

Sine and Cosine Graphs and Law of Sines using Spaghetti

*Lesson Plans by Michael D. Sturdivant
Mathematics Instructor
Arkansas School for Math & Science*

May, 2002

Please feel free to contact me at (501) 622-5168 or email me at mdstermite@hotmail.com

Various Topics in Trigonometry

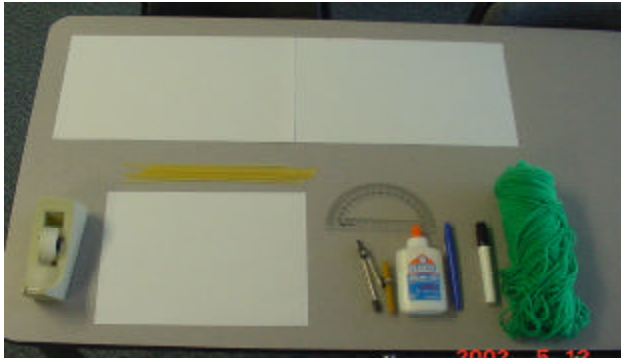
Lesson 1 – Spaghetti Trig – Developing the graphs of sine and cosine functions from the unit circle

Objectives:

Students will develop a better understanding of the unit circle.
Students will make the connection between the unit circle and the graphs of the sine and cosine trig functions.

Materials Needed:

Poster board or two legal sheets for each student
Yarn (a light color preferably)
Tape or Glue
Protractors
Compasses
Spaghetti
Markers (a different color than the yarn)
One sheet of letter-size paper for each student



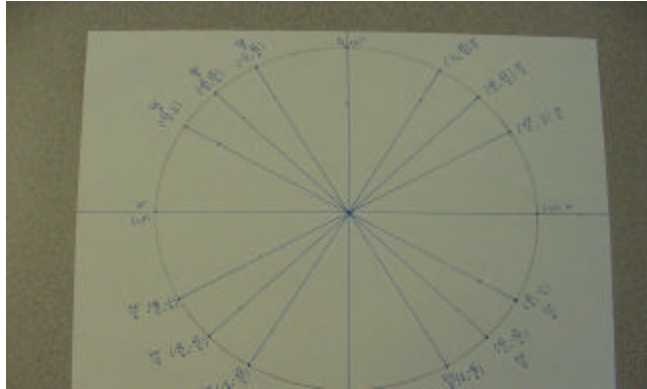
Prep before class begins:

You will need 50-65 minutes to complete the project, so the more you have prepared for the students, the better off you will be. The students will need the following items at their desks before class begins. They will need white poster board or two legal sheets taped together to make one long $8\frac{1}{2}$ " x 23" sheet, yarn (at least 3 feet for each student), tape or glue, protractor, compass, 7-10 noodles of spaghetti, and a marker.

Procedure:

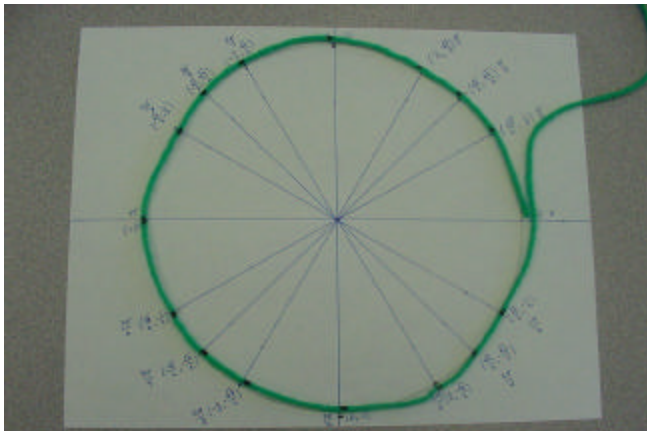
The first part of this lesson could actually be done on a separate day. Students should create a unit circle on the letter-sized paper. Begin by having the students draw an x and y axis on the letter-sized paper. The origin should be as close to the center of the paper as possible. Use the compass to create a circle with a center at the origin and a radius of at

least 3 1/2". Use the protractor to measure angles of $\pi/6$, $\pi/4$, and $\pi/3$. The point where the terminating side of these angles intersect with the unit circle will have ordered pairs of $(\frac{\sqrt{3}}{2}, \frac{1}{2})$, $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ respectively.



Then have students measure the same reference angles in each of the remaining quadrants, marking the coordinates of the ordered pairs as they go (paying careful attention to sign). The students should also mark the ordered pairs for the points where the unit circle intersects the x and y axes as well.

Now have the students wrap the string around the unit circle. One end of the string should be placed at (1, 0). Use the marker to mark points on the string at each of the special angles.



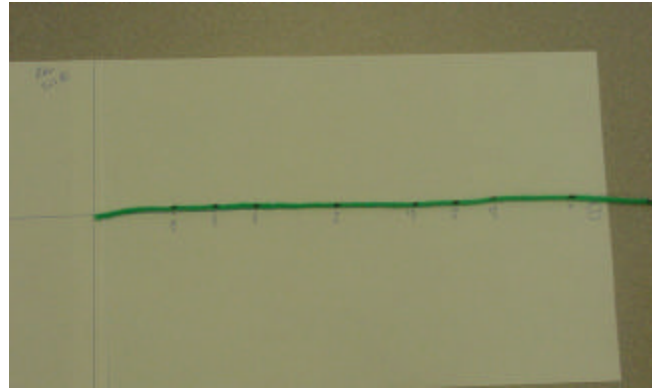
It is important to note to the students at this point that we are interested in finding a function in terms of the angle θ that will give us the value of the x coordinate and a function in terms of the angle θ that will give us the value of the y coordinate. From previous study, the students should know that the functions we are looking for are sine and cosine.

Remind the students that $x = \cos(\theta) = f(\theta)$ and $y = \sin(\theta) = g(\theta)$. This means that if we are to graph the sine function our new independent variable is the angle θ . The remainder of this lesson plan will discuss completing the project for graphing the sine function. It is easily adapted to graphing the cosine function.

Have the students draw a coordinate axis on their poster board or legal paper. Since θ is the independent variable, the horizontal axis should be labeled θ and the vertical axis should be labeled $g(\theta) = \sin \theta$.

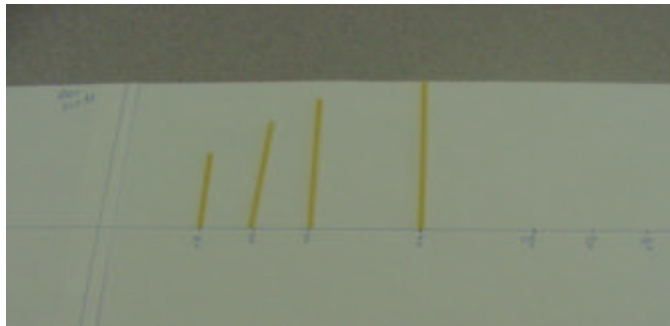
The yarn can now be placed along the horizontal axis with the end of the string that started at the point (1,0) now placed at the origin since this represents $\theta = 0$. The marks that were made on the yarn will give us the angle measures that we need. Thus the first mark can be transferred to the poster board and labeled $\pi/6$. The second mark can be transferred to the poster board and labeled $\pi/4$, etc. Once the students have labeled the

positive horizontal axis, they can flip the string over and label the negative horizontal axis using the same marks. Note to the students that this is using negative angle measures.



We will now go back to our unit circle. Since we are interested in the sine function which corresponds to the y value, we are interested in the distance a given point is from the horizontal axis. This is where the spaghetti will help. At an angle measure of 0, the y value is 0, thus we will not need any spaghetti. However, at $\pi/6$, the y value is $1/2$. Have the students place a noodle of spaghetti vertically with one end on the horizontal axis and the remainder of the noodle going through the point $(\frac{\sqrt{3}}{2}, 1/2)$. They should break the noodle off at that ordered pair. Thus

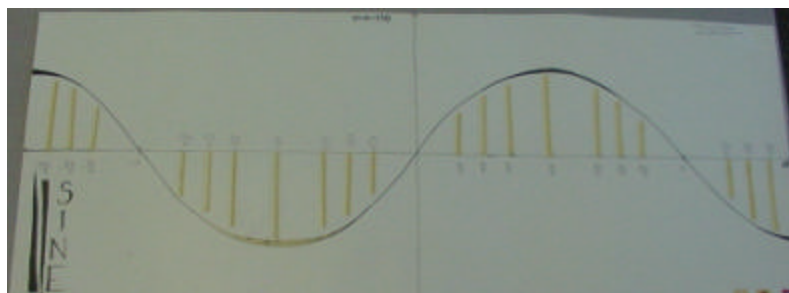
the noodle gives us a visual image of the y value ($\sin \theta$) at $\pi/6$. Take that piece of spaghetti and glue it on the poster board at $\pi/6$. Be sure to tell the students that since the y value was positive, then the spaghetti should be placed above the horizontal axis. Repeat this process at each special angle. When the students get to the third and fourth quadrants, remind them that they should now be placing their



spaghetti below the horizontal axis.

Be sure to have the students also work with the negative angles as well, reminding them that $-\pi/6$ will be in the 4th quadrant and so the spaghetti will be placed below the horizontal axis. When the students finish they should see a perfect sine curve. You can fill in the gaps by having the students measure angles of $\pi/12$ and $5\pi/12$ and their corresponding reference angles around the unit circle.

It may be a good idea to have half of your students graph the sine function and half graph the cosine function. The activity is identical with the exception that the spaghetti will be used to measure the ordered pairs distance from the vertical axis in the unit circle.



Lesson 2 – Using spaghetti to discuss the Law of Sines

Objective: This lesson will help students gain a better understanding of the law of sines. In particular, this will help students understand the ambiguous case and see why it is ambiguous.

Materials Needed:

Overhead projector
Spaghetti
Protractor

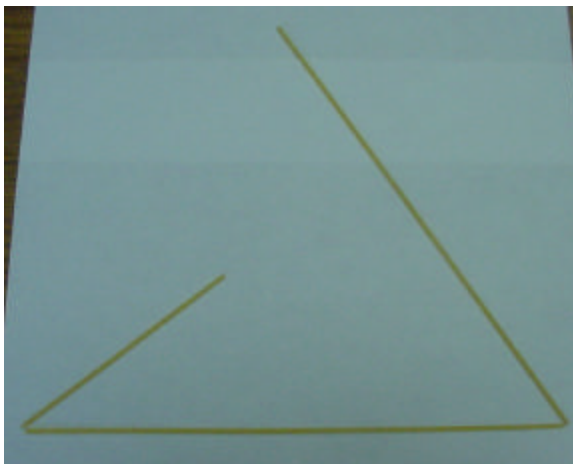
Procedure:

When discussing the Law of Sines, students sometimes have difficulty understanding why this method works. This lesson gives a visual image that can be manipulated so the students can see what is happening.

There are three cases that need to be discussed when covering the Law of Sines, AAS, ASA, and SSA. We will take these in that order. The first two are relatively straightforward and will not require a great deal of explanation. I have found that if you place the spaghetti illustrations on the overhead it is easy to manipulate and all of the students can easily see what is happening.

AAS

To illustrate AAS you will need two full noodles of spaghetti and one shorter noodle. The given information in this case is two angles and a non-included side. The shorter noodle of spaghetti will serve as our given side.



Place a full length of spaghetti horizontally on the overhead. With the second full noodle, create an acute angle with the horizontal noodle. You may wish to have a student give you an angle measurement less than 180° , preferably around 60° just to make the illustration easier to see. You can make the correct angle using the protractor. Have another student give you a second angle measurement less than 180° - the first angle. Create this angle with the shorter noodle. I recommend that you originally

create the angle at the opposite end of the horizontal noodle. This will originally cause some confusion because it will likely not form a triangle. Ask the students if this means

that no triangle can be created with those measurements. After you have given them time for discussion, remind them that the horizontal noodle does not represent a given side. Therefore we can change the length of that side. Maintaining the given angle measurement, slide the shorter noodle across the horizontal noodle until a triangle is formed. Illustrate to the students that the original angle chosen could have been acute or obtuse and the triangle would still be created. Also note to the students that if we are all given the same pieces of information, then we will all end with congruent triangles.



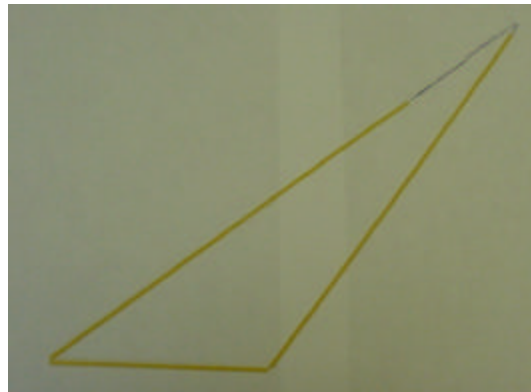
ASA

You can use the same noodles of spaghetti to illustrate this case. However, remind the students that we are now given an included side. To illustrate this, I recommend placing the shorter noodle horizontally. Since this side length is now given, the two angles must be created at the endpoints of the noodle. Again ask the students to give two angle measurements whose sum is less than 180° . I suggest you work this illustration twice. Once with two acute angles and once with an obtuse angle.



For the case with two acute angles, use the protractor to create the correct angle measurements. The students will see that the two unknown sides will clearly intersect at a point, forming the triangle. Remind the students that any excess length is not a problem since the lengths of those two sides are not given.

For the case with an obtuse angle, the students may claim that they cannot make the triangle, however, remind them that the



side lengths are not given and they can extend those sides as long as necessary to form the triangle. Again, point out to the students that in either case, if we are given the same information, we will obtain congruent triangles.

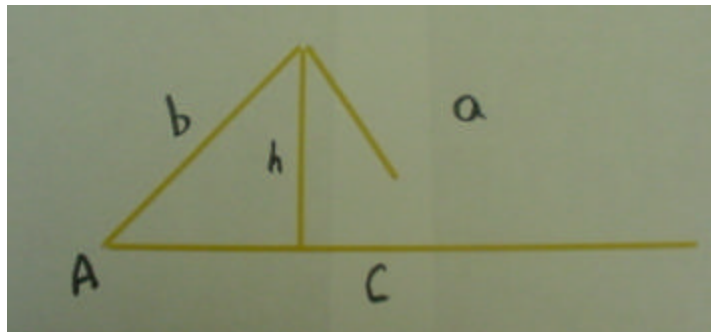
SSA

Not only is this illustration excellent for demonstrating the ambiguous case, it will convince students why we do not have a SSA congruency theorem.

For this illustration you will need four noodles of spaghetti, one full length and three shorter noodles of varying lengths. You will likely want to break these pieces off as you go since often the length you need depends on the angle the students choose. This case has multiple sub-cases, we will go through an illustration for each of them.

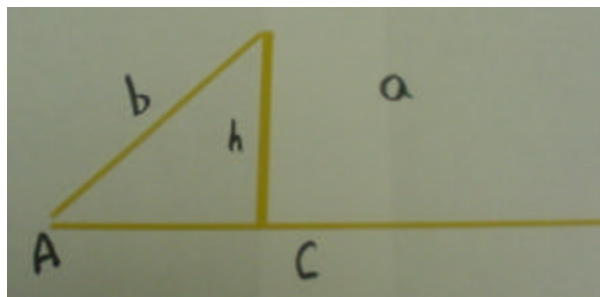
Case 1 – given angle A is acute and $a < h$ (side opposite angle a is less than the height of the triangle.)

Note that in this case, nothing is said about how b relates to a and h . Thus we can choose b to be any length we want. Break off a noodle at any length at place it at the left endpoint of the longer noodle. The angle formed here is the given angle (chosen by the students if desired) so you will need to use the protractor to measure the angle. Since b is a given side length we will break off another noodle and place it at the opposite endpoint of b and perpendicular to the unknown side. This represents the height, h . We have now restricted our options for a . We know that its length must be less than h . Break off a noodle to represent a that will be shorter than h . Show the students that there is no where to place a so that a triangle is formed.

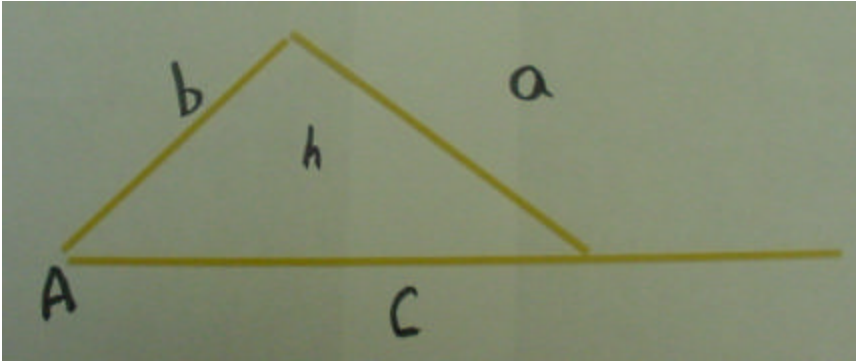


Case 2 – given angle A is acute and $a=h$.

This case is very similar to the previous case. The only change that will need to be made is to break off a noodle so that a is the same length as h rather than being shorter than h . The students should see that there is only one triangle that can be formed and that is a right triangle.



Case 3 – given angle A is acute with $a > b$.

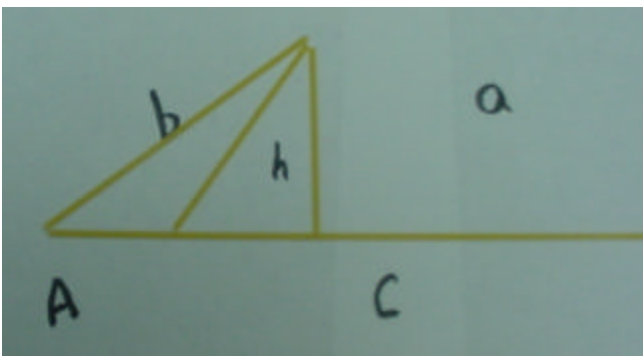
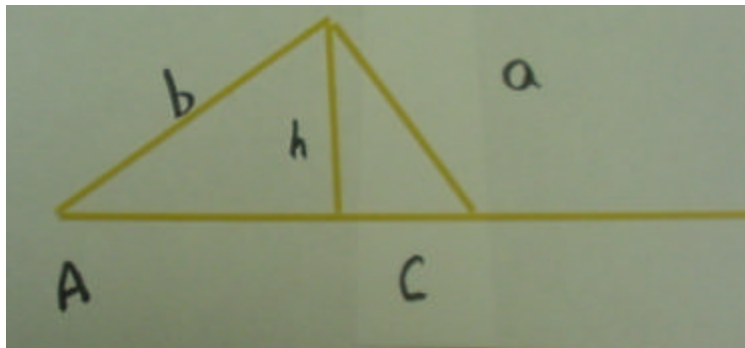


This case may be the easiest to illustrate. Since the height does not matter, simply choose two noodles of different lengths. Choose the shorter of the two to represent b and

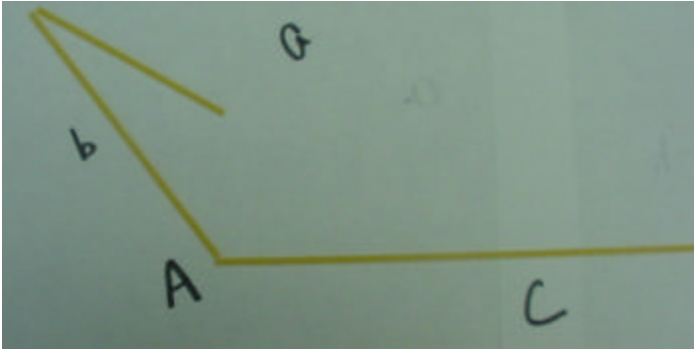
make the appropriate angle A . It will be easy for the students to see that there is only one position for a that will make a triangle. That is the correct position.

Case 4 – given angle A is acute with $h < a < b$

In this case you will have to be more careful. The easiest way to illustrate this is to choose three noodles of spaghetti of varying lengths. The longest of these noodles is to be side b . The shortest will be the height. I recommend that you do not choose a particular angle for this case. Create a right triangle with h , b and the unknown side. The angle between b and c is now set and cannot be changed. This will be easier than trying to break spaghetti off at the needed points.



This case could result in two triangles. Demonstrate to the students that side a could be placed on either side of the height to form a triangle. This is also an excellent time to discuss why we do not have a SSA congruency theorem.



Case 5 – given angle A is obtuse with $a \leq b$

In this case you will need to create an obtuse angle A and illustrate that if $a \leq b$ we will not be able to form a triangle.

Case 6 – given angle A is obtuse with $b < a$

For this final case you can use the same noodles that were used in the previous example. By switching a and b the students will see that one and only one triangle can be formed.

