Math 30-2: U8L3 Teacher Notes

The Graphs of Sinusoidal Functions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Describe, orally and in written form, the characteristics of sinusoidal functions by analyzing their graphs.
- Interpret the graph of a sinusoidal function that models a situation, and explain the reasoning.

Graphing Sinusoidal Functions

So far, you have determined characteristics of a sinusoidal function from a graph. Sometimes, it is also useful to sketch a graph from given information. Graphing a sinusoidal function from bits of information requires you to think about many things at once.

The following example shows how the graph of a function can be drawn given pieces of information. Sketch the graph of a sinusoidal function with the following characteristics:

- The domain is $\{x \mid 0° \le x \le 360°, x \in R\}$.
- The range is $\{y \mid -1 \le y \le 5, y \in R\}$.
- The period is 90°.
- The y-intercept is 2.

Video

Click the icon to watch a video on how to graph this sinusoidal function

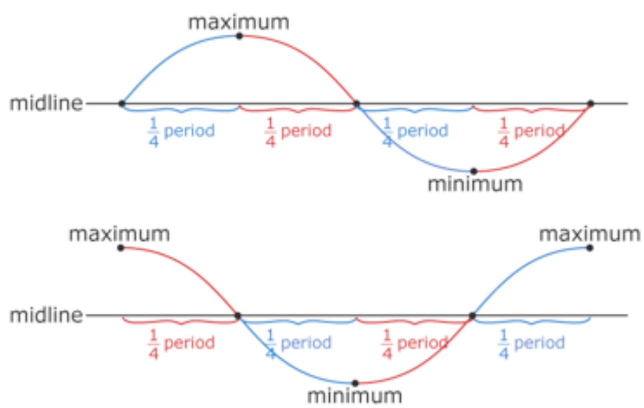
Example

Draw at least two cycles of a sinusoidal function with the following characteristics.

Period	12 rad
Midline	$y = -1$
Maximum	2
Passes Through the Point	$(0, -4)$

Hints:

Graph Size: Start by determining the size of graph you will need. Think about the minimum and maximum values as well as the period.

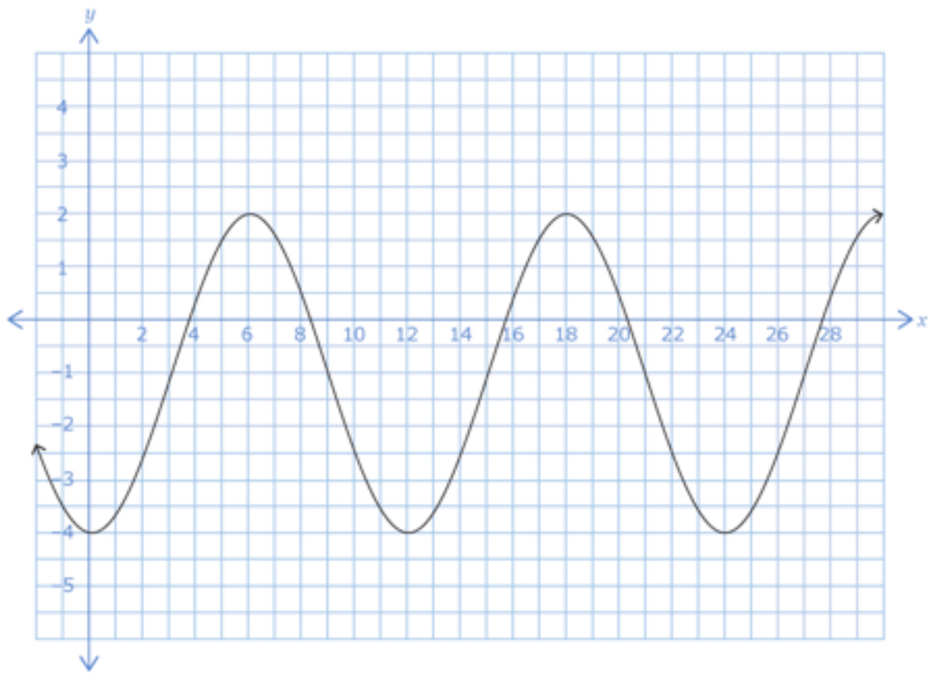


Midline: Next, draw the midline.

Sketching the Graph:

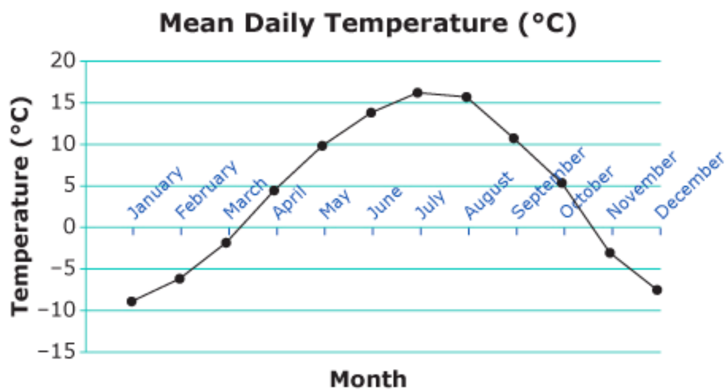
The spacing between a maximum or minimum and an intersection with the midline is always of a period. Use these key points to sketch your graph.

Solution:



Example

At the beginning of the lesson, you saw a mean daily temperature graph for Calgary. Use this information to answer the following questions.



- Predict two months where the mean daily temperature is 5°C.
- What is the mean daily temperature for March?
- Estimate the maximum and minimum values for the graph. Describe what each of these represents.
- Estimate the period of the graph. What does this represent?
- Estimate an equation for the midline. What does this represent?
- Sketch another cycle of this graph. How is this cycle related to the original?
- Describe how this graph would differ from one drawn for Whitehorse and one drawn for Miami.

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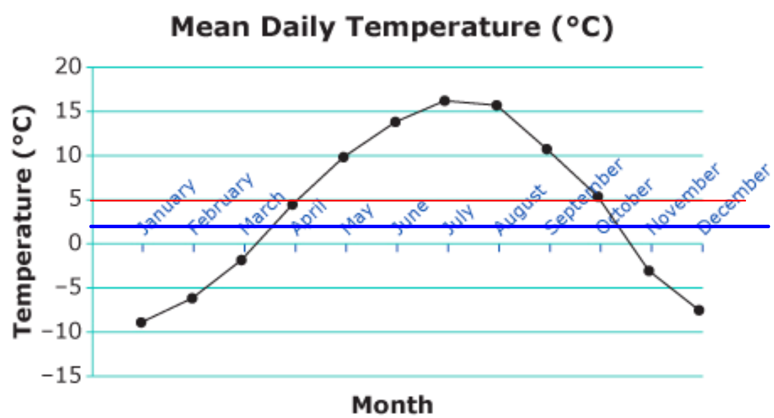


a. April and October are months that are approximately 5 degrees celsius. (Use the red line on the graph above.)

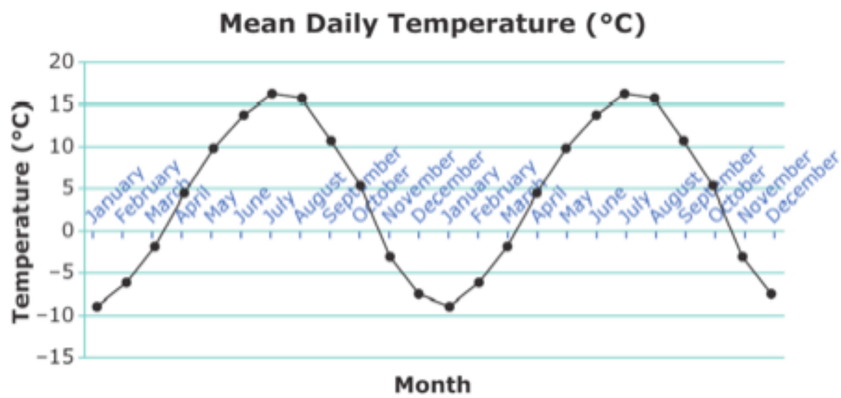
b. The mean average for March is approximately -2 degrees celsius.

c. The maximum value of the graph is approximately 16 degrees celsius and the minimum value of the graph is -9 degrees celsius. These are the highest and lowest average temperatures in the year.

d. The period of the graph is 12 months, which represents one complete year.

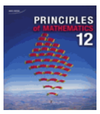
e. The midline of the graph is approximately $y = 2$. This would represent the average temperature of the year.

f.



g. The graph would be moved up in Miami since the average temperatures there are warmer than here. The graph also might not have as big of an an amplitude since the temperatures in Miami are a bit more consistent.

The graph would be moved down in Whitehorse, since the average temperatures there are colder than here.



Practice Problem:

Complete “Check your Understanding” question 4 on page of your textbook.

Solution:

4. a) Maximum value = 3 cm, Minimum value = –7 cm

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{3-(-7)}{2}$$

$$\text{Amplitude} = \frac{10}{2}$$

$$\text{Amplitude} = 5$$

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{3+(-7)}{2}$$

$$y = \frac{-4}{2}$$

$$y = -2$$

$$\text{Period} = \text{second max} - \text{first max}$$

$$\text{Period} = 225° - 45°$$

$$\text{Period} = 180°$$

The range of this graph is $\{y \mid -7 \le y \le 3, y \in R\}$, and its amplitude is 5 cm. The equation of the midline is $y = -2$ cm, and the period is 180°.

b) Maximum value = 6.5 cm
Minimum value = –0.5 cm

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{6.5-(-0.5)}{2}$$

$$\text{Amplitude} = \frac{7}{2}$$

$$\text{Amplitude} = 3.5$$

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{6.5+(-0.5)}{2}$$

$$y = \frac{6}{2}$$

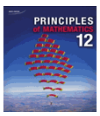
$$y = 3$$

$$\text{Period} = \text{second max} - \text{first max}$$

$$\text{Period} = 5 - 0$$

$$\text{Period} = 5$$

The range of this graph is $\{y \mid -0.5 \le y \le 6.5, y \in R\}$, and its amplitude is 3.5 cm. The equation of the midline is $y = 3$ cm, and the period is 5.



Practice Problem:

Complete “Check your Understanding” question 5 on page of your textbook.

Solution:

5. Maximum value = 3 cm
Minimum value = –2 cm

$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$

$\text{Amplitude} = \frac{3 - (-2)}{2}$

$\text{Amplitude} = \frac{5}{2}$

$\text{Amplitude} = 2.5$

Equation of the midline:

$y = \frac{\text{max} + \text{min}}{2}$

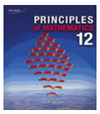
$y = \frac{3 + (-2)}{2}$

$y = \frac{1}{2}$

$y = 0.5$

Period = second max – first max
Period = 6.45 – 2.2
Period = 4.25

The range of this graph is $\{y \mid -2 \le y \le 3, y \in R\}$, and its amplitude is 2.5 cm. The equation of the midline is $y = 0.5$ cm, and the period is 4.25.



Practice Problem: **(KEY QUESTION)**

Complete "Check your Understanding" question 8 on page of your textbook.

Solution:

8. a) The depth of the water with no waves is represented by the equation of the midline of the graph.

$$y = \frac{\text{max}+\text{min}}{2}$$

$$y = \frac{2.4+0.8}{2}$$

$$y = \frac{3.2}{2}$$

$$y = 1.6$$

The depth of the water when no waves are being generated is 1.6 m.

b) The height of each wave is represented by the amplitude of the graph.

$$\text{Amplitude} = \frac{\text{max}-\text{min}}{2}$$

$$\text{Amplitude} = \frac{2.4-0.8}{2}$$

$$\text{Amplitude} = \frac{1.6}{2}$$

$$\text{Amplitude} = 0.8$$

The height of each wave is 0.8 m.

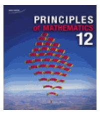
Solution:

c. The amount of time it takes for one complete wave to pass is represented by the period of the graph.

$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 3.125 - 0.625$$
$$\text{Period} = 2.5$$

It takes 2.5 s for one complete wave to pass.

d. After 4 seconds, the water will be about 1.2 m deep, as can be seen from the graph. Since the depth of the water at 5 s is 1.6 m, and the period is 2.5 s, we can assume that at 7.5 s, the water will also be 1.6 m deep.



Practice Problem:

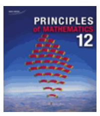
Complete “Check your Understanding” question 10 on page of your textbook.

Solution:

a. Both: the increased amplitude means that the breathes are deeper and the decreased period means more breaths per minute.

b. The amplitude and period of this graph has changed compared to the graph in Question 9.

c. The maximum velocity of air entering the lungs has increased to 1.2 L/s



Practice Problem:

Complete “Check your Understanding” question 12 on page of your textbook.

Solution:

12. Range: $\{y \mid -7 \le y \le 3, y \in \mathrm{R}\}$

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{3-(-7)}{2}$$

$$\text{Amplitude} = \frac{10}{2}$$

$$\text{Amplitude} = 5$$

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{3+(-7)}{2}$$

$$y = \frac{-4}{2}$$

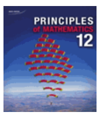
$$y = -2$$

$$\text{Period} = \text{second max} - \text{first max}$$

$$\text{Period} = 0.875 - 0.375$$

$$\text{Period} = 0.5$$

The range of this graph is $\{y \mid -7 \le y \le 3, y \in \mathrm{R}\}$ and the amplitude is 5. The equation of the midline is $y = -2$, and the period is 0.5.



Practice Problem:

Complete “Check your Understanding” question 13 on page of your textbook.

Solution:

13. a) Pendulum A:

$\text{Period} = \text{second max} - \text{first max}$

$\text{Period} = 1 - 0$

$\text{Period} = 1$

Minimum value = 10 cm

Maximum value = 26 cm

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{26 - 10}{2}$$

$$\text{Amplitude} = \frac{16}{2}$$

$$\text{Amplitude} = 8$$

Pendulum B:

$\text{Period} = \text{second max} - \text{first max}$

$\text{Period} = 0.5 - 0$

$\text{Period} = 0.5$

Minimum value = 10 cm

Maximum value = 18 cm

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

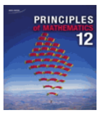
$$\text{Amplitude} = \frac{18 - 10}{2}$$

$$\text{Amplitude} = \frac{8}{2}$$

$$\text{Amplitude} = 4$$

The period for Pendulum A is 1 second, and the period for pendulum B is only 0.5 seconds. Pendulum B is swinging twice as fast as Pendulum A. Both pendulums have a minimum value of 10 cm above the table. Pendulum A reaches a maximum height of 26 cm, while Pendulum B reaches a height of 18 cm. The amplitude of graph A is 8 cm, and of graph B is 4 cm.

b) Pendulum A is longer because its amplitude is greater. The amplitude of graph A is twice as large as the amplitude of graph B.



Practice Problem:

Complete “Check your Understanding” question 15 on page of your textbook.

Solution:

eg., The period and midline stay the same, while the amplitude decreases.

eg., The range is the difference of the maximum and minimum y-values. The amplitude is half the difference of the maximum and minimum values. The equation of the midline is y = average of the minimum and maximum values.

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