Math 30-2: U7L2 Teacher Notes

Evaluating Logarithmic Functions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Express a logarithmic equation as an exponential equation and vice versa.
- Determine the value of a logarithmic expression without technology.
- Determine the approximate value of a logarithmic expression without technology.
- Solve problems that involve logarithmic scales, such as the Richter scale and pH scale.

Logarithmic Functions are Equivalent to Exponential Functions

From the first lesson, we learned that the **logarithm** of a number is the **exponent** by which another fixed value, the base, has to be raised to produce that number.

For example, the logarithm of 1000 to base 10 is 3, because $10^3 = 1000$

Therefore, the **logarithmic function is equivalent to an exponential function**.

$y = \log_b x$ is equivalent to $x = b^y$

Remember that the common logarithmic function has a base of 10, therefore:

$y = \log x$ is equivalent to $x = 10^y$

And that the natural logarithmic function has a base of *e*, therefore:

$y = \ln x$ is equivalent to $x = e^y$

Evaluating Logarithmic Functions

The value of a logarithm can be determined in one of the following ways:

- Set the logarithmic expression equal to y, and write the equivalent exponential form. Then determine the exponent to which the base must be raised to get the required number.

- If the base of logarithm is 10 or *e*, you can use a scientific or graph calculator.

Does the Logarithm of a Negative Number Exist?

The logarithm of a negative number does not exist.

Example:

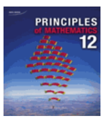
Calculate $\log_5(-25)$

Solution:

Write the logarithm as an exponent.

$$5^x = -25$$

There is no exponent that will give you a negative answer. This logarithm is UNDEFINED. Remember that the logarithmic function has a domain of $x > 0, \text{ where } x \in R$



Practice Problem:

Complete "Check your Understanding" question 5 on page of your textbook.

Solution:

5. a) I estimate that $2 < y < 3$ since $10^2 = 100$ is less than 250 and $10^3 = 1000$ is greater than 250.

$250 = 10^y$

$y = \log 250$

$y = 2.397\ldots$

It is approximately 2.4.

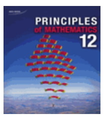
b) I estimate that $2 < y < 3$ since $e^2 < 9 < e^3$.

$9 = e^y$

$y = \ln 9$

$y = 2.197\ldots$

It is approximately 2.2.



Practice Problem:

Complete “Check your Understanding” question 6.

Solution:

6. a) $\frac{1}{9} = 3^y$

$y = \log_3 \frac{1}{9}$

b) $1\ 000\ 000 = 2^y$

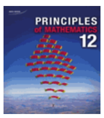
$y = \log_2 1\ 000\ 000$

c) $5 = 7^y$

$y = \log_7 5$

d) $x = a^y$

$y = \log_a x$



Practice Problem:

Complete “Check your Understanding” question 7.

Solution:

7. a) $x = \log_{\frac{1}{2}} 100$

$\left(\frac{1}{2}\right)^x = 100$

b) $x = \log_{20} 40$

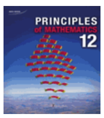
$20^x = 40$

c) $x = \ln 0.25$

$e^x = 0.25$

d) $x = \ln 1000$

$e^x = 1000$



Practice Problem:

Complete “Check your Understanding” question 8.

Solution:

8. a) Let $y = \log_3 81$

$y = \log_3 81$

$3^y = 81$

$3^y = 3^4$

$y = 4$

$\log_3 81 = 4$

b) Let $y = \log_4 16$

$y = \log_4 16$

$4^y = 16$

$4^y = 4^2$

$y = 2$

$\log_4 16 = 2$

c) Let $y = \log_8 64$

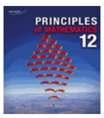
$y = \log_8 64$

$8^y = 64$

$8^y = 8^2$

$y = 2$

$\log_8 64 = 2$



Practice Problem:

Complete “Check your Understanding” question 11.

Solution:

11. a) $\log_3 81 + \log_3 3$
First term: $\log_3 81$
$3^4 = 81$, so the first term in this expression is equal to 4.
Second term: $\log_3 3$
$3^1 = 3$, so the second term in this expression is equal to 1.

$$\begin{aligned}\log_3 81 + \log_3 3 &= 4 + 1 \\ &= 5\end{aligned}$$

b) $\log_2 64 - \log_2 4$
First term: $\log_2 64$
$2^6 = 64$, so the first term in this expression is equal to 6.
Second term: $\log_2 4$
$2^2 = 4$, so the second term in this expression is equal to 2.

$$\begin{aligned}\log_2 64 - \log_2 4 &= 6 - 2 \\ &= 4\end{aligned}$$

c) $(\log_4 1)\left(\log_5\left(\frac{1}{5}\right)\right)$

First term: $\log_4 1$

$4^0 = 1$, so the first term in this expression is equal to 0.

Second term: $\log_5\left(\frac{1}{5}\right)$

$5^{-1} = \frac{1}{5}$, so the second term in this expression is equal to –1.

$$(\log_4 1)\left(\log_5\left(\frac{1}{5}\right)\right) = (0)(-1)$$
$$= 0$$

d) $\log_3 27 \div \log_3\left(\frac{1}{9}\right)$

First term: $\log_3 27$

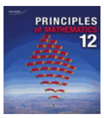
$3^3 = 27$, so the first term in this expression is equal to

Second term: $\log_3\left(\frac{1}{9}\right)$

$5^{-2} = \frac{1}{9}$, so the second term in this expression is equal to –2.

$$\log_3 27 \div \log_3\left(\frac{1}{9}\right) = 3 \div (-2)$$

$$\log_3 27 \div \log_3\left(\frac{1}{9}\right) = -\frac{3}{2}$$



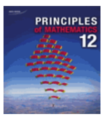
Practice Problem:

Complete “Check your Understanding” question 12.

Solution:

12. A: $\log_2 32 - \log_2 8$
First term: $\log_2 32$
$2^5 = 32$, so the first term in expression A is equal to 5.
Second term: $\log_2 8$
$2^3 = 8$, so the second term in expression A is equal to 3.
$\log_2 32 - \log_2 8 = 5 - 3$
$= 2$
B: $\log 85 + \log 5 = 2.628\ldots$

Since $2.628\ldots > 2$, B is greater than A.



Practice Problem:

Complete “Check your Understanding” question 13.

Solution:

13. A: $\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{16}\right)+\log_2\left(\frac{1}{8}\right)$

First term: $\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{16}\right)$

$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$, so the first term in expression A is equal to 4.

Second term: $\log_2\left(\frac{1}{8}\right)$

$2^{-3} = \frac{1}{8}$, so the second term in expression A is equal to –3.

$$\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{16}\right)+\log_2\left(\frac{1}{8}\right)=4+(-3)$$

$$\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{16}\right)+\log_2\left(\frac{1}{8}\right)=1$$

B: $\log_3 27 + \ln 2$

First term: $\log_3 27$

$3^3 = 27$, so the first term in expression B is equal to 3.

$\log_3 27 + \ln 2 = 3 + 0.693\ldots$

$= 3.693\ldots$

C: $\log 100 + \log_8 8$

First term: $\log 100$

$10^2 = 100$, so the first term in expression C is 2.

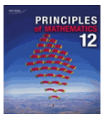
Second term: $\log_8 8$

$8^1 = 8$, so the second term in expression C is 1.

$\log 100 + \log_8 8 = 2 + 1$

$= 3$

The order of the expressions from least to greatest is A, C, B.



Practice Problem:

Complete "Check your Understanding" question 16.

Solution:

16. a) Car battery acid has a pH of 0, and distilled water has a pH of 7.

Car battery acid:

$pH = -\log[H^+]_C$

$0 = -\log[H^+]_C$

$0 = \log[H^+]_C$

$[H^+]_C = 10^0$

$[H^+]_C = 1$

Distilled water:

$pH = -\log[H^+]_D$

$7 = -\log[H^+]_D$

$-7 = \log[H^+]_D$

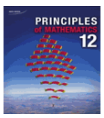
$[H^+]_D = 10^{-7}$

$$\frac{[H^-]_C}{[H^-]_D} = \frac{1}{10^{-7}}$$

$$\frac{[H^-]_C}{[H^-]_D} = 10^7$$

$$\frac{[H^-]_C}{[H^-]_D} = 10\,000\,000$$

The acid used in car batteries is 10 million times more acidic than distilled water.



Practice Problem:

Complete “Check your Understanding” question 21

Solution:

21. e.g., To evaluate $\log_b A$, determine the exponent needed that when applied to the base b gives the result A. If A is a power of the base b, the exponent can be determined without technology.