Math 30-2: U7L5 Teacher Notes

Modelling Data Using Logarithmic Functions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Graph data and determine the logarithmic functions, that best approximates the data.
- Interpret the graph of a logarithmic function that models a situation, and explain the reasoning.
- Solve, using technology, a contextual problem that involves data that is best represented by graphs of logarithmic functions, and explain the reasoning.

Logarithmic Regression

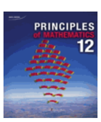
Most graphing calculators provide the logarithmic regression equation in the form

$$y = a + b \ln x$$

The abbreviation "ln" is referring to log and is usually called a **natural logarithm**. The number *e* is an irrational number, like π, and is approximately 2.718. Natural logarithms are important in advanced mathematics, so most regression programs use it instead of log10.

A **logarithmic regression** equation

- can be of the form $y = a + b \ln x$
- has end behaviour that extends from quadrant IV to quadrant I (increasing) or quadrant I to quadrant IV (decreasing)



Read “Example 2” on pages 463 to 465 of your textbook to see how a logarithmic regression can be performed and used to solve problems. Also, read and note the section titled “Key Idea” on the top of page 466, which summarizes logarithmic regressions.

Example

The Iverson family ordered some trees from the provincial shelterbelt program to plant on their acreage. The following data shows the average growth rate of 20 weeping willow trees after planting. At the time of planting, all the trees were 2 ft in height and were 1 y old.

Age of Trees (Years)	Average Height (Feet)
1	2
2	4.8
3	6.4
4	9.5
5	13
6	15.2
7	16.6
8	17.8
9	18.5
10	19
11	19.4
12	19.6

a. Use your graphing calculator to plot the data.

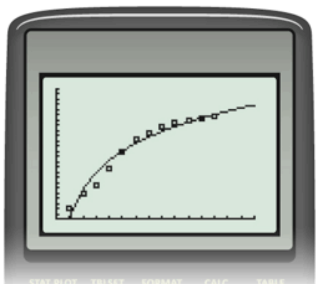
b. Is the average height increasing or decreasing?

c. What is the shape of your scatter plot? What kind of model do you expect to use with this data?

d. What is an appropriate domain and range in this situation?

Solution:

a.



b. The average height is increasing.

c. The scatter plot appears to be in the shape of a logarithmic function. A logarithmic regression is reasonable for the data.

d. An appropriate domain could be, $1 \le x \le 12$ where *x* is the age of the trees. An appropriate range could be $1 \le y \le 20$ where *y* is the height of the trees.

Example:

a. Determine a logarithmic regression equation for the data in the Example above.

Using your logarithmic regression model from the weeping willow tree example, answer the following questions.

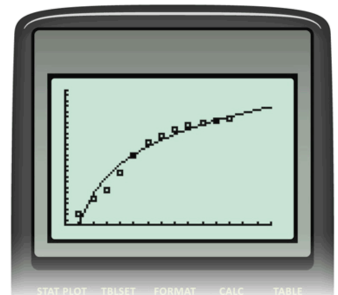
b. Interpolate: What was the average height of the trees at 2.5 y of age (to the nearest tenth of a foot)?

c. Extrapolate: What is the predicted average height of the trees at 20 y of age (to the nearest tenth of a foot)? Is this prediction realistic?

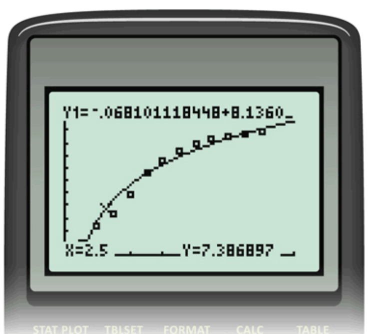
d. According to your model, if the average height of the trees is 17 ft, what is the age of the trees to the nearest tenth of a year?

Solution:

a. If you are unsure how to calculate a logarithmic regression, see your calculator manual or contact your teacher. The logarithmic regression for the tree data is $y = 0.068 + 8.136 (\ln x)$. Make sure that you are able to do this on your calculator. Graph this function on the same grid as the scatter plot. The screen on your graphing calculator should look similar to the following:

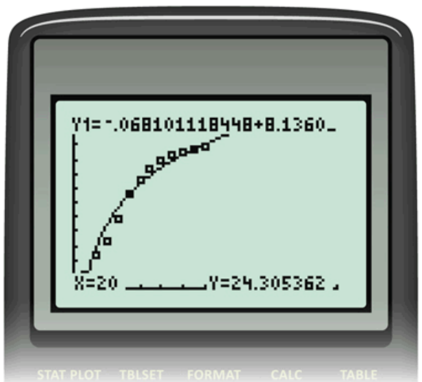


b. (To find the value of "y" when you have the value of "x", press 2nd TRACE, 1:Value and then put in x = 2.5



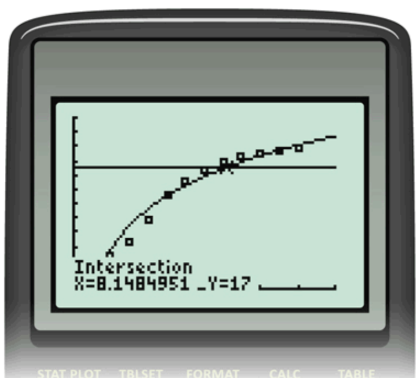
The average height of the trees at 2.5 y would be about 7.4 ft.

c.

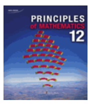


The average height of the trees at 20 y would be about 24.3 ft.

d.



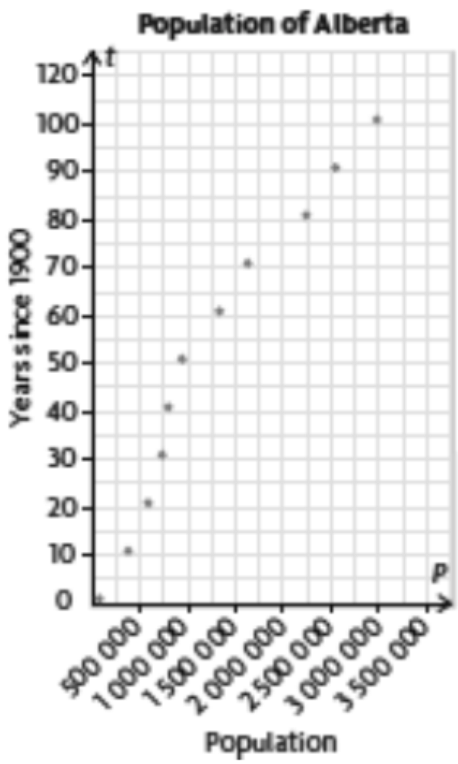
If the average of the trees is 17 ft, then the age of the trees is 8.1 y.



Practice Problem:

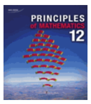
Complete “Check your Understanding” question 3 on page 467 of your textbook.

Solution: **3. a)**



b) The regression equation that models the data is
$t = -346.090\ldots + 28.957\ldots \ln P$
P-intercept: 155 076.923…
t-intercept: none
End behaviour: QIV to QI
Domain: $\{P \mid P > 0, P \in W\}$
Range: $\{t \mid t \geq 0, t \in W\}$
Function: increasing

c) $t = -346.090\ldots + 28.957\ldots \ln P$
$t = -346.090\ldots + 28.957\ldots \ln (2\,000\,000)$
$t = 74.043\ldots$
The population exceeded 2 000 000 in 1974.



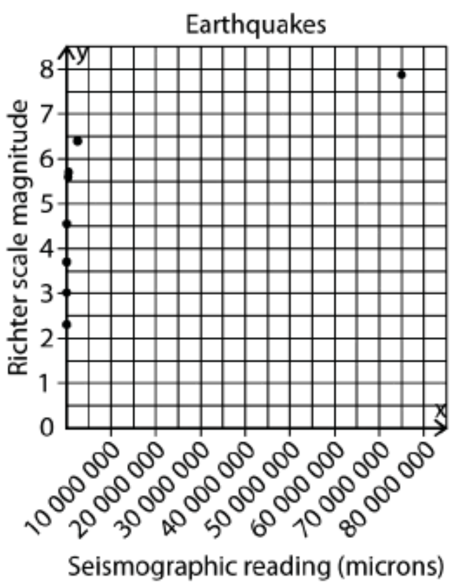
Practice Problem:

Complete “Check your Understanding” question 4 on page 468 of your textbook.

Solution:

4. a) The independent variable is seismographic reading and the dependent variable is Richter Scale magnitude.

b)



c) The regression equation for the data is
$M = -0.006\ldots + 0.434\ldots \ln r$.
d) $M = -0.006\ldots + 0.434\ldots \ln r$

Let $M = 5.7$

$$5.7 = -0.006\ldots + 0.434\ldots \ln r_1$$
$$5.706\ldots = 0.434\ldots \ln r_1$$
$$13.122\ldots = \ln r_1$$
$$r_1 = e^{13.122\ldots}$$

Let $M = 4.5$

$$4.5 = -0.006\ldots + 0.434\ldots \ln r_2$$
$$4.506\ldots = 0.4348 \ln r_2$$
$$10.362\ldots = \ln r_2$$
$$r_2 = e^{10.362\ldots}$$

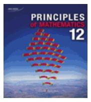
$$\frac{r_1}{r_2} = \frac{e^{13.122\ldots}}{e^{10.362\ldots}}$$

$$\frac{r_1}{r_2} = e^{(13.122\ldots - 10.362\ldots)}$$

$$\frac{r_1}{r_2} = e^{2.759\ldots}$$

$$\frac{r_1}{r_2} = 15.794\ldots$$

A magnitude 5.7 earthquake is about 15.8 times more intense than a magnitude 4.5 earthquake.



Practice Problem: (KEY QUESTION)

Complete "Check your Understanding" question 5 on page 468 of your textbook.

Solution:

5. a) The independent variable is pressure and the dependent variable is altitude.

b) The regression equation for this data is $h = 30\ 665.960\ldots - 6640.436\ldots \ln P$.

c) P-intercept: 101.297…
h-intercept: none
End behaviour: QI to QIV
Domain: $\{P \mid P > 0, P \in \mathrm{R}\}$
Range: $\{h \mid h \in \mathrm{R}\}$
Function: decreasing

d)

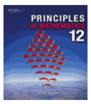
$$\begin{aligned} y &= 30\ 665.960\ldots - 6640.436\ldots \ln x \\ 139 &= 30\ 665.960\ldots - 6640.436\ldots \ln x \\ -30\ 526.960\ldots &= -6640.436\ldots \ln x \\ 4.597\ldots &= \ln x \\ x &= e^{4.597\ldots} \\ x &= 99.199\ldots \end{aligned}$$

The pressure setting that Michael will need to use is 99.2 kPa.

e)

$$\begin{aligned} y &= 30\ 665.960\ldots - 6640.436\ldots \ln x \\ 8848 &= 30\ 665.960\ldots - 6640.436\ldots \ln x \\ -21\ 817.960\ldots &= -6640.436\ldots \ln x \\ 3.285\ldots &= \ln x \\ x &= e^{3.285\ldots} \\ x &= 26.725\ldots \end{aligned}$$

The atmospheric pressure at the summit of Mt. Everest is 26.7 kPa.



Practice Problem:

Complete “Check your Understanding” question 7 on page 469 of your textbook.

Solution:

7. a) The exponential regression equation that represents this data is $A = 14\,999.826\ldots(1.045\ldots)^t$.

b) The logarithmic regression equation that represents this data is $t = -218.442\ldots + 22.717\ldots \ln A$.

c) Exponential:

$$A = 14\,999.826\ldots(1.045\ldots)^t$$
$$25\,000 = 14\,999.826\ldots(1.045\ldots)^t$$
$$1.666\ldots = (1.045\ldots)^t$$
$$\log 1.666\ldots = \log (1.045\ldots)^t$$
$$\log 1.666\ldots = t \log 1.045\ldots$$
$$\frac{\log 1.666\ldots}{\log 1.045\ldots} = t$$
$$11.604\ldots = t$$

Logarithmic:

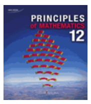
$t = -218.442\ldots + 22.717\ldots \ln A$

$t = -218.442\ldots + 22.717\ldots \ln (25\,000)$

$t = 11.604\ldots$

It will take about 12 years for the balance to equal \$25 000 according to both equations.

e.g., I prefer using the logarithmic because it makes the calculations simpler.



Practice Problem:

Complete “Closing” question 10 on page 471 of your textbook.

Solution:

10. Enter the data in my calculator, perform a logarithmic regression, graph the data points in a scatter plot and the logarithmic function on the same axes, and identify the point with the known x-value and the unknown y-value.

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