

MATH 20-2 FINAL EXAM STUDY GUIDE

Use the following study guide to help you prepare for your final exam. It is taking place on _____ and is worth _____ of your final Math 20-2 grade.

Helpful hints:

- ✓ We have used the textbook and the workbook to complete the course. You should use both resources to study.
- ✓ Make a study plan. Try to study for 20 minutes then take a break then study for 20 minutes more at least 5 days a week. Cramming does not work and will make your brain mush.
- ✓ Your textbook has 'study aid' pages that are helpful to read through
- ✓ Do the practice questions at the end of each chapter.
- ✓ Don't underestimate the power of the digital age... various websites can help you with these concepts. Try <http://www.khanacademy.org> or simply search a topic in *youtube*.
- ✓ Check off each learning objective below when you feel you thoroughly understand it.

Math 20-2 Unit 1: Inductive and Deductive Reasoning

Outcome 1: Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems.

1.1 I can make conjectures by observing patterns and identifying properties, and justify the reasoning.

1. What is a conjecture? Provide an example.
2. Emma works part-time at a bakery shop in Saskatoon. Today, the baker made 20 apple pies, 20 cherry pies, and 20 bumbleberry pies. Which conjecture is Emma most likely to make from this evidence?
 - a. People are more likely to buy cherry pie than any other pie.
 - b. Each type of pie will sell equally as well as the others.
 - c. People are more likely to buy bumbleberry pie than any other pie.
 - d. People are more likely to buy apple pie than any other pie.
3. Which conjecture, if any, could you make about the sum of two odd integers and one even integer?
 - a. The sum will be an even integer.
 - b. The sum will be an odd integer.
 - c. The sum will be negative.
 - d. It is not possible to make a conjecture.
4. Guilia created the following table to show a pattern.

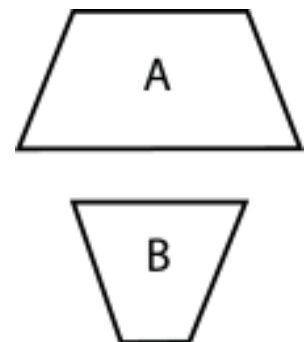
Multiples of 9	18	27	36	45	54
Sum of the Digits	9	9	9	9	9

Which conjecture could Guilia make, based solely on this evidence? Choose the best answer.

- a. The sum of the digits of a multiple of 9 is equal to 9.
- b. The sum of the digits of a multiple of 9 is an odd integer.
- c. The sum of the digits of a multiple of 9 is divisible by 9.
- d. Guilia could make any of the above conjectures, based on this evidence..

1.2 I can explain why inductive reasoning may lead to a false conjecture.

5. What is inductive reasoning?
6. Which figure has the longer top side, A or B? Make a conjecture and check the validity of your conjecture.



7. Mick and Sue were discussing what conjecture could be made regarding the prime number pattern shown.

They both agreed to the following conjecture: "The sum of two prime numbers is an even number."

$$\begin{aligned}3 + 5 &= 8 \\5 + 7 &= 12 \\7 + 11 &= 18 \\13 + 17 &= 30 \\19 + 23 &= 42 \\29 + 31 &= 60\end{aligned}$$

- a) Provide a counterexample which shows their conjecture is false.
b) Revise their conjecture so it holds true for prime numbers.
c) Make a conjecture based on the number pattern if the condition "prime number" was removed.

1.3 I can compare, using examples, inductive and deductive reasoning.

8. What is the difference between inductive and deductive reasoning?
9. All camels are mammals. All mammals have lungs to breathe air. Humphrey is a camel. What can be deduced about Humphrey?
10. Prove, using deductive reasoning, that the product of an even integer and an even integer is always even.
11. Write a conclusion which can be deduced from each pair of statements.
a) Leona lives in 100 Mile House. 100 Mile House is in British Columbia.
b) Joan is taller than Stefan. Stefan is taller than Patrick.
c) The sides of a rhombus are equal. PQRS is a rhombus.
d) Prime numbers have two factors. 13 is a prime number.

1.4 I can provide and explain a counterexample to disprove a given conjecture.

12. What is a counterexample?
13. Austin told his little sister, Celina, that horses, cats, and dogs are all mammals. As a result, Celina made the following conjecture:
All mammals have four legs.
Use a counterexample to show Celina her conjecture is not valid.
14. Abby made the following conjecture:
The sum of a multiple of 7 and a multiple of 8 will be an odd number.
Do you agree or disagree? Briefly justify your decision with a counterexample if possible.

1.5 I can prove algebraic and number relationships such as divisibility rules, number properties, mental mathematics strategies or algebraic number tricks.

15. What is a proof and how do you set up a 2 column proof?

16. Determine the error in reasoning to show that the proof of $5 = 4$ is invalid.

Proof that $5 = 4$

Step 1: Let $a + b = c$.

Step 2: $5a - 4a + 5b - 4b = 5c - 4c$

Step 3: $5a + 5b - 5c = 4a + 4b - 4c$

Step 4: $5(a + b - c) = 4(a + b - c)$

Step 5: Divide both sides by $a + b - c$.

Therefore $5 = 4$.

1.6 I can Prove a conjecture, using deductive reasoning.

17. Prove that the product of an even number and an odd number is always odd.

1.7 I can Determine if a given argument is valid, and justify the reasoning.

18. What type of error occurs in the following deduction?

Briefly justify your answer.

Felix is a barber.

Felix has a very good haircut.

Therefore, Felix is a very good barber.

1.8 I can Identify errors in a given proof; e.g., a proof that ends with $2 = 1$.

19. What type of error occurs in the following proof? Briefly justify your answer.

$$\begin{aligned}2 &= 2 \\4(2) &= 4(1 + 1) \\4(2) + 3 &= 4(1 + 1) + 3 \\8 + 3 &= 6 + 3 \\11 &= 9\end{aligned}$$

1.9 I can Solve a contextual problem that involves inductive or deductive reasoning.

20. Determine the unknown term in this pattern. 2, 2, 4, 6, _____, 16, 26, 42

21. What number should appear in the centre of Figure 4?

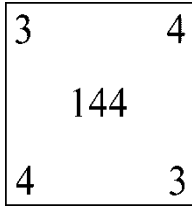


Figure 1

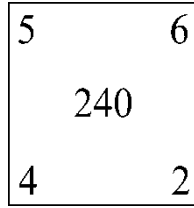


Figure 2

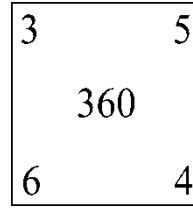


Figure 3

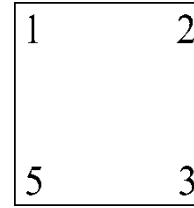


Figure 4

Outcome 2: Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

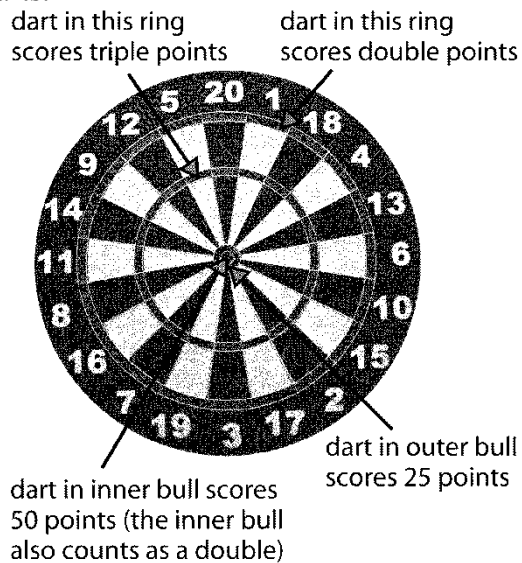
2.1 I can Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., guess and check, look for a pattern, make a systematic list, draw or model, eliminate possibilities, simplify the original problem, work backward, develop alternative approaches.

22. What number(s) could go in the grey square in this Sudoku puzzle? How did you come to this solution?

1		5		3		2	6	
7		3			8		4	
			9					
	1							2
	3	7	5		6	8		4
4								9
	7				3			
	4		2			7		1
	6	2		5		4		9

23. Emma and Alexander are playing darts.

Emma has a score of 48. To win, she must reduce her score to zero and have her last counting dart be a double. Give a strategy that Emma might use to win.



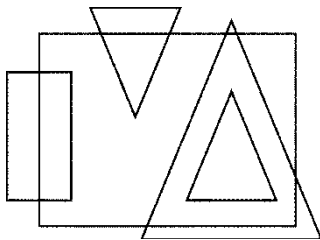
Unit 2: Properties of Angles and Triangle.

Outcome 1: Derive proofs that involve the properties of angles and triangles.

Outcome 2: Solve problems that involve properties of angles and triangles.

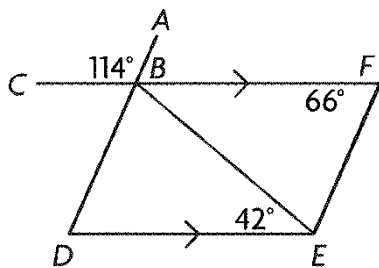
1.1 I can find solutions, using inductive reasoning, the relationships between pairs of angles formed by transversals and parallel lines.

1. Use a pencil to thicken one transversal and a parallel line to that transversal in this pattern.

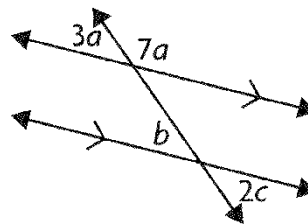


1.2 I can prove, using deductive reasoning, properties of angles formed by transversals and parallel lines, including the sum of the angles in a triangle.

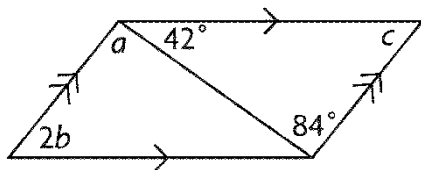
2. Describe and draw examples of the F, Z and C angles.
3. What is the sum of the angles in a triangle? Is this true for all triangles?
4. Determine the measure of $\angle ABF$ and $\angle BEF$.



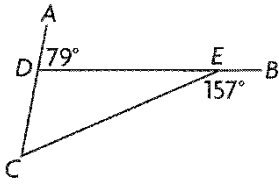
5. Determine the values of a , b , and c .



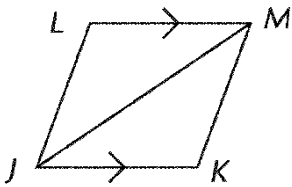
6. Determine the values of a , b , and c .



7. State the correct measures of the interior angles of $\triangle CDE$?



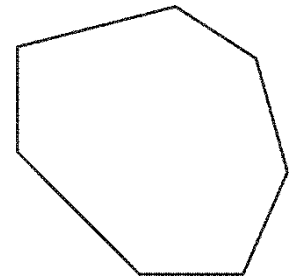
8. Given $LM \parallel JK$ and $\angle LMJ = \angle KMJ$, prove (using a 2 column proof) $JK = KM$.



1.3 I can generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides (n) in a polygon.

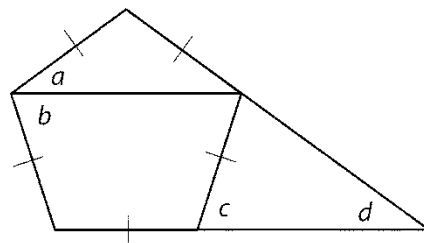
9. What is the formula for determining the sum of interior angles in a polygon? Does the polygon need to be regular?

10. Determine the sum of the measures of the interior angles of this seven-sided polygon. Show your calculation.



11. Each interior angle of a regular convex polygon measures 150° . How many sides does the polygon have?

12. Determine the value of c .



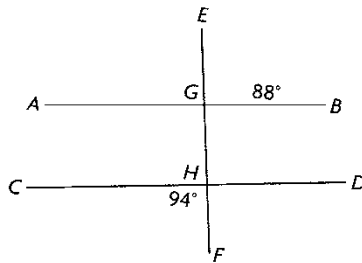
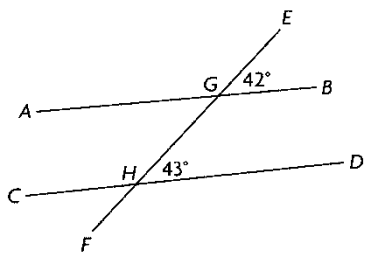
1.4 I can identify and correct errors in a given proof of a property that involves angles.

13. Saito and Dileep determined different measures of the external angles of a regular 32-sided figure. Check each calculation. Identify any errors made and correct the work.

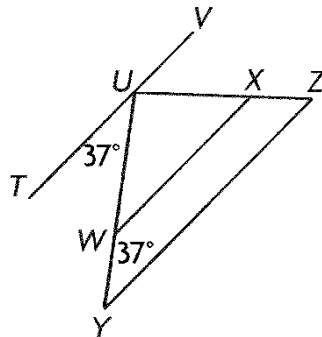
Saito's Solution	Dileep's Solution
In a regular polygon with n sides, when you multiply the measure of the external angles by $n - 2$, the product is 360° .	Determine the interior angle, x , and subtract that value from 360° .
Let x represent the measure of an exterior angle and n represent the number of sides.	$x = \frac{n-2}{n} (180^\circ)$
$x = \frac{360^\circ}{n-2}$	$x = \frac{32-2}{32} (180^\circ)$
$x = \frac{360^\circ}{32-2}$	$x = \frac{30}{32} (180^\circ)$
$x = \frac{360^\circ}{30}$	$x = \frac{5400^\circ}{32}$
$x = 12^\circ$	$x = 168.75^\circ$
The measure of an external angle is 12° .	The measure of an external angle is: $180^\circ - 168.75^\circ = 11.25^\circ$.

1.5 I can verify if lines are not parallel.

14. In which diagram(s) is AB parallel to CD ? Why?



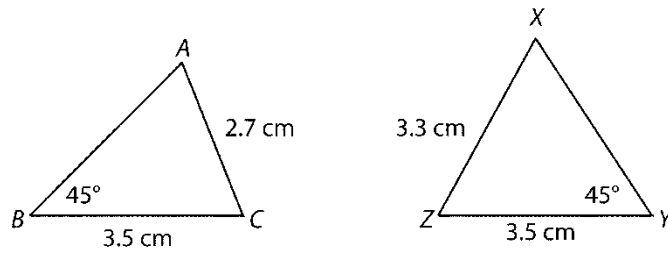
15. Given $\triangle UWX \sim \triangle UYZ$, prove: $WX \parallel YZ$



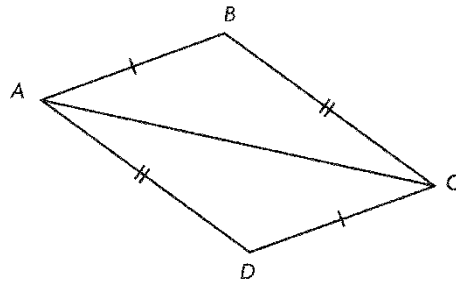
1.6 I can prove that two triangles are congruent.

16. What criteria can be used to prove if a triangle is congruent? Draw examples of each situation.

17. Can you conclude that $\triangle ABC$ is congruent to $\triangle XYZ$? Explain your answer briefly.



18. Joan drew these congruent triangles. List the equal angles.



Unit 3: Acute Triangle Trigonometry

Specific Outcome 3: Solve problems that involve the cosine law and the sine law, excluding the ambiguous case.

3.1 I can draw diagrams to represent a problem that involves the cosine law or the sine law.

1. Sketch a triangle that corresponds to the equation. Then, determine the third angle measure and the third side length.

$$\frac{25.0}{\sin 45^\circ} = \frac{32.0}{\sin 65^\circ}$$

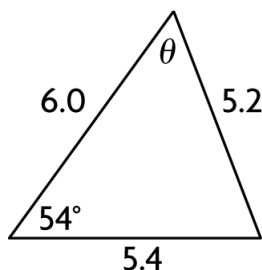
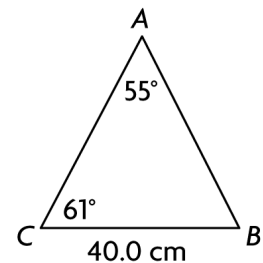
2. In $\triangle RST$, the values of s and $\angle T$ are known. What additional information do you need to know if you want to use the sine law to solve the triangle?
3. In $\triangle WXY$, the values of w , x , and y are known. Write the form of the cosine law you could use to solve for the angle opposite w .

3.2 I can explain the steps in a given proof of the sine law or cosine law.

4. *** This outcome will not be assessed on the final exam.

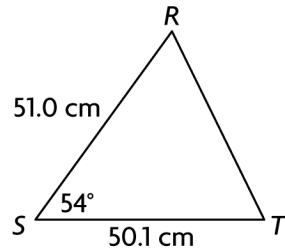
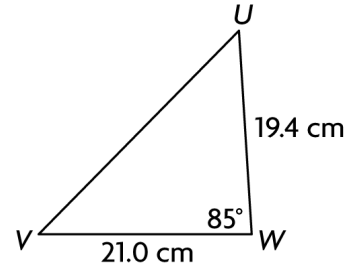
3.3 I can solve a problem that requires the use of the sine law or cosine law, and explain the reasoning.

5. What are the formulas for sine and cosine law?
6. In general, what pieces of information do you need to know in order for sine law to work?
7. Describe a situation where you could not use sine law but you could use cosine law to solve a problem.
8. Determine the length of c to the nearest tenth of a centimeter.
9. Determine the measure of θ to the nearest degree.



10. Determine the length of w to the nearest tenth of a centimetre.

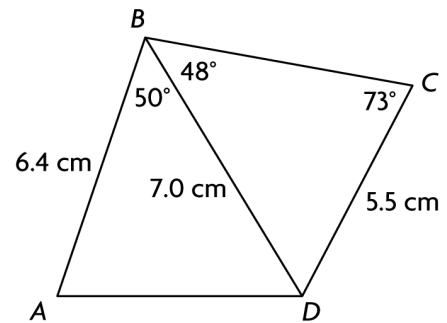
11. Determine the length of s to the nearest tenth of a centimetre.



12. In $\triangle ABC$, $a = 108$ cm, $b = 100$ cm, and $c = 124$ cm. Determine the measure of $\angle C$ to the nearest degree.

3.4 I can solve a problem that involves more than one triangle.

13. Determine the perimeter of this quadrilateral to the nearest tenth of a centimetre.



Specific Outcome 2: Solve problems that involve properties of angles and triangles.

2.3 I can solve a contextual problem that involves angles or triangles.

14. A canoeist leaves a dock on Lesser Slave Lake in Alberta, and heads in a direction $S20^\circ W$ from the dock for 1.5 km. The canoeist then turn and travels north until he is directly west of the dock. How you can determine the distance between the dock and the canoe?

15. A kayak leaves a dock on Lake Athabasca, and heads due north for 2.8 km. At the same time, a second kayak travels in a direction $N70^\circ E$ from the dock for 3.0 km. How you can determine the distance between the kayaks?

Unit 4: Radical Numbers and Problems

Specific Outcome 3. Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands (limited to square roots).

3.1 I can compare and order radical expressions with numerical radicands.

1. Which number is **not** expressed in the simplest form? How can you tell? Simplify the expression.

$$\sqrt{31} \quad 5\sqrt{71} \quad \sqrt{9} \quad 8\sqrt{2}$$

2. Lists these numbers in increasing order?

$$4\sqrt{8}, 2\sqrt{35}, \sqrt{33}, 3\sqrt{12}$$

3.2 I can express an entire radical with a numerical radicand as a mixed radical.

1. List the rules for converting entire radicals to mixed radicals.
2. Express $\sqrt{60}$ as a mixed radical in simplest form.

3.3 I can express a mixed radical with a numerical radicand as an entire radical.

3. List the rules for converting mixed radicals to entire radicals.
4. Express $7\sqrt{20}$ as an entire radical.

3.4 I can +, -, x, and ÷ to simplify radical expressions.

5. Describe the rules for adding and subtracting radical numbers.
6. Describe the rules for multiplying and dividing radical numbers.
7. What is the simplest form of $2\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}$?
8. Express $\sqrt{400}$ as a product of two radicals.
9. Convert $\sqrt{24} - \sqrt{150} - \sqrt{54}$ into mixed radical form. Then simplify.
10. Add $4\sqrt{63} + 4\sqrt{112} + 4\sqrt{175}$.
11. Express $2\sqrt{3} \cdot \sqrt{26}$ in mixed radical form and entire form.
12. Expand the expression $\sqrt{5}(2\sqrt{7} - \sqrt{8})$.
13. Write $\frac{\sqrt{700}}{\sqrt{35}}$ in simplest form.

3.5 I can rationalize the monomial denominator of a radical expression.

14. What does it mean to 'Rationalize a denominator' when dealing with radical fractions? Why would we do it?
15. Write out a set of rules for rationalizing a denominator.
16. Rationalize the denominator in $\frac{\sqrt{190}}{3\sqrt{6}}$.

3.6 I can Identify values of the variable for which the radical expression is defined.

17. State any restrictions on the variable, then expand. $\sqrt{x+3} \cdot \sqrt{x+3}$

Specific Outcome 4. Solve problems that involve radical equations (limited to square roots or cube roots).

4.1 Determine any restrictions on values for the variable in a radical equation and 4.4 I can explain why some roots determined in solving a radical equation are extraneous.

18. Can the total value of an expression in a radical be less than zero? Why or why not? Provide an example.
19. What is the value of x in $\sqrt{x} = 5$?
20. How many solutions are there for $\sqrt{x-4} = -3$? Explain your reasoning.

4.2 Determine, algebraically, the roots of a radical equation, and explain the process used to solve the equation.

21. What is the value of x in $6\sqrt{x} = 18$?
22. What is the value of x in $\sqrt{\frac{17-x}{2}} = 3$?

4.3 I can verify, by substitution, that the solution of a radical expression is true. And 4.5: I can solve problems by modelling a situation with a radical equation and solving the equation.

23. Joss has been contracted to water lawns for the summer using circular sprayers. The radius of sprayed water, r , in metres, is modelled by $r = \sqrt{0.25A}$, where A is the area of grads watered by in square metres. Determine the area covered for a circle with a radius of 12 m. Verify your answer by substitution.
24. State any restrictions on x , then solve $\sqrt{8+x} = 3$.