Math 30-2: U7L3 Teacher Notes

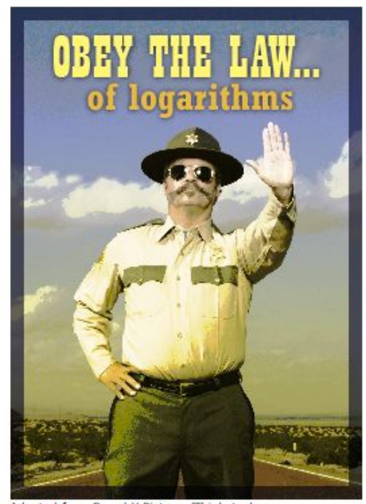
Laws of Logarithms

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Develop the laws of logarithms, using numeric examples and the exponent laws.
- Determine an equivalent expression for a logarithmic expression by applying the laws of logarithms.

Log of a Quotient and Product Law



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From the Investigation, you may have found that logarithms can be added or subtracted using laws similar to

$$\log M + \log N = \log(M \bullet N)$$

$$\log M - \log N = \log\left(\frac{M}{N}\right)$$

These rules can be extended to any base to give

$$\log_b M + \log_b N = \log_b(M \bullet N)$$

$$\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$$

Notice that the bases must be the same to use these laws. The same restrictions apply to these rules as to other logarithms: *b*, *M*, and *N* are real numbers greater than 0, and $b \neq 1$.

Comparing Logarithmic Laws to Exponential Laws

The following table provides an explanation of how the logarithmic laws are related to exponential laws. Remembering that a logarithm is an exponent may help you interpret the table.

Law of Logarithms	Law Expressed Mathematically	Pattern Explained by the Law of Powers
product law of logarithms	$\log_b (M \times N) = \log_b M + \log_b N$	In the product law of powers, $(b^m)(b^n) = b^{m+n}$, the exponents are added. In the product law of logarithms, $\log_b (M \times N) = \log_b M + \log_b N$, when you are multiplying two terms in a single logarithm, it is the same as adding the logarithms of each term.
quotient law of logarithms	$\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$	In the quotient law of powers, $\frac{b^m}{b^n} = b^{m-n}$, the exponents are subtracted. In the quotient law of logarithms, $\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$, when you are dividing two terms in a single logarithm, it is the same as subtracting the logarithms of each term.

The product and quotient laws of logarithms can be used to manipulate an expression into a different form. The following examples show how two logarithms can be combined into a single logarithm.

Example:

Write an equivalent expression to $\log_7 6 + \log_7 9$

Solution:

The logarithms have the same base, so they can be combined.

$= \log_7(6 \times 9)$ ⬅ The logarithms are added, so use the product of laws of logarithms.

$= \log_7 54$ ⬅ Determine the product and write a single logarithm.

Example:

Write an equivalent expression to $\log_3 72 - \log_3 8$

Solution:

The logarithms have the same base, so they can be combined.

$= \log_3\left(\frac{72}{8}\right)$ — The logarithms are subtracted, so use the quotient law of logarithms.

$= \log_3 9$ — Determine the quotient and write a single logarithm.

$= 2$ — This logarithm can be evaluated to a whole number.

The process shown in the examples can be reversed to write a logarithm as two separate logarithms, as shown in the following examples:

$$\log_5 (8 \times 12) = \log_5 8 + \log_5 12$$

$$\log\left(\frac{97}{1000}\right) = \log 97 - \log 1000$$
$$= \log 97 - 3$$

Power Law of Logarithms

The logarithm of a power can be rewritten as the product of a number and a logarithm using the law

$$n \log_b M = \log_b (M)^n$$

The same restrictions apply to these rules as to other logarithms: *b*, *M*, and *N* are real numbers greater than 0, and $b \neq 1$.

The following explains the relationship between this law and the power law of powers:

power law of logarithms	$\log_b (M^n) = n \log_b M$	In the product law of powers, $(b^m)^n = b^{m \times n}$, the exponents are multiplied. In the power law of logarithms, $\log_b (M^n) = n \log_b M$, when you have a power in a single logarithm, it is the same as multiplying the exponent to the logarithm of the base of the power.

Please note that this is a very important law to know when solving exponential equations when the bases are different. This concept will be covered in the next lesson.

Example:

Write an equivalent expression to. $\log 4^{x}$

Solution:

Using the power log we can write $x \log 4$ as an equivalent expression.

Example:

Apply the power log to evaluate the following $\log_3 9^4$

Solution:

$4\log_3 9$ ← Using the power law

$4\log_3\left(3^2\right)$ ← Using the power law again

$4(2)\log_3 3$ ← Since $\log_3 3 = 1$

= 8

Example:

Apply the power log to evaluate the following $\log_5 \sqrt{125}$

Solution:

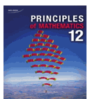
$$= \log_5(125^{\frac{1}{2}})$$

$$= \frac{1}{2}\log_5 125$$

$$= \frac{1}{2}\log_5(5^3)$$

$$= \frac{1}{2}(3)\log_5(5)$$

$$= \frac{3}{2}$$



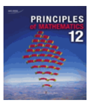
Practice Problem:

Complete “Check your Understanding” question 4 on page of your textbook.

Solution:

4. a) $\log_3 15 - \log_3 11 = \log_3\left(\frac{15}{11}\right)$

b) $\log_b s - \log_b r = \log_b\left(\frac{s}{r}\right)$

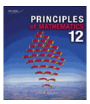


Practice Problem:

Complete “Check your Understanding” question 5 on page 446 of your textbook.

Solution:

5. a) $\log_6 7^3 = 3 \log_6 7$
b) $\log_z x^y = y \log_z x$



Practice Problem:

Complete “Check your Understanding” question 7 on page 447 of your textbook.

Solution:

7. a) $\left(\frac{1}{2}\right)\log_3 36 - \log_3 2 = \log_3 6 - \log_3 2$

$= \log_3\left(\frac{6}{2}\right)$

$= \log_3 3$

$= 1$

b) $\log 625 + 2\log 4 = \log 625 + \log 16$

$= \log(625 \cdot 16)$

$= \log 10\,000$

$= 4$

c) $\log_2 40 - \log_2 2.5 = \log_2\left(\frac{40}{2.5}\right)$

$= \log_2 16$

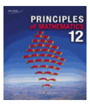
$= 4$

d) $\log_3 27 - \left(\frac{1}{3}\right)\log_3 27 = \log_3 27 - \log_3 3$

$= \log_3\left(\frac{27}{3}\right)$

$= \log_3 9$

$= 2$



Practice Problem:

Complete “Check your Understanding” question 8 on page 447 of your textbook.

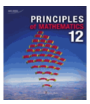
Solution:

8. a) e.g., $\log_2 32 = \log_2 8 + \log_2 4$

$\log_2 32 = \log_2 64 - \log_2 2$

b) e.g., $\log 125 = \log 25 + \log 5$

$\log 125 = \log 250 - \log 2$



Practice Problem:

Complete “Check your Understanding” question 11 on page 447 of your textbook.

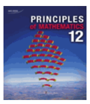
Solution:

11. a) $\log_3 2 + \log_3 4.5 = \log_3 (2 \cdot 4.5)$
$= \log_3 9$
$= 2$

b) $\log_3 162 - \log_3 2 = \log_3 \left(\frac{162}{2}\right)$
$= \log_3 81$
$= 4$

c) $\log_2 16^3 = 3 \log_2 16$
$= 3 \cdot 4$
$= 12$

d) $\log_5 10 - \log_5 1250 = \log_5 \left(\frac{10}{1250}\right)$
$= \log_5 \left(\frac{1}{125}\right)$
$= -3$



Practice Problem:

Complete “Check your Understanding” question 12 on page 447 of your textbook.

Solution:

12. a) $\log 40 - \log 8 = \log\left(\frac{40}{8}\right)$

$= \log 5$

$= 0.698\ldots$

It is approximately 0.70.

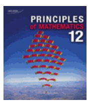
b) $\log 5 + 2\log 2 = \log 5 + \log 4$

$= \log(5 \cdot 4)$

$= \log 20$

$= 1.301\ldots$

It is approximately 1.30.

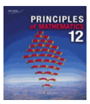


Practice Problem:

Complete “Check your Understanding” question 13 on page 447 of your textbook.

Solution:

13. $$3\log_8 2 - \left(\frac{1}{2}\right)\log_8 64 = \log_8 8 - \log_8 8$$
$$= 0$$



Practice Problem:

Complete "Check your Understanding" question 14 on page 447 of your textbook.

Solution:

14. e.g., Wayne made an error when he simplified $\log_3 27 - \log_3 9$ as $\dfrac{\log_3 27}{\log_3 9}$. The solution should be

$$\begin{aligned}\log_3 27 - \log_3 9 &= \log_3\left(\frac{27}{9}\right)\\ &= \log_3 3\\ &= 1\end{aligned}$$

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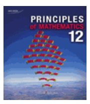

Practice Problem:

Complete “Check your Understanding” question 17 on page 447 of your textbook.

Solution:

17. e.g., Because exponents and logarithms are two ways to express the same meaning, the laws are related. They follow the same pattern.

The product law of logarithms is related to the product law for exponents.

$\log_a mn = \log_a m + \log_a n \rightarrow a^{xy} = a^x a^y$; multiply two powers, add the exponents

$\log (100 \cdot 10) = \log 100 + \log 10$, which corresponds to $10^2 \cdot 10^1 = 10^{2+1}$

The quotient law of logarithms is related to the quotient law for exponents.

$\log_a \frac{m}{n} = \log_a m - \log_a n \rightarrow \frac{a^x}{a^y} = a^{x-y}$; divide two powers, subtract the exponents

$\log \frac{100}{10} = \log 100 - \log 10$, which corresponds to

$\frac{10^2}{10^1} = 10^{2-1}$

The power law of logarithms is related to the power law for exponents.

$\log_a m^n = n \log_a m \rightarrow (a^x)^y = a^{xy}$; power of a power, multiply the exponents $\log_2 16^3 = 3 \log_2 16$, which corresponds to $(2^4)^3 = 2^{4 \cdot 3}$