What is Math
Math is a study of relationships. It is a way to view the world.

What are students doing if they are doing mathematics?

"As part of the study of mathematics, students look for relationships among numbers, sets, shapes, object and concepts.
The search for possible relationship involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form. P. 11)

"Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion" (p. 9)

"Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true." (p. 9)

"Students must feel comfortable taking intellectual risks, asking questions and posing conjectures (p.2)"

Visualization, mental imagery and spatial reasoning .are central to the understanding of mathematics!!!

The test of an activity or assessment: It engages students in the process skills:
Communicating, Connecting, Reasoning, Problem Solving, Visualizing, Applying Mental Math and Estimation, Applying technology to build and examine mathematical relationships. If students can **demonstrate an understanding of a concept** they can:

<div align="center">

Build
Explain
Represent
Compare
Synthesize

</div>

Demonstrate an Understanding of Multiplication as Equal Groups, as Arrays and Areas.

Strategies for multiplication facts MUST include: Commutative Property, Distributive Property, Properties of zero and one for multiplication, Multiplication and Division as inverse operations. Situations with fractins or divisiosn should trigger thinking about multiplication.

Grade 4
Build and label an array. Preferably one that is in their repertoire.
Remember Grade 3 work with facts to 5 x 5.

I like to work with 3 x 4
It is small enough to visualize
I is small enough for students to have as a given fact 3 x 4 equals 12.
It is a manageable size for drawing and estimating.
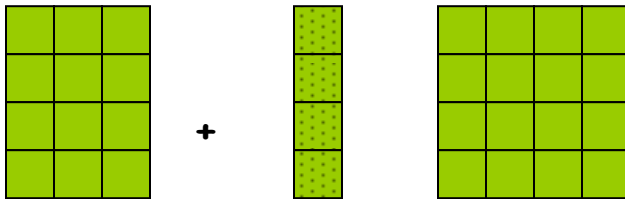**It has one odd and one even factor.**
It can be used to build the threes, the fours, the sixes and the eights.

Strategies: Are based on the distributive and commutative properties

Add on to a know fact:

(3 x 4) + (1 x 4) = 4 x 4
(4 x 4) +(1 x 4) = 5 x 5

(3 x 4) + (1 x 4) = 4 x 4
(4 x 4) + (1 x 4) = 5 x 4
(5 x 4) + (1 x 4) = 6 x 4
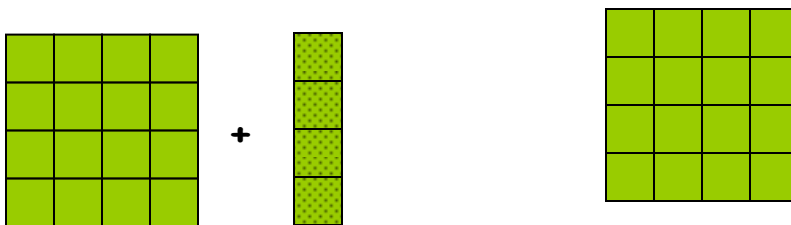
| 3 x 4 | + | 1 x 4 | = | 4 x 4 |

| 4 x 4 | + | 1 x 4 | = | 5 x 4 |

Build and Describe the next one.

(3 x 4) + (1 x 4) = 4 x 4   therefore 12 + 4 = 16
(4 x 4) + (1 x 4) = 5 x 4   therefore 16 + 4 = 20
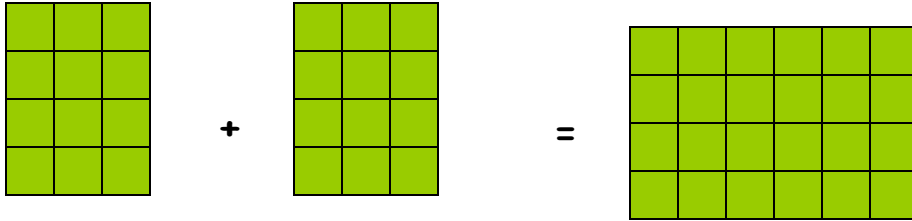(5 x 4) + (1 x 4) = 6 x 4   therefore 20 + 4 = 24

Doubles

$3 \times 4 + 3 \times 4 = 6 \times 4$ $6 \times 4 + 6 \times 4 = 12 \times 4$
$4 \times 3 + 4 \times 3 = 8 \times 3$ $8 \times 3 + 8 \times 3 = 16 \times 3$
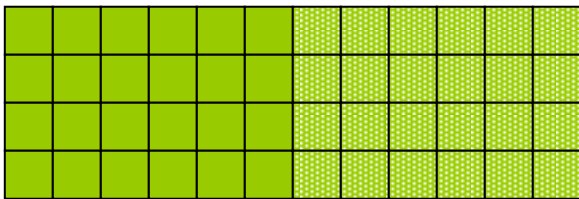
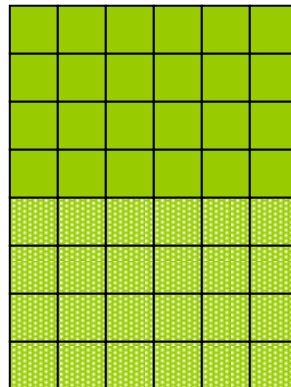| 3 x 4 | | + | | 3 x 4 | | = | | 6 x 4 |

OR $3 \times 4 + 3 \times 4 = 3 \times 8$
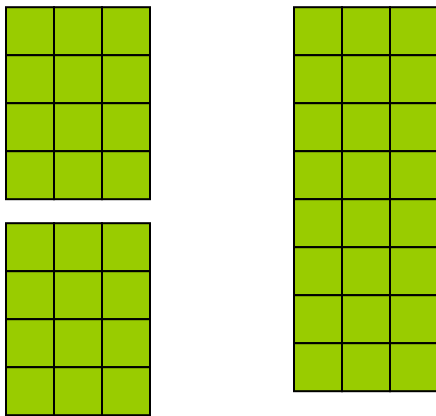
Multiplication is dimensional.

Now $6 \times 4 + 6 \times 4 = 12 \times 4$ **OR $6 \times 4 + 6 \times 4 = 6 \times 8$**

Adding known facts

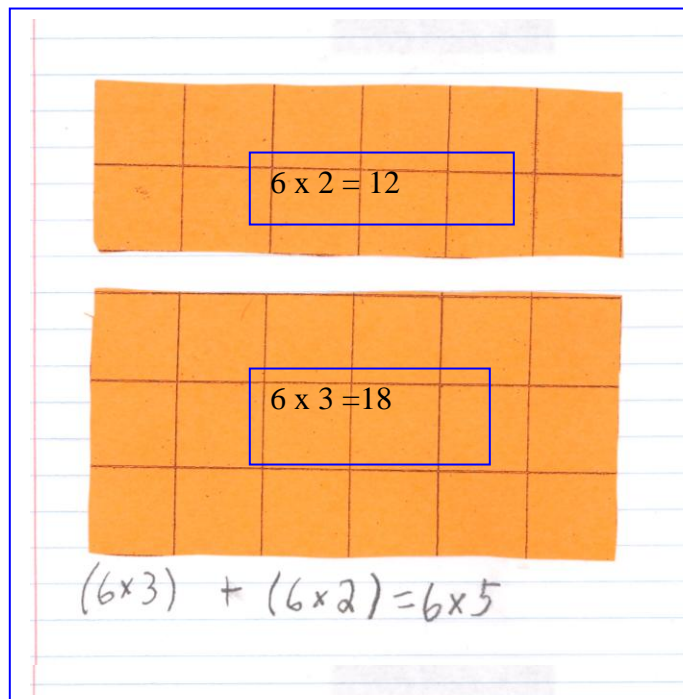**3 x 4 + 4 x 4 = 7 x 4**
3 x 4 + 3 x 5 = 3 x 8

While working with concrete models and representations is important in the early stages, a key aspect in crossing the 'abstracting barrier' appears to be the capacity to work with mental images and strategies based on doubling and known facts without physical objects. A critical step in this process appears to be the shift from counting groups to seeing the number of groups as a factor and generalising. That is, from counting groups, for example, 1 three, 2 threes. 3 threes, 4 threes,
..., to the consideration instead of 3 ones, 3 twos, 3 threes, 3 fours, 3 fives, ... This shifts the focus from the number in each group to the number of groups which supports more efficient strategies.

Students must Practise:
Decomposing, recomposing facts
Building the timestable in pieces that are related

Build

Permission to reproduce is only granted to the teacher viewing this. These materials are provided courtesy of NRLC & thinking 101 2010

4

If 7 x 7 = 49 then 49 divided into 7 groups must be 7 in each group.
If the array covers 49 squares and one side is 7 then the other side must be 7 so 49 divided by 7 = 7
52 ÷ 7 =   7 x ? = 52 nothing but 7 x 7 = 49 and that is 3 away so 7 remainder 3 is the answer.

I saw 7 x 7 is 49 then add another 7 on so 56.
Using the benchmarks of 50 on the hundred chart keeps counting to a minimum and we are in the same space we would be on the multiplication grid. Students use the grid of 100 squares as a tool for thinking.
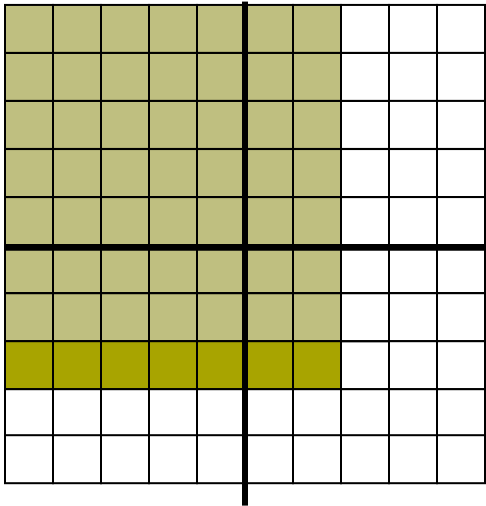
7 x 10 - 7 x 2 = 7 x 8
Working from a known fact or referent The use of brackets is a natural extension in the discussion. We want people to know we multiplied first, or we get the wrong answer.
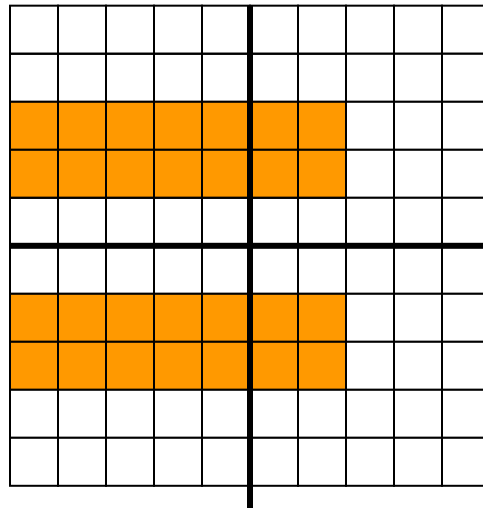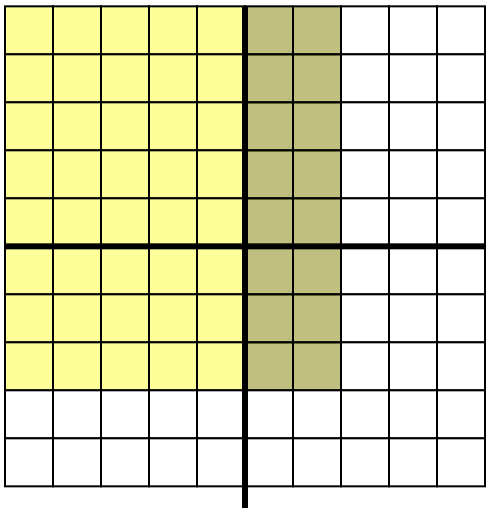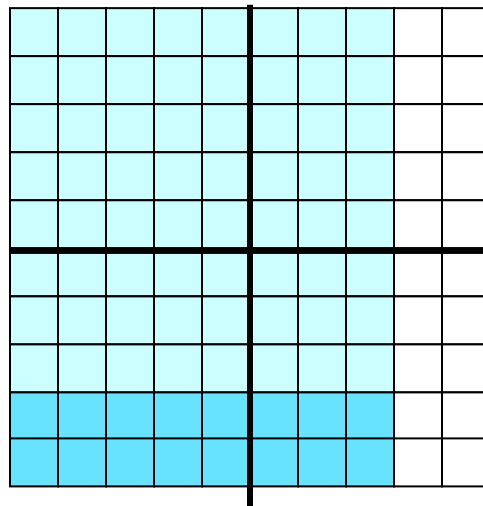
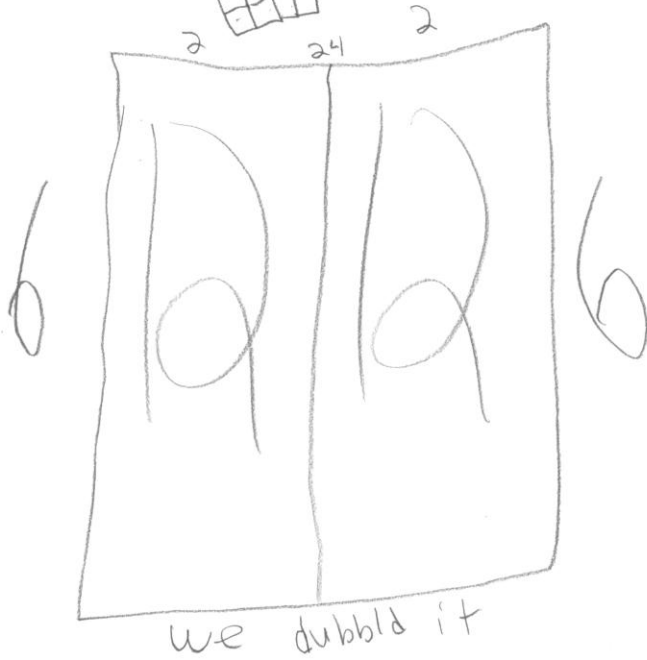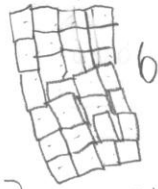7 x 8 is the same as 5 x 8 + 2 x 8 or (5 + 2) x 8. Can you see the two rectangles?
I used the distributive property
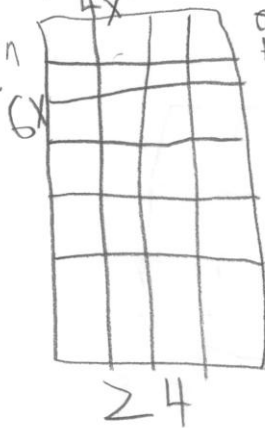
I see 7 x 2 four times so I can double, doubles.

7 x 2 = 14 then 14 x 2 = 28 then 28 x 2 = 56

or 7 x 2 x 4

I bilt a 4 by 6
in strouiter   4

I jist listed to the
4 X 6

6

2   24   2

6   6

we dubbld it

8 blues 16 greens 9 reds 24 yellow
I added 4 tiles in 3a row 6 times, so I got
this from that. But if you turn it fits the
same   eqasion backwards but
24 is in   the same spot. If you
double it 6x   it eqals 48.

4x

6x

24

I added 4 tiles in a row, three times so I got this from that. But if you turn it it's the
same equation backwards but 24 is in the same spot. If you double it it equals 48

5   3

7

$3 \times 7 = 21$
$5 \times 7 = \underline{35}$
56
$(3 * 5) \times 7 = 56$
$7 \times 8 = 56$

4   4

7   7

$(4 \times 7) + (4 \times 7) = 56$
$28 + 28 = 56$

Distribution is a number property that students must master if they are to be successful with algebra. Exploring it in Grade 4 puts the focus on thinking and problem solving, not just getting answers.

I have a collection of 24 teddies. My dad made me a special shelf with 6 levels. I want to arrange the bears so that there is the same number of bears on each level.
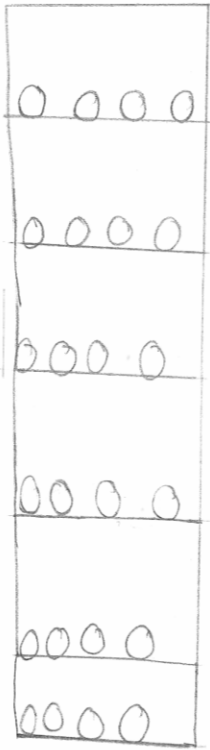**How many will be on each level?**

Thinking multiplication for division. Can you see the array?
24 divided by 6 triggers 4 times 6 as a solution.

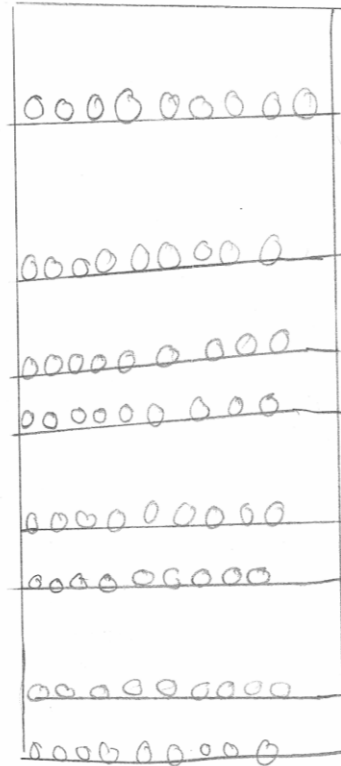42 divided by 7 triggers a picture of 6 times 7 as a solution.

Permission to reproduce is only granted to the teacher viewing this. These materials are provided courtesy of NRLC & thinking 101 2010

8

Erin

I have a collection of 81 teddy bears. I have 9 shelves. How many on each shelf?

$$\begin{array}{r} 24 \\ 9\overline{)\,6} \\ \hline 4 \end{array}$$

9 on each

Are we helping students link the imagery they are developing for multiplication to the imagery that helps them solve division?
Does this student see the connection in his pictures?
Did he need the picture to solve 81 divided by 9? Or could he have written 9 x 9 = 81 therefore 81 divided by 9 = 9

6×6=12 and thers 24 owers a day
that makes 4 groops of six

Double

6×2=2×2=24÷6=4

Brandon

It appears she was thinking about 6 + 6 as 12 and 12 times 2 as 24. Not sure where 24 hours in a day fits in but it seems to be a significant piece of information for her.

Then she repeats as 6 x 2 =12 x 2 = 24 and 24 divided by 6 equals 4.

It is clear this student can use the relationships between 2 x 6 and 4 times 6 to find 24.

It seems also clear this student has linked multiplication to division.

However, I would question the value of the picture. Does the picture further the understanding or does it simply fill up the page and keep the student busy?
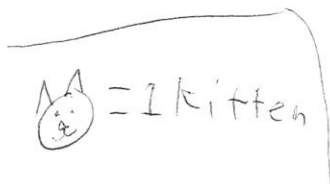
Let's try a different question and see if doubles appear again.

How about 35 teddies and 7 shelves.

24÷6—4

0600
0060
0000
0000
0000
0000
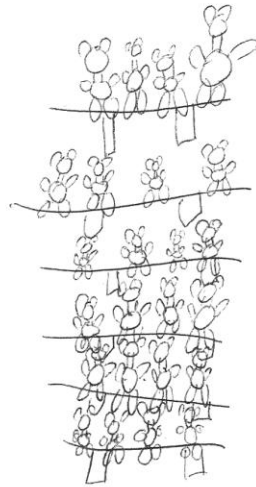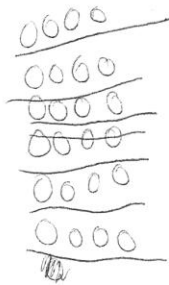
Alex
I have 50 kittens my Dad bilt 5 Jumbo
cages. How maney kittens can go
in each cage?
50÷5=10

🐱 = 1 kitten

This student has drawn the solution with dots then gone back and drawn actual bears. A chance to discuss with students the value of diagrams in math and what is sufficient.
Did this student count or link 4 times 6 to the question? We have no evidence without asking.
The problem she chose to build is with a division fact that is likely not a challenge to her so again we do not have evidence that she used 4 x 6 to solve this problem. She may have just counted as she put out bears on shelves.