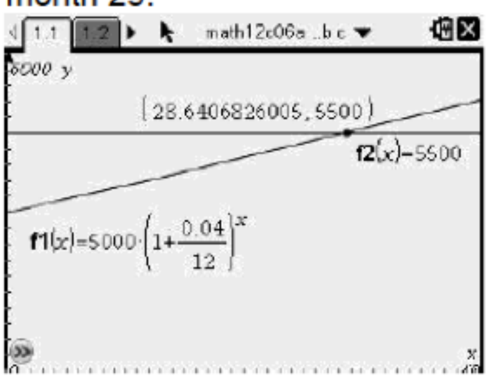
Complete “Practising” question 11a and b on page 397 of your textbook.

Solution:

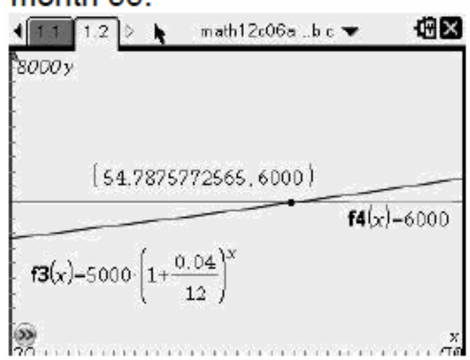
13. a)

Number of Months	Value of Investment
0	\$5000
1	\$5016.67
2	\$5033.39
3	\$5050.17
4	\$5067.00
5	\$5083.89

b) i) The investment will reach \$5500 during month 29.



ii) The investment will reach \$6000 during month 55.



Solution:

c) i) $y = 5000\left(1+\dfrac{0.04}{12}\right)^{x}$

$y = 5000(1.003...)^{x}$

$y = 5000(1.003...)^{60}$

$y = 5000(1.220...)$

$y = \$6104.982...$

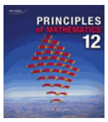
There will be \$6104.99 in the account after 5 years.

ii) $y = 5000(1.003...)^{120}$

$y = 5000(1.490...)$

$y = \$7454.163...$

There will be \$7454.17 in the account after 10 years.



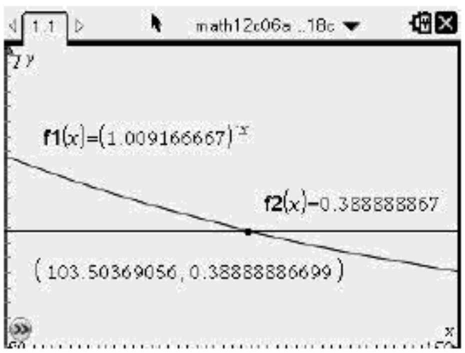
Practice Problem: **(KEY QUESTION)**

Complete “Practising” question 18 on page 398 of your textbook.

Solution:

18. a) Emma paid \$24 000 for her new car.

b) 1 – 0.85 = 0.15



(0.15)(100%) = 15%

Emma's grandfather applied an annual depreciation rate of 15% to her car.

c) It will take Emma 103.503… or 104 months to pay off the loan.

d) $A = 300(103.503...)$

$A = \$31051.107...$

$I = A - P$

$I = 31\,051.107... - 20\,000$

$I = \$11\,051.107...$

Emma will end up paying \$11 051.11 in interest on the loan.

Solution:

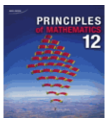
e) $A\left(\frac{104}{12}\right) = 24\,000(0.85)^{\frac{103.503...}{12}}$

$A\left(\frac{104}{12}\right) = 24\,000(0.85)^{8.625...}$

$A\left(\frac{104}{12}\right) = 24\,000(0.246...)$

$A\left(\frac{104}{12}\right) = \$5907.827...$

Emma's car will be worth $5907.83.



Practice Problem:

Complete “Practising” question 22 on page 398 of your textbook.

Solution:

22. a) e.g. $P = \dfrac{5000}{\left(1+\dfrac{0.12}{12}\right)^{12}}$

b) e.g., You could calculate the value of an investment in later years once you have calculated the principal in part a). For example, you could determine the principal for an investment that is worth $10 000 after 10 years where interest is 9% compounded monthly. The solution is $4079.37.