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Dream Job

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Many people dream of becoming millionaires these days. In order to make this dream come true, people play the stock market, play the lottery, and appear on television game shows designed for this purpose. Counting on the stock market requires placing money at risk and playing the lottery involves playing against the odds. And honestly, the television programs place contestants in the risky position of looking foolish by missing an easy question in front of millions of viewers.

Let us examine a plan for earning a million dollars based on a contract between an employee and an employer. After all, this way sounds like a respectable method -- working for your wages. First let us agree upon a contract.

Contract for Employment

Employee _____(enter your name)

Employer _____(a company agreeing with these terms)

Points of Agreement

1. The employee will work a five-day work week.
2. The employee will be paid for the week's wages each Friday.
3. The employee will be hired for a minimum of 30 work days.
4. The salary schedule is as follows:
 - The base pay for Day 1 is one penny.
 - Each subsequent day, the salary is double that of the previous day.

Signed _____(Employee)

Signed _____(Employer)

Date: _____

Is this a good deal? It certainly looks as if this employee has made a bad deal. Just to test our expectations, guess how much money this employee will have earned in the 30 working days:

My Guess \$_____. Calculate the amount one would earn working six weeks (40-hours a week) at minimum wage? Minimum wage salary (before taxes and other deductions) \$_____. Now let's calculate the earnings for this contract and see whether the employer or the employee has made the better deal.

Week 1

Day	Salary Earned	
	Dollars	Cents
Day 1 -- Monday	0	1
Day 2 -- Tuesday	0	2
Day 3 -- Wednesday	0	4
Day 4 -- Thursday	0	8
Day 5 -- Friday	0	16

Paycheck for week one	0	31
Total wages earned	0	31

What do you think? You worked hard all week. You spend the weekend soaking your feet and resting your bones. Maybe you made a bad deal -- you are under contract for 30 days. Chin up - - only 25 more days to go!

Week Two

Day	Salary Earned	
	Dollars	Cents
Day 6 -- Monday	0	32
Day 7 -- Tuesday	0	64
Day 8 -- Wednesday	1	28
Day 9 -- Thursday	2	56
Day 10 -- Friday	5	12

Paycheck for week two	9	92
Total wages earned	10	23

After two exhausting weeks you have only earned a total of \$10.23! Certainly your employer is laughing behind your back!

Week Three

Day	Salary Earned	
	Dollars	Cents
Day 11 -- Monday	10	24
Day 12 -- Tuesday	20	48
Day 13 -- Wednesday	40	96
Day 14 -- Thursday	81	92
Day 15 -- Friday	163	84

Paycheck for week three	317	44
Total wages earned	327	67

The weekly wage is getting better, but still, you have worked three whole weeks for only \$327.67. What a shame you made such an agreement!

Week Four

Day	Salary Earned	
	Dollars	Cents
Day 16 -- Monday	327	68
Day 17 -- Tuesday	655	36
Day 18 -- Wednesday	1310	72
Day 19 -- Thursday	2621	44
Day 20 -- Friday	5242	88

Paycheck for week four	10,158	08
Total wages earned	10,485	75

Oh my! This is interesting! Where can I get an application for employment? By the way, I heard that your employer has been in touch with the corporate law firm about the consequences of breaking a contract. And just think -- you still have to work for 10 more days!

Week Five

Day	Salary	Earned
	Dollars	Cents
Day 21 -- Monday	10,485	76
Day 22 -- Tuesday	20,971	52
Day 23 -- Wednesday	41,943	04
Day 24 -- Thursday	83,886	08
Day 25 -- Friday	167,772	16

Paycheck for week five	325,058	46
Total wages earned	335,544	31

You must be a genius! How did you find this job? Do they have any openings? Can I apply for the job when your contract is complete? You only have one more week to go!

Week Six

Day	Salary Earned	
	Dollars	Cents
Day 26 -- Monday	335,544	32
Day 27 -- Tuesday	671,088	64
Day 28 -- Wednesday	1,342,177	28
Day 29 -- Thursday	2,684,354	56
Day 30 -- Friday	5,368,709	12

Paycheck for week six	10,401,873	92
Total wages earned	10,737,418	23

You became a millionaire on day # 27 (total salary) and made over \$1 million on day 28 alone! By the end of 30 days, you have accumulated over \$10 million!

Background

Do you think there is a formula that might allow one to calculate a particular day's salary without having to calculate every step? Of course there is! The dream job calculation is a classic example of a **geometric progression**.

A geometric progression is a sequence of numbers in which the ratio of any number to the number before it is a constant amount, called the **common ratio**. For example, the sequence of numbers 1, 2, 4, 8, 16, ... has a common ratio of 2. A geometric progression may be described by calling the first term in the progression X (in our case X is one cent), the common ratio as R (in our case, R=2), and in a finite progression, the number of terms as n. The nth term of a geometric progression is given by:

$$X_n = X_1 R^{n-1}$$

Obviously this problem could have been presented using other setups (not necessarily money), as long as the geometric progression is shown. A legendary treatment of the problem (according to a source at <http://www.np.ac.sg/~bms.SereBino/19-2GePr.htm>) involves a king who promised a prince anything he wanted because the prince saved the princess's life. The prince requested one grain of rice on the first square of a chessboard, 2 on the 2nd, 4 on the 3rd, etc. Check the web site to see how much rice (by weight) the king would eventually need to place on the chessboard to satisfy this agreement.

Another source discusses it just the way we did, starting with one penny and doubling the amount everyday. You may see that treatment at "Doubling Pennies":

<http://forum.swarthmore.edu/dr.math/problems/valerie11.26.96.html>.

The key for clever math students is realizing how rapidly the sum grows and understanding the geometric progression formula(s). Hope you enjoyed it!

Dream Job Questions

1. How does the total amount compare with your original guess?
2. Suppose you wanted to buy a car. On which day could you purchase your dream car and pay in cash?
3. This calculation does not take into account income taxes! Also, contributions must be made to Social Security and to Medicare! Look up the tax tables on the web and determine how much income tax and other contributions you will need to pay!
4. What percentage of the total salary (before taxes and withholding) was earned in week 1? In week 6?
5. Can you state a formula for the daily salary?

Answer: Daily Salary = $2^{n-1} X$

where n = the number of days you've been working and X = your base salary on day one

6. Can you state a formula for the accumulated salary on a given day?

Answer: Accumulated Salary = $2^{n-1} X$ (from $n = 1$ to N)

where n = the number of days you've been working; N = the number of given day for the calculation and X = your base salary on day one

7. Did you notice anything special about the relationship between Friday's paycheck of one week and Monday's paycheck of the next week? State a formula giving this connection.

Answer: $2^n X$ (from $n = 0$ to N) = $(2^{n+1} - 1) X$

where n = the number of days you've been working; N = the number of given day for the calculation and X = your base salary on day one

8. Write a computer program to tell you daily salary and accumulated salary on a given day.

9. Using the geometric series described by 1, 3, 9, 27,... determine the 8th term. What is the formula for the n th term?

Answer: 2187; $X_N = 3^{n-1}$ (from $n = 0$ to N)

Application of this counting principal to a social cause

There are many worthy social issues that deserve the attention of politicians and the general public. You may feel very strongly about discouraging the use of illegal drugs or about the treatment of wildlife. No matter how strongly you feel about any of the countless issues in our world today, you may also feel overwhelmed and discouraged about the impact of one lone person. The above illustration shows that it is not impossible for one person committed to a cause to make a difference. Let's think about this:

What can you do to raise social consciousness about this very important issue? You have no money to launch a major campaign. You have no provision to make a television plea or to reach a

large number of people in the newspapers or magazines. Social issues, however, are often affected by “grass-roots” efforts -- with it all started by a scant few individuals who are committed to a cause.

Suppose you tell one person a day about your issue. One on one you can make a much more convincing argument than you could in a 15-second television spot. A one-on-one plea will be much more effective in convincing the listener.

One the second day, there will be two of you who can approach two more people. On day three, there are four of you to approach four more people. On day five, the eight of you convince eight more people and so forth. By day 12, there are over 2,000 people who know about your cause and who are each taking the time every day to tell just one more person about the cause! On day 30, over one billion people are talking about the issue that is so close to your heart!

You personally talked to 30 people. **YOU CAN DO THAT!** You have impacted a billion people! Imagine that. Even if only a fraction of the contacts is truly successful in convincing the listener, still you can make a difference!

Social Impact Questions

1. What are some successful grass-roots efforts that have resulted in increased awareness or social reform?
2. If you could successfully perform the social campaign discussed above, how many days would it take to reach the population of your state?
3. Suppose you win an award and you tell one new person an hour. Everyone who knows the news also tells one new person an hour. How long would it take everyone in your class to hear your news? How long before everyone in your state heard the news? Your state? The United States? The World? Do you understand how gossip or unfounded rumors spread so rapidly?

Conclusion

Let's face it -- none of us are likely to get the dream job described above, but we can make a difference in the world when it comes to issues we really care about -- one person at a time. Let your friends and family know about the issues that are important to you! You *can* change the world! You *can* make a difference!