Math 30-2: U7L4 Teacher Notes

Solving Exponential Equations Using Logarithms

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Determine the approximate value of a logarithmic expression with technology
- Determine the solution of an exponential equation in which the bases are not powers of one another.
- Solve problems that involve the application of exponential equations to loans, mortgages and investments.

Solving Exponential Equations

Previously, we studied how to solve exponential equations. At that time, we said that there are two different types of exponential equations.

a) where the bases can be changed into the same base

- we already know that we can just change both sides to the same base, and then let the exponents equal each other and solve.

Steps To Solving Exponential Equations with the Same Base
• change both sides to the same base • let the exponents equal • solve for x.

Solving Exponential Equations Using Logarithms

In this lesson we will be learning how to solve exponential equations, where the bases cannot be changed into a power with the same base.

Steps To Solving Exponential Equations Where the Powers are of Different Bases
• Isolate the power • Take the log of both sides • Then isolate the variable • Solve

Example:

Solve for *x* in the equation $19 = 2^x$

Solution:

$$19 = 2^x$$

We cannot change both 19 and 2, to the same base, so take the log of both sides

$$\log 19 = \log 2^x$$

Using the the power law, we can move the exponent (*x*) down in front to become the coefficient of log2.

$$\log 19 = x \log 2$$

$$x = \frac{\log 19}{\log 2}$$

Divide both sides by log 2, in order to isolate x.

$$x = 4.25$$

Example:

Solve for *x* in the equation $5\left(2^x\right)=52$

Solution:

$$5\left(2^x\right)=52$$

⬅ Isolate the power 2^x , by dividing both sides by 5.

$$\left(2^x\right)=\frac{52}{5}$$

⬅ We cannot change both 2 and 10.4, to the same base, so take the log of both sides

$$\log\left(2^x\right)=\log 10.4$$

⬅ Using the the power law, we can move the exponent (*x*) down in front to become the coefficient of log2.

$$x\log 2=\log 10.4$$

⬅ Divide both sides by log 2, in order to isolate x.

$$x=\frac{\log 10.4}{\log 2}$$

$$x=3.38$$

Example:

Solve for *x* in the equation $3^{x+1}=21$

Solution:

$$3^{x+1}=21$$

We cannot change both 3 and 21, to the same base, so take the log of both sides

$$\log 3^{(x+1)}=\log 21$$

Using the the power law, we can move the exponent (*x +1*) down in front to become the coefficient of log3.

$$(x+1)\log 3=\log 21$$

Divide both sides by log 3, in order to isolate x.

$$x+1=\frac{\log 21}{\log 3}$$

$$x+1=2.77...$$

$$x=1.77$$

Example:

Solve for *x* in the equation $2^{x+1} = 3^{x-1}$

Solution:

Method 1: Algebraically

$$2^{x+1} = 3^{x-1}$$

We cannot change both 2 and 3, to the same base, so take the log of both sides

$$\log 2^{x+1} = \log 3^{x-1}$$

Using the the power law, we can move the exponents (*x* +*1*) and (*x* - 1) down in front to become the coefficient of log 2 and log3.

$$(x+1)\log 2 = (x-1)\log 3$$

$$x\log 2 + \log 2 = x\log 3 - \log 3$$

Collect like terms (xlog 2 and xlog 3 are like terms).

$$x\log 2 - x\log 3 = -\log 3 - \log 2$$

Factor out an x.

$$x\left(\log 2 - \log 3\right) = -\log 3 - \log 2$$

Divide both sides by log 2 - log 3, in order to isolate x.

$$x = \frac{-\log 3 - \log 2}{\left(\log 2 - \log 3\right)}$$

$$x = 4.419...$$

$$x = 4.42$$

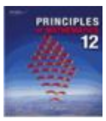
Method 2: Graphically
Input the following equations into the graphing calculator

$$y_1 = 2^{x+1}$$
$$y_2 = 3^{x-1}$$

Set the WINDOW so you can see where the two equations intersect.

Press 2nd, TRACE, Intersection

Solution x = 4.42



Practice Problem:

Complete “Check your Understanding” question 5 on page 456 of your textbook.

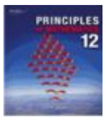
Solution:

. **a)** $$\log_5 50 = \frac{\log 50}{\log 5} = 2.430\ldots = 2.431$$

b) $$\log_2\left(\frac{5}{8}\right) = \frac{\log\left(\frac{5}{8}\right)}{\log 2} = -0.678\ldots = -0.678$$

c) $$\log_3 1000 = \frac{\log 1000}{\log 3} = 6.287\ldots = 6.288$$

d) $$\log_{0.1} 200 = \frac{\log 200}{\log 0.1} = -2.301\ldots = -2.301$$



Practice Problem:

Complete “Check your Understanding” question 6 on page 456 of your textbook.

Solution:

6. a)

$$5^{x+1} = 24$$
$$\log 5^{x+1} = \log 24$$
$$(x + 1) \log 5 = \log 24$$
$$x + 1 = \frac{\log 24}{\log 5}$$
$$x + 1 = 1.974\ldots$$
$$x = 0.974\ldots$$
$$x = 0.97$$

b)

$$3 = 8^{2x-1}$$
$$\log 3 = \log 8^{2x-1}$$
$$\log 3 = (2x - 1) \log 8$$
$$2x - 1 = \frac{\log 3}{\log 8}$$
$$2x - 1 = 0.528\ldots$$
$$x = 0.764\ldots$$
$$x = 0.76$$

c) $\left(\frac{1}{3}\right)^{x} - 40 = 0$

$$\left(\frac{1}{3}\right)^{x} = 40$$

$$\log\left(\frac{1}{3}\right)^{x} = \log 40$$

$$x\log\left(\frac{1}{3}\right) = \log 40$$

$$x = \frac{\log 40}{\log\left(\frac{1}{3}\right)}$$

$$x = -3.357...$$

$$x = -3.36$$

d) $\left(\frac{2}{5}\right)^{-x} = 5$

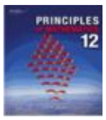
$$\log\left(\frac{2}{5}\right)^{-x} = \log 5$$

$$-x\log\left(\frac{2}{5}\right) = \log 5$$

$$-x = \frac{\log 5}{\log\left(\frac{2}{5}\right)}$$

$$-x = -1.756...$$

$$x = 1.76$$



Practice Problem:

Complete “Check your Understanding” question 7 on page 456 of your textbook.

Solution:

7. $I = 10^{(1-0.13x)}$

$4.2 = 10^{(1-0.13x)}$

$\log 4.2 = \log 10^{(1-0.13x)}$

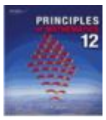
$\log 4.2 = (1-0.13x)\log 10$

$\frac{\log 4.2}{\log 10} = 1 - 0.13x$

$0.623\ldots = 1 - 0.13x$

$2.898\ldots = x$

Plants will receive enough light at a depth of about 2.9 m.



Practice Problem:

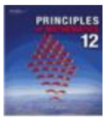
Complete “Check your Understanding” question 9 on page 457 of your textbook.

Solution:

9. Let x represent the number of washings.
Let y represent the percentage of the original dye remaining.

$$y = (1 - 0.022)^x$$
$$0.3 = 0.978^x$$
$$\log 0.3 = \log 0.978^x$$
$$\log 0.3 = x \log 0.978$$
$$\frac{\log 0.3}{\log 0.978} = x$$
$$54.121\ldots = x$$

Since you can't have part of a wash, 55 washes are required to give a pair of jeans the well-worn look.



Practice Problem: **(KEY QUESTION)**

Complete “Check your Understanding” question 12 on page 458 of your textbook.

Solution:

12. The investment has tripled when the investment is worth \$3000.

$$A = P(1 + i)^n$$

$$3000 = 1000(1 + \frac{0.06}{12})^n$$

$$3 = (1.005)^n$$

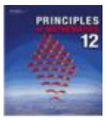
$$\log 3 = \log (1.005)^n$$

$$\log 3 = n \log(1.005)$$

$$\frac{\log 3}{\log 1.005} = n$$

$$220.271\ldots = n$$

Therefore the investment tripled in about 221 months or 18 years 5 months.



Practice Problem:

Complete “Check your Understanding” question 13 on page 458 of your textbook.

Solution:

13. His debt will have doubled when the balance is $4000.

$$A = P(1 + i)^n$$

$$4000 = 2000(1 + \frac{0.185}{12})^n$$

$$2 = (1.015\ldots)^n$$

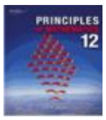
$$\log 2 = \log (1.015\ldots)^n$$

$$\log 2 = n \log(1.015\ldots)$$

$$\frac{\log 2}{\log 1.015\ldots} = n$$

$$45.306\ldots = n$$

Therefore his debt will have doubled in about 4 years.



Practice Problem:

Complete “Check your Understanding” question 15b and c on page 458 of your textbook.

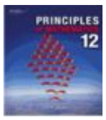
Solution:

b)

$$\begin{aligned}
6^{x-6} &= 3^{x+1} \\
\log 6^{x-6} &= \log 3^{x+1} \\
(x-6)\log 6 &= (x+1)\log 3 \\
x\log 6 - 6\log 6 &= x\log 3 + \log 3 \\
x\log 6 - x\log 3 &= 6\log 6 + \log 3 \\
x(\log 6 - \log 3) &= 6\log 6 + \log 3 \\
x &= \frac{6\log 6 + \log 3}{\log 6 - \log 3} \\
x &= 17.094\ldots \\
x &= 17.09
\end{aligned}$$

c)

$$\begin{aligned}
10^{x+1} &= 5^{x-1} \\
\log 10^{x+1} &= \log 5^{x-1} \\
(x+1)\log 10 &= (x-1)\log 5 \\
x\log 10 + \log 10 &= x\log 5 - \log 5 \\
x + 1 &= x\log 5 - \log 5 \\
x - x\log 5 &= -1 - \log 5 \\
x(1 - \log 5) &= -1 - \log 5 \\
x &= \frac{-1 - \log 5}{1 - \log 5} \\
x &= -5.643\ldots
\end{aligned}$$



Practice Problem:

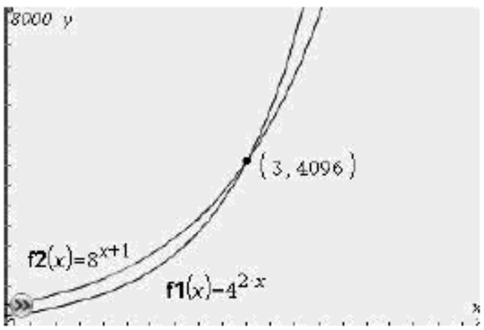
Complete "Closing" question 17 on page 458 of your textbook.

Solution:

17. Strategy 1: Write both sides of the equation as powers with equivalent bases, then solve for the unknown by comparing exponents.

$$4^{2x} = 8^{x+1}$$
$$2^{2(2x)} = 2^{3(x+1)}$$
$$2^{4x} = 2^{3x+3}$$
$$4x = 3x + 3$$
$$x = 3$$

Strategy 2: Use a system of equations and graphing technology to determine the intersection of the two curves.



Therefore, $x = 3$

Strategy 3: Take the common logarithm of both sides of the equation, and then use the logarithm laws to solve for the unknown.

$$4^{2x} = 8^{x+1}$$
$$\log 4^{2x} = \log 8^{x+1}$$
$$(2x)\log 4 = (x+1)\log 8$$
$$\frac{x+1}{2x} = \frac{\log 4}{\log 8}$$
$$\frac{1}{2} + \frac{1}{2x} = 0.666...$$
$$\frac{1}{2x} = 0.666... - \frac{1}{2}$$
$$x = \frac{1}{2(0.166...)}$$
$$x = 3$$