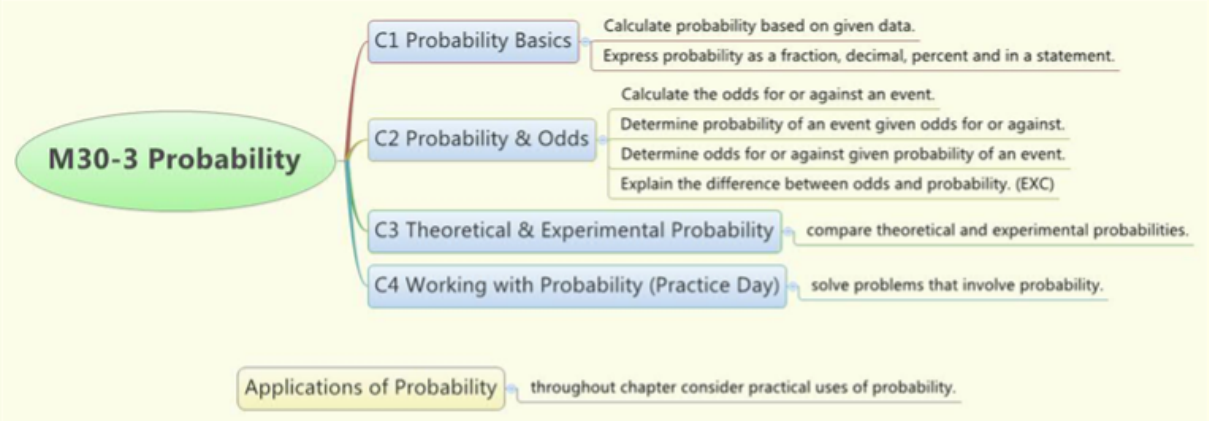
M30-3 Probability Notes



C1 - PROBABILITY BASICS

Probability: the mathematical likelihood of something happening.

What are some practical uses of probability?

Lotteries

Insurance

Warranties

Vehicle Recalls

Weather Predictions

Failure of a Product

Gambling

Investor

Calculate Probability

$$Probability = \frac{\#\ of\ possible\ successful\ outcomes}{total\ \#\ of\ possible\ outcomes}$$

Example: What is the probability of selecting a queen from a standard deck of 52 cards?

successful outcomes = 4 (4 Queens)

possible outcomes = 52 (Total Cards)

$P(\text{Queen}) = \frac{4}{52} = \frac{1}{13} \approx 0.08 = 8\%$

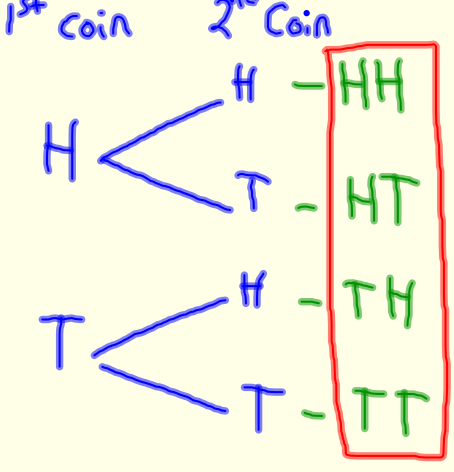
Probability may be expressed as a:
- fraction in lowest terms
- decimal
- percentage
- statement.

Tree Diagrams & Grids

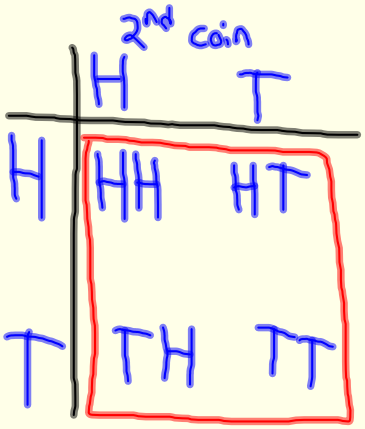
A tree diagram or a grid can be useful ways to list all possible outcomes of an event.

Example: What are all the possible outcomes when flipping a coin two times?

Tree Diagram



Grid



The problem below is now easier to solve since we have used a tree-diagram or a grid to list all the possible outcomes.

Example: In an experiment you flip a coin twice and record the result after each toss. What is the probability of:

a) tossing two heads?

HH HT
TH TT

$P(HH) = \frac{1}{4}$
$= 0.25$
$= 25\%$

b) tossing exactly one head?

HH HT
TH TT

$P(1H) = \frac{2}{4} = \frac{1}{2}$
$= 0.5$
$= 50\%$

c) tossing no heads?

HH HT
TH TT

$P(\text{no } H) = \frac{1}{4}$
$= 0.25$
$= 25\%$

d) tossing at least one head?

HH HT
TH TT

$P(\text{at least } 1H) = \frac{3}{4}$
$= 0.75$
$= 75\%$

Probability of Precipitation (POP)

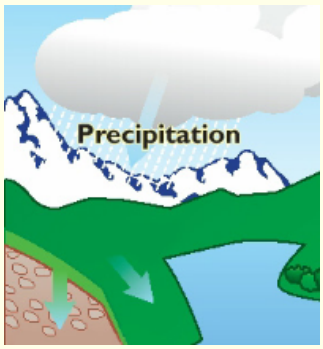
Part of a meteorologist's job is to predict the probability of precipitation (POP). In Gander, NL, one day the POP is 60%. What is the probability as a fraction and as a decimal? (MathAtWork12 pg. 25)

Fraction

$60\% = \frac{60}{100}$
$= \frac{3}{5}$

Decimal

Calc. → $3 \div 5 = 0.6$
or $60 \div 100$



Defective Tires

A tire manufacturing company does random testing of tires coming off the production lines to ensure that they are produced correctly. After testing, the quality control manager calculates that the experimental probability of a tire having a defect is 0.003.

a) What is the probability as a fraction and as a percentage?

b) In a production run of 30 000 tires, how many tires would you expect to be defective? Non-defective? (MathWorks12 pg. 168)

a) Fraction

$0.003 = \frac{3}{1000}$

3 thousandths

Percentage

$0.003 \times 100 = 0.3\%$

b) Defective

$0.003(30\,000) = 90$

Non-defective

$30\,000 - 90 = 29\,910$

C2 - PROBABILITY & ODDS

Odds are another way to measure the likelihood of something happening.

Odds: a ratio that compares the # of possible successful outcomes to the # of possible unsuccessful outcomes.

Odds = # successful outcomes : # of unsuccessful outcomes

Example: When rolling a six-sided die, the odds of rolling a 4 are 1:5 .

1:5 = 1 successful outcome : 5 unsuccessful outcomes

Example

A standard deck has 52 cards. What are the odds of choosing

a) the 7 of spades?

1 : 51

b) a queen?

4 : 48

= 1 : 12

c) a club?

13 : 39

= 1 : 3

d) a red card?

26 : 26

= 1 : 1

Example

You have one six-sided die. What is the probability and the odds of rolling

a) a 2?

Prob. = $\frac{1}{6}$

Odds = 1 : 5

b) a 5 or a 6?

Prob. = $\frac{2}{6} = \frac{1}{3}$

Odds = 2 : 4 = 1 : 2

c) an odd number?

Prob. = $\frac{3}{6} = \frac{1}{2}$

Odds = 3 : 3 = 1 : 1

d) any number but a 3?

Prob. = $\frac{5}{6}$

Odds = 5 : 1

C3 - THEORETICAL & EXPERIMENTAL PROBABILITY

Theoretical Probability: probability determined by reason or calculation.

$$Theoretical\ Probability = \frac{\#\ of\ possible\ successful\ outcomes}{total\ \#\ of\ possible\ outcomes}$$

Experimental Probability: probability determined by experiment.

$$Experimental\ Probability = \frac{\#\ of\ times\ event\ occurs\ in\ experiment}{total\ \#\ of\ trials\ in\ experiment}$$

Example - Rolling a Die

Theoretical

# on die	Prob.
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

In theory, each # should be rolled 1 in 6 times.

Experimental

Roll a die 50 times.

# on die	tally	times rolled	Prob.
1	𝍸 ||	7	7/50
2	𝍸 |||	8	8/50
3	𝍸 |	7	7/50
4	𝍸 |||	9	9/50
5	𝍸 |	8	8/50
6	𝍸 𝍸 |	11	11/50

Experimental results do not always exactly match theoritical results.

Example - Flip a Coin

Suppose you flip a coin 40 times.

a) Theoretically, how many tails should you get?

$P(T) = \frac{1}{2}$ ... so I expect tails 20 times.

b) In an actual experiment, how many tails would you get?

The # of tails I get could vary but I expect it would be close to 20.