Math 30-2: U8L5 Teacher Notes

Modelling Data with Sinusoidal Functions

Key Math Learnings:

By the end of this lesson, you will learn the following concepts:

- Graph data and determine the sinusoidal functions, that best approximates the data.
- Interpret the graph of a sinusoidal function that models a situation, and explain the reasoning.
- Solve, using technology, a contextual problem that involves data that is best represented by graphs of sinusoidal functions, and explain the reasoning.

What is the Sinusoidal Best Fit Curve

It is possible to sketch a curve of best fit for sinusoidal data. The idea of curve fitting is to find a mathematical model that fits your data. The curve fit finds the specific parameters which makes the sinusoidal function match your data as closely as possible.

Example:

A clock with a pendulum sits above a counter. The height of the pendulum above the counter is measured at various time intervals.

Time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height (cm)	22.0	16.0	10.0	16.0	22.0	16.0	10.0	16.0	22.0

a. Plot the data.

b. Sketch a curve of best fit for the data.

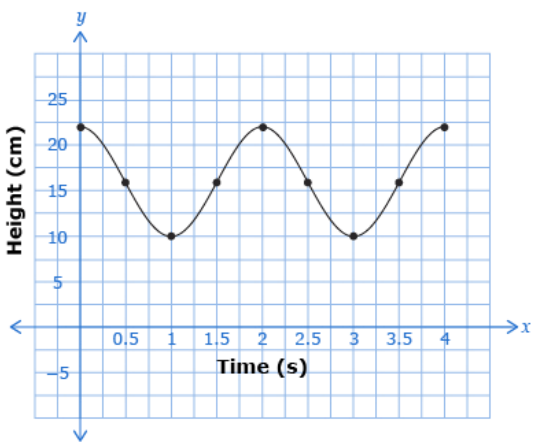
c. What is the period of the graph? What does it represent in this scenario?

d. Use your graph to predict the height of the pendulum at 1.7 s

e. Use your graph to predict the height of the pendulum at 12.5 s

Solution:

a. and b.



c. The period is 2 s. This is the amount of time for the pendulum to complete one full cycle of movement (high to low to high). The period is the time for the pendulum to swing from left to right or from right to left.

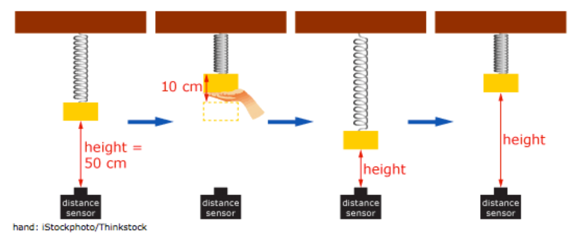
d. The height of the pendulum at 1.7 s is 20 cm.

e. The height of the pendulum at 12.5 s is16 cm.

Although creating a curve of best fit by hand can be done, it is often easier and more accurate to create a curve by using a graphing calculator or some other graphing technology. You can also use this technology to find the regression equation that best fits the data given. A regression is actually a statistical analysis that assesses the association between two variables. It is very difficult to calculate a regression by hand, so you can use technology to help.

Example:

Hagan is performing an experiment with a spring attached to a weight. She places a distance sensor 50 cm below the weight. She then lifts the weight 10 cm and turns on the distance sensor as she drops the weight.



Hagan received the following information from the sensor.

Time (s)	Height (cm)	Time (s)	Height (cm)
0	60.0	2.2	53.2
0.2	57.7	2.4	46.0
0.4	53.2	2.6	41.7
0.6	46.9	2.8	39.8
0.8	41.9	3.0	43.2
1.0	40.0	3.2	51.5
1.2	42.8	3.4	58.1
1.4	50.9	3.6	59.8
1.6	56.9	3.8	58.2
1.8	59.6	4.0	51.4
2.0	59.4		

a. Make a scatter plot of the data using a graphing calculator.

b. Use your calculator to determine a sinusoidal regression equation for the data. Record this equation.

c. Graph the regression equation on the same grid as your scatter plot. How well does the regression equation match the data?

d. What is the period of this graph? Explain how you determined the period.

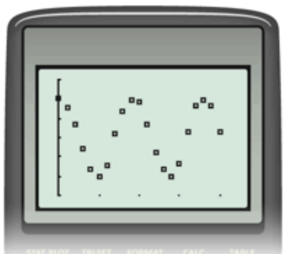
e. Determine the height at 1.34 s.

f. Determine the height at 200.0 s.

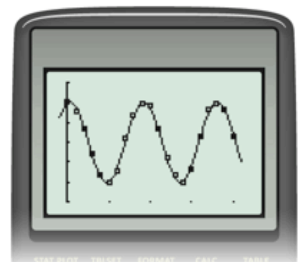
g. Determine the first three points where the weight is at a height of 55.0 cm.

Solution:

a.



b and c.



d. The period of this graph is approximately 1.776 484 55. The period can be calculated using p = 2pi/b or it can be measured from the graph using the distance between the maximums or the distance between the minimums.

e. The height at 1.34 s is $y = 10.1046\,\mathit{sin}[3.5369\ (1.34) + 1.3526] + 49.9940 = 48.074$.

f. The height at 200.0 s is $y = 10.1046\,\mathit{sin}[3.5369\ (200.0) + 1.3526] + 49.9940 = 40.35$.

g. The three points where the weight is at a height of 55.0 cm can be found graphically by finding the points of intersection of $y = 10.1046\,\mathit{sin}(3.5369x + 1.3526) + 49.9940$ and $y = 55$.

The three times are 0.359 s, 1.541 s, and 2.136 s.

Graphical Method to Interpolating

Suppose you have a regression equation of $y = 3.1\sin[0.59(x - 7.60)] + 4.0$ **where** *x* represents time and *y* represents height. If you want to determine the times when *y* = 4.5, it is possible to substitute *y* = 4.5 into the equation and solve as follows:

$$y = 3.1\sin[0.5(x - 7.6)] + 4.0$$
$$4.5 = 3.1\sin[0.5(x - 7.6)] + 4.0$$
$$\vdots$$
$$x = \_\_\_$$

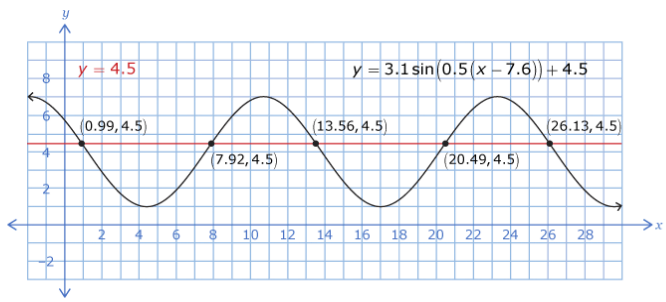
But this process is somewhat complicated. An easier strategy is to graph the equations

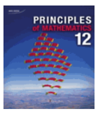
$$y = 3.1\sin[0.5(x - 7.6)] + 4.0$$

y = 4.5

and then find the *x*-value of the intersections.

From the intersections in the graph, you can see that possible *x*-values include 0.99, 7.92, 13.56, 20.49, and 26.13.





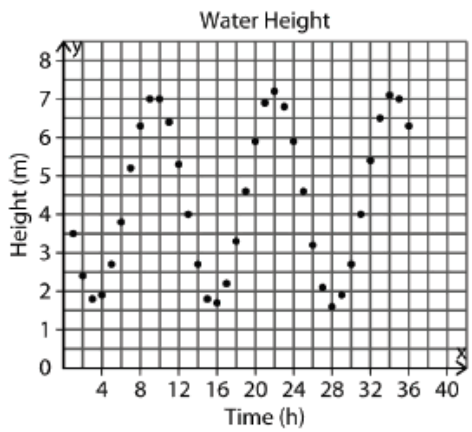
Practice Problem:

Complete “Check your Understanding” question 4 on page of your textbook.

Solution:

4. a) e.g., Tide heights could be sinusoidal, because the tide moves in a cyclical pattern and the data follows a sinusoidal pattern.

b) The equation of the sinusoidal regression function is
$y = 2.726\ldots \sin(0.505\ldots x + 3.016\ldots) + 4.426\ldots$,
which matches the data fairly closely.



Solution:

c) Maximum = $d + a$
Maximum = 4.426… + 2.726…
Maximum = 7.152…
Minimum = $d - a$
Minimum = 4.426… – 2.726…
Minimum = 1.700…
The height at high tide is about 7.2 m. The height at low tide is about 1.7 m.

d) $\text{Period} = \dfrac{2\pi}{b}$

$\text{Period} = \dfrac{2\pi}{0.505...}$

$\text{Period} = 12.418...h$

$\text{Period} = 12\text{ h, } 25.084...\text{ min}$

It takes the tide 12 h, 25 min to cycle from high tide to low tide and back again.

e) Substituting $x = 50$:

$y = 2.726...\sin(0.505...x + 3.016...) + 4.426...$

$y = 2.726...\sin(0.505...(50) + 3.016) + 4.426...$

$y = 2.726...\sin(25.298... + 3.016) + 4.426...$

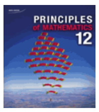
$y = 2.726...\sin(28.315...) + 4.426...$

$y = 2.726...(-0.040...) + 4.426...$

$y = -0.111... + 4.426...$

$y = 4.314...$

At hour 50, the tide will be 4.3 m high.

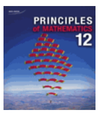


Practice Problem:

Complete “Check your Understanding” question 5 on page of your textbook.

Solution:

At t = 6.145. . . h, the tide is coming in, and is 4 m above the seabed. At t = 12.975...,
the tide is going out and is 4 m above the seabed. The length of time it is safe to sail is
12.975... - 6.145... = 6.83 hours or 6 hours and 50 minutes.



Practice Problem:

Complete “Check your Understanding” question 6 on page of your textbook.

Solution:

a. The regression equation for the average high temperatures is y = 19.582 sin (0.462x -1.623) + 6.239.

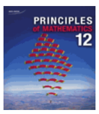
b. The regression equation for the average low temperatures is y = 16.993 sin (0.470x - 1.752) - 4.739.

c. The regression equation for the average temperatures is y = 18.388 sin (0.465x - 1.695) + 0.687.

d. The amplitudes, phase shifts, and periods are very close (i.e., similar), but the vertical shift or the equation of the midline is different.

e. The record daily high temperatures could be represented by a sinusoidal function, but the data is more scattered than the other temperature patterns.

f. The approximate average high temperature for October 1 occurs on month 9.5, so the approximate average high temperature is $y = 19.582 \sin[0.462(9.5) - 1.623] + 6.239 = 14.1°C.$



Practice Problem: (KEY QUESTION)

Complete "Check your Understanding" question 8 on page of your textbook.

Solution:

8. a) The equation of the sinusoidal regression function is
$y = 4.207\ldots \sin(0.017\ldots x - 1.398\ldots) + 12.411\ldots$.
b) The midline is $y = 12.411\ldots$ because $y = d$.
The amplitude is $4.207\ldots$ since a is equal to the amplitude.
Maximum = $d + a$
Maximum = $12.411\ldots + 4.207\ldots$
Maximum = $16.619\ldots$
Convert from decimal hours to hours and minutes.
Maximum = 16 h $(0.619 \cdot 60)$ min
Maximum = 16 h 37 min

Minimum = $d - a$
Minimum = $12.411\ldots - 4.207\ldots$
Minimum = $8.204\ldots$
Convert from decimal hours to hours and minutes.
Minimum = 8 h $(0.204\ldots \cdot 60)$ min
Minimum = 8 h 12 min
The range of the data is from 8 h 12 min to 16 h 37 min.
c) The day with the most hours of sunlight is the x value for the maximum value. The day with most hours of sunlight is day 171.
d) $y = 4.207\ldots\sin(0.017\ldots x - 1.398\ldots) + 12.411\ldots$

$y = 4.207\ldots\sin(0.017\ldots(30) - 1.398\ldots) + 12.411\ldots$

$y = 4.207\ldots\sin(0.520\ldots - 1.398\ldots) + 12.411\ldots$

$y = 4.207\ldots\sin(-0.878\ldots) + 12.411\ldots$

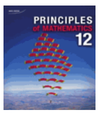
$y = 4.207\ldots(0.769\ldots) + 12.411\ldots$

$y = -3.238\ldots + 12.411\ldots$

$y = 9.173\ldots$

$y = 9$ h, $10.409\ldots$ min

On day 30, Regina will get 9 h 10 min of daylight.
e) To determine which days get 15 h of sunlight, graph the horizontal line $y = 15$. This line crosses the graph twice, at day 119 and at day 224.
Days 119 and 224 will get 15 h of sunlight.



Practice Problem:

Complete “Check your Understanding” question 10 on page of your textbook.

Solution:

10. a) e.g.

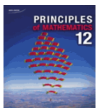
Time (s)	Height (in.)
0	70
0.25	55
0.5	40
0.75	55
1	70
1.25	55
1.5	40
1.75	55
2	70
2.25	55
2.5	40
2.75	55
3	70

b) $y = 15 \sin(2\pi x + 0.5\pi) + 55$

c) e.g., $y = 15\sin(2\pi x - 0.5\pi) + 55.$

$$y = 15\sin(2\pi(7.25) - 0.5\pi) + 55$$
$$y = 15\sin(14.5\pi - 0.5\pi) + 55$$
$$y = 15\sin(14\pi) + 55$$
$$y = 15(0) + 55$$
$$y = 55$$

After 7.25 s, the pendulum will be 55 cm high.



Practice Problem:

Complete “Check your Understanding” question 11 on page of your textbook.

Solution:

11. a) The equation of the sinusoidal function that models the data is
$y = 6 \sin(\pi x - 1.570\ldots) + 11$.

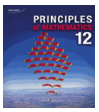
b) At 0.75 s:

$$y = 6\sin(\pi x - 1.570...) + 11$$
$$y = 6\sin(\pi(0.75) - 1.570...) + 11$$
$$y = 6\sin(2.356... - 1.570...) + 11$$
$$y = 6\sin(0.785...) + 11$$
$$y = 6(0.707...) + 11$$
$$y = 4.242... + 11$$
$$y = 15.242...$$

At 5.3 s:

$$y = 6\sin(\pi x - 1.570...) + 11$$
$$y = 6\sin(\pi(5.3) - 1.570...) + 11$$
$$y = 6\sin(16.650... - 1.570...) + 11$$
$$y = 6\sin(15.079...) + 11$$
$$y = 6(0.587...) + 11$$
$$y = 3.526... + 11$$
$$y = 14.526...$$

After 0.75 s and 5.3 s, the spring will be 15.2 in. and 14.5 in. above the table, respectively.



Practice Problem:

Complete “Check your Understanding” question 12 on page of your textbook.

Solution:

12. e.g., No, because the regression function takes into account the effects of other months. The temperature likely will be close to –1.6 °C, but it could be that that temperature is an outlier. If you do a sine regression, to see how regular the pattern is, the results should be more reliable. The data was graphed using 1 for January, 2 for February, and so on. For November, that is, when $x = 11$, $y = –0.695...°$ To the nearest tenth, this is –0.7 °C. This is close to –1.6 °C, but not exact.