Math 30-2: U8L1 Teacher Notes

Understanding Angles

Key Math Learnings:

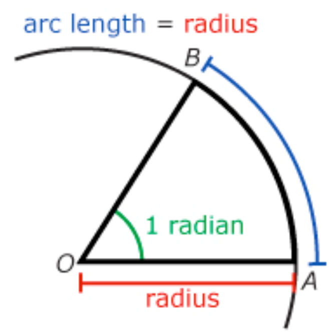
By the end of this lesson, you will learn the following concepts:

★ Understand that angles can be measured in both radian and degrees and that you can convert between the two measures.

What is a Radian Measure?

Just like the length of a pencil can be measured in centimeters or inches, angles can be measured in more than one unit. We have learned that the measurement for angles is degrees. In this lesson we will learn how to measure an angle in radians and how to convert from degrees to radians and vice versa.

The diagram show a **radian** measure. The measure of angle *AOB* is 1 rad (radian) since the measure of arc *AB* is equal to the radius of the circle.



When an angle measurement is given and there are no units written after the measurement, you can assume the units are in radians. When you're writing an angle measurement in degrees, the degree symbol must be included to indicate the measurement is in degrees. For example,

75° indicates 75 degrees

110 indicates 110 radians (rad)

Converting Degrees to Radians

To convert from degrees to radians, we multiply the angle by $\frac{\pi}{180}$.

Example:

Convert the following degree to radian $135°$.

Solution:

$$135° \times \frac{\pi}{180}$$
$$= \frac{135\pi}{180}$$
$$= \frac{3\pi}{4}$$

Converting Radians to Degrees

To convert from radians to degrees, we multiply the angle by $\frac{180}{\pi}$.

Example:

Convert the following radian to degree 1.24 rad .

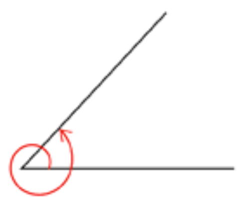
Solution:

$$1.24 \times \frac{180}{\pi}$$

$$= 71^\circ$$

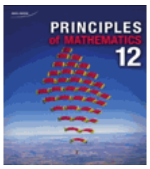
Angles Bigger than 360 Degrees

So far, you have looked at angles between 0° and 360° or 0 and 2π. It is also possible to express angles larger than 360°. Angles Larger than 360° can be used to describe objects that rotate. A spiral is often used to show this in a diagram.



This is an angle larger than 360°.

In general, angles larger than 360° can be treated the same as smaller angles. Converting between radians and degrees follows the same procedure as the one used in previous examples.



Practice Problem:

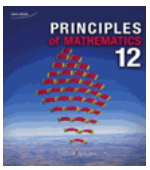
Complete “Check your Understanding” question 2a, b and c on page of your textbook.

Solution:

a. $1.6 \times 60 = 96°$

b. $0.5 \times 60 = 30°$

c. $2.4 \times 60 = 144°$



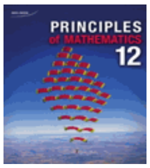
Practice Problem:

Complete “Check your Understanding” question 5 page of your textbook.

Solution:

a. $8.1 \times 60° = 486°$, so 8.1 rad is approximately 486°.

b. $10.5 \times 60 = 630°$, so 10.5 rad is approximately 630°.



Practice Problem:

Complete "Check your Understanding" question 6 page of your textbook.

Solution:

6. a) A clock has 60 minutes in a full circle. That means that each minute is $\dfrac{360°}{60\text{ min}} = 6°$.

i) $\dfrac{120°}{6°} = 20$ minutes. It will be 9:20 am.

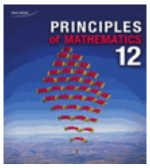
ii) $\dfrac{330°}{6°} = 55$ minutes. It will be 9:55 am.

iii) $\dfrac{690°}{6°} = 115$ minutes. It will be 10:55 am.

b) i) e.g., 120° = 180° – 60°
180° is about 3.1 radians.
60° is about 1 radian.
3.1 – 1 = 2.1 radians
Therefore I estimate that 120° is about 2.1 radians.

ii) e.g., 330° = 360° – 30°
360° is about 6.3 radians.
30° is about 0.5 radians.
6.3 – 0.5 = 5.8 radians
Therefore I estimate that 330° is about 5.8 radians.

iii) e.g., 690° = 360° + 360° – 30°
360° is about 6.3 radians.
30° is about 0.6 radians.
6.3 + 6.3 – 0.6 = 12.0 radians
Therefore I estimate that 690° is about 12.0 radians.



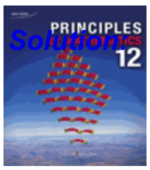
Practice Problem: **(KEY QUESTION)**

Complete “Check your Understanding” question 8 page of your textbook.

Solution:

a. Converting radians to degrees, you find $2 \times 60° = 120°$. Thus, 2 rad is greater than $100°$.

c. Converting radians to degrees, you find $0.5 \times 60° = 30°$. Thus, $45°$ is greater than 0.5 rad.



Practice Problem:

Complete “Check your Understanding” question 10 page of your textbook.

Solution:

10. e.g., Use benchmarks to estimate the radian measure equivalent of angles greater than 360°, where 1 radian is about 60°, 3.2 radians is about 180° and 6.3 radians is about 360°. For example, determine the radian measure of

i) 480° **ii)** 525° **iii)** 650°

i) 480° = 8 · 60°, and 8 · 1 = 8
OR 480° = 360° + 90° + 30°, and
6.3 + 1.6 + 0.5 = 8.4

ii) 525° < 540°
540° = 9 · 60°, and 9 · 1 = 9
OR 540° = 360° + 180°, and 6.3 + 3.2 = 9.5

ii) 650° < 660°
660° = 11 · 60°, and 11 ·1 = 11
OR 660° = 360° + 180° + 120°, and

$$2\pi + \pi + \frac{\pi}{3} = \frac{10\pi}{3}$$