Math 30-2: U7L1 Teacher Notes

Characteristic of Logarithmic Functions with Base 10 and Base e

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Express a logarithmic equation as an exponential equation and vice versa.
- Describe, orally and in written form, the characteristics of logarithmic functions by analyzing their graphs.
- Match equations to a given set to their corresponding graphs

What is Logarithm?

The **logarithm** of a number is the **exponent** by which another fixed value, the base, has to be raised to produce that number.

For example, the logarithm of 1000 to base 10 is 3, because $10^3 = 1000$

The formal definition of a **logarithmic function** is:

Logarithmic Function
$y = a\log_b x$ Where b > 0, $b \neq 1$, $a \neq 0$ and *a* and *b* are real numbers.

Click the icon to watch a Youtube video on the Definition of a Logarithm

Euler's Number

The number ***e*** is a famous irrational number, and is one of the most important numbers in mathematics.

The first few digits are:

2.7182818284590452353602874713527 (and more ...)

It is often called **Euler's number** after Leonhard Euler

Screen snapped from http://www.mathsisfun.com

The Common and Natural Logarithms

When the base of a logarithmic function is 10, we call that the **common logarithm** and it is often written without the base. For example, $y = \log_{10} x$ is commonly written as $y = \log x$.

A logarithm with base *e* (Euler's number) is called the **natural logarithm** and is written as ln x. For example, $y = \log_e x$, $y = \ln x$ and $x = e^y$.

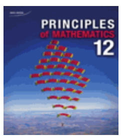
Characteristics of Logarithmic Functions

After doing the Investigation in the Assessment For Learning Questions you will have noticed the following characteristics of Logarithmic Functions and how they compare to Exponential Functions.

	Form	y-intercepts	x-intercepts	Graph Increases	Graph Decreases
Exponential Functions	$y = a(b)^x$	one y-intercept that equals the a-value	no x-intercepts	If $b > 1$, the graph is increasing.	If $0 < b < 1$, the graph is decreasing.
Logarithmic Functions	$y = a + b \ln x$	no y-intercepts	one x-intercept	If $0 > b > 1$, the graph is increasing.	If $0 > b < 1$, the graph is decreasing.

Video

Click the icon to enter a multimedia piece that compares Exponential and Logarithmic Functions.

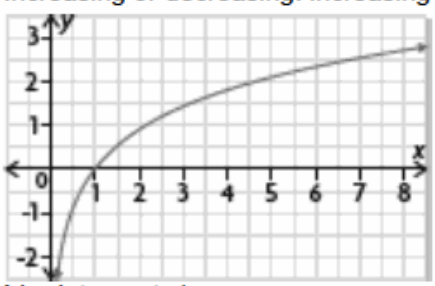


Practice Problem:

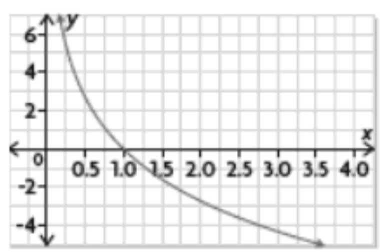
Complete "Practising" question 5 on page 421 of your textbook.

Solution:

5. a) x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing

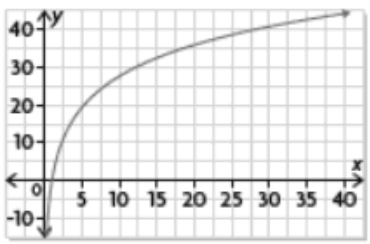


b) x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: decreasing

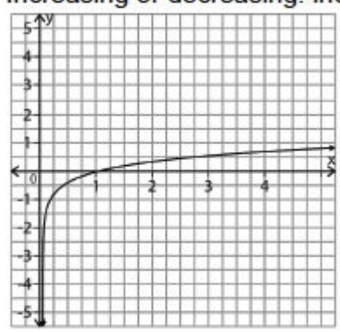


Solution:

c) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing

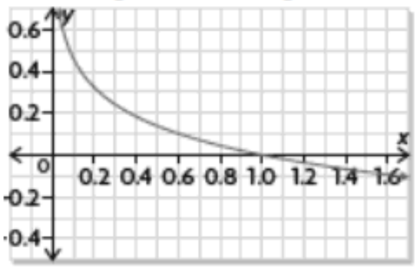


d) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing

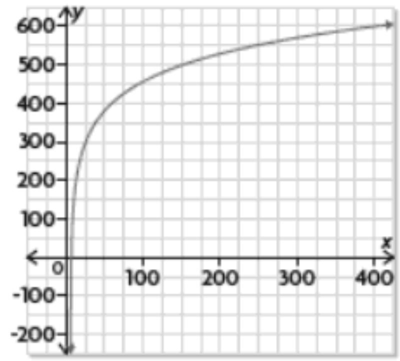


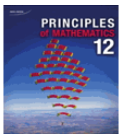
Solution:

e) x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: decreasing



f) x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing



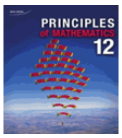


Practice Problem:

Complete "Practising" question 6 on page 421 of your textbook.

Solution:

6. e.g., one x-intercept of 1, no y-intercepts, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$.

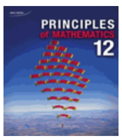


Practice Problem:

Complete “Practising” question 8 on page 422 of your textbook.

Solution:

8. i) b, e.g., *x*-intercept is 1, no *y*-intercept, graph extends from QIV to QI. Thus, the function is logarithmic so b and c are the only options. The function is increasing so the correct graph is b.
ii) c, e.g., *x*-intercept is 1, no *y*-intercept, graph extends from QI to QIV. Thus, the function is logarithmic so b and c are the only options. The function is decreasing so the correct graph is c.
iii) d, e.g., no *x*-intercept, *y*-intercept is 1, graph extends from QII to QI. Thus, the function is exponential so a and d are the only options. The function is increasing so the correct graph is d.
iv) a, e.g., no *x*-intercept, *y*-intercept is 2, graph extends from QII to QI. Thus, the function is exponential a and d are the only options. The function is decreasing so the correct graph is a.

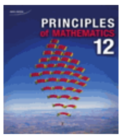


Practice Problem:

Complete “Practising” question 9 on page 423 of your textbook.

Solution:

9. Yes, e.g., An exponential function has no *x*-intercepts, and a logarithmic function has one *x*-intercept.

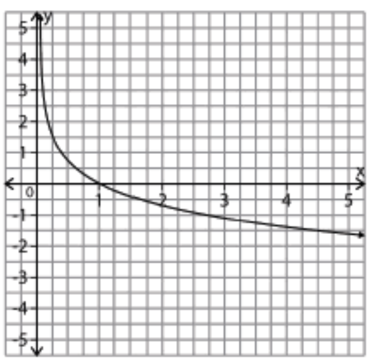


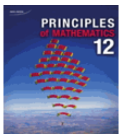
Practice Problem:

Complete “Practising” question 10 on page 423 of your textbook.

Solution:

10. As hydrogen ion concentration increases, pH decreases.





Practice Problem:

Complete "Closing" question 13 on page 424 of your textbook.

Solution:

13. a) When $a < 0$, each function will be decreasing.
b) The domain of each function is restricted as for each function x must be greater than 0.
c) The range is unrestricted as all values of y are possible.