



Mathematics 10-3

Unit 5

2-D and 3-D Measurements



Lesson A Area Referents



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Unit 5 2-D and 3-D Measurements



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Unit 1:
Money Math

Unit 2:
Personal Finances

Unit 3:
Linear
Measurements

Unit 4:
Measurement
Systems

Unit 5:
2-D and 3-D
Measurements

Unit 6:
Lines, Angles, and
Shapes

Unit 7:
Pythagorean
Theorem and Right
Triangles

Unit 8:
Introduction to
Trigonometry

Instructions for Submitting Assignment Booklets

1. Submit Assignment Booklets regularly for correction.
2. **Submit only one Assignment Booklet at one time.** This allows your teacher to provide helpful comments that you can apply to subsequent course work and exams (if applicable).
3. **Check the following** before submitting each Assignment Booklet:
 - Are all assignments complete?
 - Have you edited your work to ensure accuracy of information and details?
 - Have you proofread your work to ensure correct grammar, spelling, and punctuation?
 - Did you complete the Assignment Booklet cover and attach the correct label?

Postal Mail

Determine sufficient postage by having the envelope weighed at a post office. (Envelopes less than two centimetres thick receive the most economical rate.)

Alternate Methods

Contact your teacher if you require an alternate method for submission of assignments.

Mathematics 10-3

Unit 5, Lesson A

Assessment

For successful completion of this course, you must do the following:

- 1. Complete all questions in each Assignment Booklet to the best of your ability.** Incomplete Assignment Booklets will be returned unmarked.
- 2. Achieve a Final Exam mark of at least 40%.**
- 3. Achieve a final course mark of at least 50%.**

Process

- **Read the course material** and complete the practice questions as well as the assignments in this booklet.
- **Proceed carefully** through each assignment. Reflect upon your answers and prepare your written responses to communicate your thoughts effectively. Time spent in planning results in better writing.
- **Proofread your work** before submitting it for marking. Check for content, organization, paragraph construction (if applicable), grammar, spelling, and punctuation.
- **If you encounter difficulties or have any questions**, contact your course teacher at Alberta Distance Learning Centre for assistance.

Format

If you choose to **use a word processor** for your written work,

- format your work using an easy-to-read 12-point font such as Times New Roman
- include your full name and student file number as a document header
- double-space your final copy
- staple your printed work to this Assignment Booklet

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Although you are encouraged to work collaboratively and discuss various aspects of this course with others, **all submitted work must be your own.**

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Lesson A

Area Referents



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Lesson A:

Area Referents

Lesson B:

Area Problems in 2-D

Lesson C:

Surface Area of 3-D Shapes

Icons

The following icons will guide you through the course.



Let's Get Messy



How Does It Work?



Digging Deeper



Practice Questions



Practice Solutions



Assignment



Treasure Chest



Coins



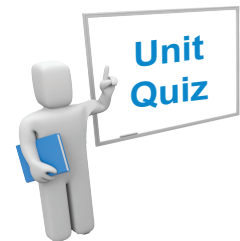
Discussions



Check It Out



How Did It Go?



Unit Quiz

Lesson A:

Area Referents



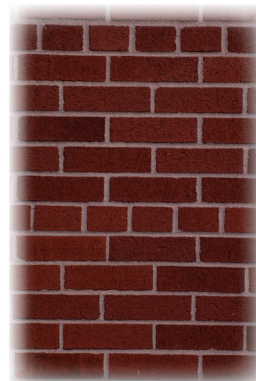
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Think back to Unit 3.... Do you remember that a referent is an item that provides a good idea of the size of an object, but the referent is not a real measuring device? In the past, hands were used to measure the height of horses, and in the Get Messy Activity, you used uniform-sized objects such as bathroom tissue to measure the length of other items.

Referents can be also be used to estimate or give a picture of how much **surface space** is covered.

Bricks make walls, drywall sheets cover walls, floor tiles cover floors, and siding covers the outside of houses. Sometimes, people can estimate very accurately the amount of material needed to cover an area without actually measuring. It is a very handy skill to be able to picture how much surface space or **area** is covered.

area:
the extent of
space or surface
space



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In this lesson, you will learn how to use referents to estimate how much area is covered by a 2-D shape. In addition, you will learn how to choose suitable referents to describe area in SI units and Imperial units. Because we use two systems of measurement, you will also learn when SI area measurements are appropriate and when Imperial area units are better.



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Here are the things that you will learn in this lesson.

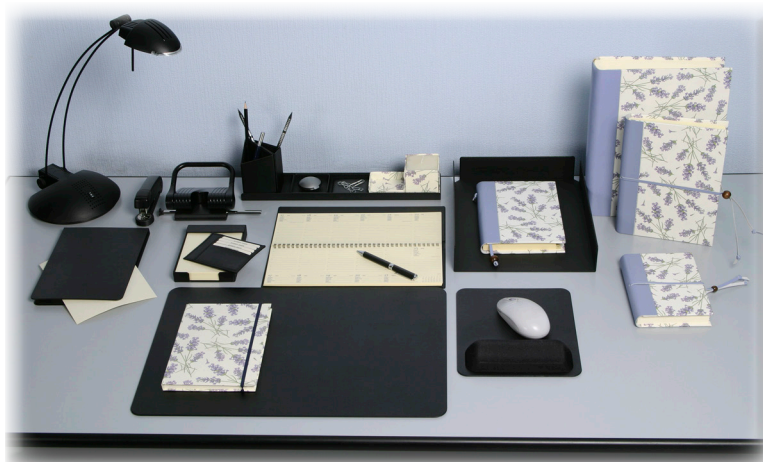
- Estimate area measurements using a referent.
- Choose a referent for SI and Imperial area units.
- Estimate areas of 2-D shapes using grid paper for SI and Imperial units.
- Recognize situations when given SI or Imperial units of area measure are used.



Let's Get Messy

Here are the items you need for this activity:

- tabletop or desktop
- toilet paper... Yup, again!
- text book, notebook, or binder
- deck of cards
- small round plate or saucer
- paper (larger than the plate)
- pencil



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Task 1

Unroll a strip of toilet paper that is as long as your desktop or tabletop.

Lay the strip across the bottom of the desktop or tabletop.

How many pieces of toilet paper are in the strip that is the same length as your desk? _____



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Task 2

Unroll enough strips that are the length of your desk so that your desk is covered by the strips.



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How many **strips** of toilet paper did it take to cover your desk? _____

How many **squares** of toilet paper would it take to cover your desktop or table? _____

Explain how you figured out how many **squares** of toilet paper it would take to cover your desk.

Notice that in the diagram, the three strips are 6 pieces long.

It takes 18 pieces of toilet paper to cover the desk in the image.

Task 3

Using your deck of cards, lay a row of cards across your binder or textbook.

How many cards did it take to make the row? _____



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Task 4



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Without removing the row of cards across the bottom, place a column of cards up the side of the book.

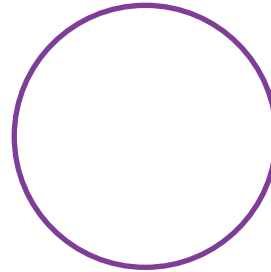
How many rows of cards will it take to cover your book? _____

How many cards will it take to cover your book? _____

How did you figure out how many cards it would take to cover the front of your book?

Task 5

Lay the plate or saucer face down on the paper. Trace around it with your pencil to make a circle on the paper.



Task 6

Think back to Unit 3 when you found the midpoint of lines and 2-D shapes and 3-D objects.

Find the midpoint or centre of your circle. (Remember your skills in finding midpoints!) Mark the centre on your diagram.



Task 7

Place a row of pennies from the centre of the circle to the edge of the circle.

Your pennies might go a little beyond the edge or be a little short of the edge. How many pennies did it take to make a radius for your circle?



Task 8

Cover your circle with pennies. Try to stay inside the circle.

How many pennies did it take to cover your circle? ____

Task 9

Multiply the number of pennies for your radius by itself.

Radius \times radius = ____

Radius \times radius, or radius squared, can also be written as radius^2 .

Think back to Unit 3 when you put Smarties around the cookies and then across the cookie.

By taking the circumference and dividing by the diameter, you found that $C \div d$ was equal to a decimal number between 3 and 4. That number is a ratio called *pi* (π).

A ratio compares the size of two things. As the circumference got bigger, so did the diameter. The ratio was always *pi*... about 3.14.

Task 10

The number of pennies needed to cover your circle is the area of the circle in pennies.

Divide the number of pennies it took to fill the circle by the radius^2 .

Area \div radius^2 = _____



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Task 11: Discussion

You can earn coins for the Mathematical Amusement park and your bonus marks by participating in this discussion.



Remember that coins add up to make amusement park attractions and that those are bonus marks in the course! Contact your teacher to discuss the following questions.

Complete the chart and answer the questions. Ask your discussion participants to do the same with their measurements.

	Squares of TP	Whose desk was bigger?	Cards used	Whose book was bigger?	Pennies used	Whose circle was bigger?
You						
Person 1						
Person 2						
You						
Person 1						
Person 2						

Who can you have for discussion participants? Here are some suggestions.

- your teacher
- parents
- friends
- classmates

1. On the chart, state the number of toilet paper pieces that it took to cover your desktop. One piece of toilet paper has become your referent to measure area. Then, find one person who needed more pieces and another person who needed fewer pieces than you needed. Try to determine if your desktop is larger or smaller than the desktops of the two people you have selected.

State why determining who has the bigger desktop might be difficult.

2. When you placed the cards along the edge of your book, did you place them sideways or up and down?

Would the number of cards needed to cover your book change if you turned your cards the other way?

3. Would it be easier to determine the size of another person's desktop using toilet paper or using cards?

4. How did you determine the number of toilet paper pieces that would cover your desk or the number of cards that would cover your book? Did you count each one, or did you use another method?

5. When you counted the pennies covering your circle and divided by the radius², what value did you get?

If the radius gets longer, what do you think will happen to the area?

Think of a way to determine how many pennies you would need to cover a circle if you knew how many pennies are in the radius.

Make a conclusion about the area referents you used for this Get Messy activity. Were they good area referents? Explain.



How Does It Work?

Lindsay is staying with her grandparents for the summer, and they have decided to build a greenhouse in the backyard. Her grandfather asked her to help plan the size of the greenhouse, and together they will estimate the cost to build it.



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First, they decided it will be rectangular and that it needs to be about as large as the two-car garage that Lindsey has at home. Lindsay did not know the actual dimensions of her garage at home, but she could picture her mom and dad's cars parked beside each other to imagine how much space the greenhouse would cover.



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She remembered talking about linear referents in math class and using common uniform items to measure the length, width, or linear distances. Her grandfather's car was about the same size as her parents' cars. Lindsay decided to use her big step as a linear referent.



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She walked along the side of her grandfather's car. Then, she walked across the front of it. The car was about 4 big steps long and 2 big steps wide. Therefore, she estimated that two cars would be 4 big steps long and about 4 big steps wide.



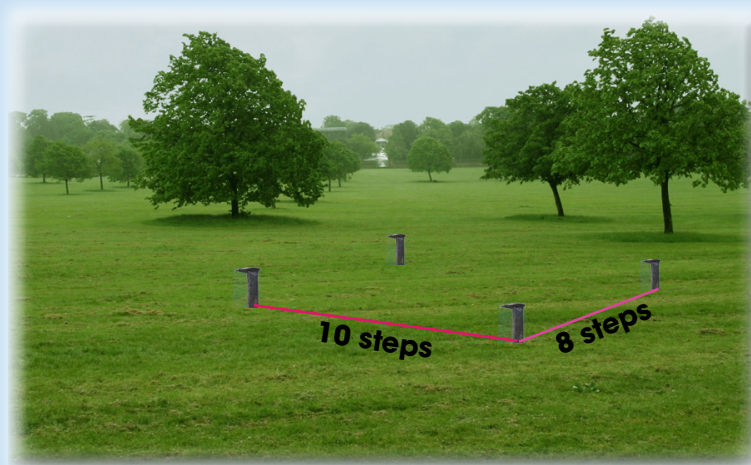
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Of course, cars cannot park tightly beside each other; there needs to be space between and around them in a garage. Lindsay decided that a two-car garage would be about 8 big steps wide and 10 big steps long to allow for car doors to open and some storage on either side of the garage and along the front.



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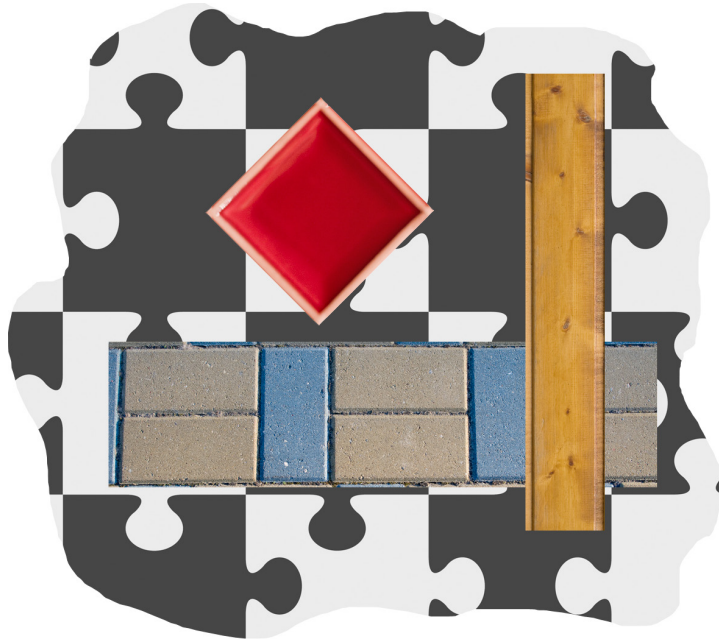
Her grandfather put some stakes in the ground to show where the corners of the greenhouse would be if it were 8 of Lindsay's big steps wide and 10 of her big steps long.



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Lindsay used a linear referent, her big step, to help construct a rectangle in her grandparents' yard that might be the right size for their greenhouse. By staking out the rectangle based on her steps, the total space that would be taken up by the greenhouse, can be seen.

Are there referents that are not linear that could be used to help picture the size and shape of the greenhouse? Area referents will take up space and not just represent a distance.



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The toilet paper was used in Unit 3 as a linear referent to represent your height. In the Get Messy in this lesson, it was used as an area referent because it also has sections that take up space.



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The toilet paper was a good referent for representing the area of the top of a desk. It has sections that are uniform in area and those sections are an appropriate size when compared to the size of the desk top.



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Toilet paper would not be a very good area referent for something as large as a rectangle that represents the space needed for a greenhouse. Can you imagine how much toilet paper would be needed to cover the area needed by a greenhouse?



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Example 1

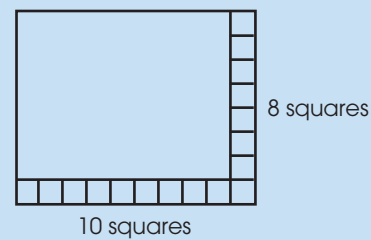
If sidewalk slabs are one of Lindsay's big steps wide and one big step long, how many slabs would they need to cover the ground inside the greenhouse?



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Solution

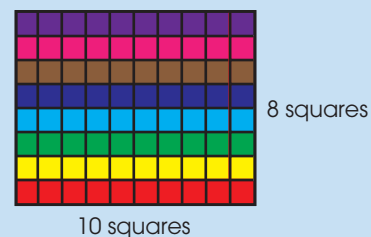
Because the slabs are one step by one step, they are squares. Eight could lie along the width and 10 could lie along the length of the greenhouse.



Now, instead of finishing the diagram and counting the spaces that would represent each slab, a shortcut can be used.

A strip 10 slabs long is needed to go along the length of the greenhouse. Then, 8 strips are needed to cover the floor of the greenhouse. The floor of the greenhouse is the shape of a rectangle.

The area of the floor is $10 \times 8 = 80$ slabs.



Can you think of a formula that will always work to find the area of a rectangle?

Sometimes our area referents are exact objects that cover space. Lindsay used a length referent, her big step, to estimate the dimensions of the greenhouse and then staked out the total area.



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Area referents can be used, too. An area referent covers space and not just distance. There are many 'areas' that may need measuring and many referents that could be used to help estimate the amount of materials needed.

Lindsay chose a sidewalk slab as an area referent. A sidewalk slab covers a certain amount of space and could be used as an area referent instead of using linear 'big steps' to get the dimensions of the greenhouse.



Treasure Chest

Bamboo is now used to make hardwood flooring because

- pandas do not eat it anymore
- it grows fast
- it is hollow

Call your instructor to give your treasure chest answer and earn coins towards bonus marks.



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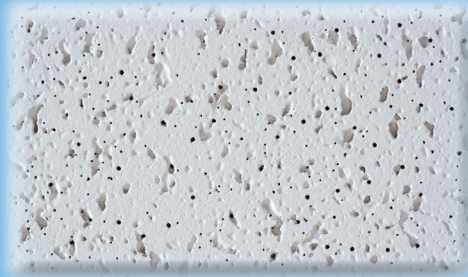
Example 2

Wendy plans to put carpet on her basement floor. She wants to get an estimate of how many square feet of carpet she needs to carpet the rectangular family room.



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The tiles in the ceiling of the family room are 1 ft x 2 ft. She counted 9 tiles across the length of the ceiling and 15 along the width of the ceiling.



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The long side of the ceiling tiles go along the length and the short side of the tiles are on the width of the ceiling.

Approximately what area of carpet does Wendy need for her family room?

Solution

Step 1: Because the ceiling is the same size as the floor, Wendy can use its size to determine how much carpet she needs.

Determine how many ceiling tiles cover the whole ceiling. Wendy could count them all, but she knows a faster way!

The area of the room 'measured' in ceiling tiles is (using the multiplication shortcut or Area formula for area of a rectangle)

$$\begin{aligned}A &= \text{length} \times \text{width} \\ &= 9 \times 15 \\ &= 135\end{aligned}$$

Wendy needs 135 tiles.

Step 2: Determine the area covered by one ceiling tile.

$$\begin{aligned}A &= l \times w \\ &= 2 \text{ ft} \times 1 \text{ ft} \\ &= 2 \text{ ft}^2\end{aligned}$$

The area of one tile is 2 ft^2 .

Step 3: Use the number of ceiling tiles and the size of each tile to determine the square footage of the room.

Area of room in square feet
= number of tiles needed \times area of each tile.

$$\begin{aligned}&= 135 \text{ tiles} \times 2 \text{ ft}^2/\text{tile} \\ &= 370 \text{ ft}^2\end{aligned}$$

The room requires about 370 ft^2 of carpet.



In the example of Wendy and her family room, the ceiling tiles were used to get an exact measurement for the area of the floor. Although normally we do not measure things with ceiling tiles, Wendy could get an exact area of carpet needed based on the area of the tiles.



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As you know, referents do not have to fit exactly into the shape of the object for which we are trying to estimate the area. As long as the referent is uniform, it can give a 'good estimation' of the real area.



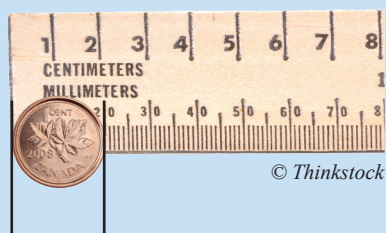
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Example 3

Measure the diameter of a penny in millimetres. Use the penny to estimate the area of a \$5 bill.

Solution

Step 1: Measure the penny at its widest part to determine the diameter of the penny.



A penny is about 20 mm wide.

Step 2: Place pennies next to each other along the length of the \$5 bill and across the width.



Step 3: Find the area of a rectangular \$5 bill in 'pennies'.

$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= 7 \times 4 \\ &= 28 \text{ pennies} \end{aligned}$$



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Step 4: Using the diameter of the penny (20mm), find the area of one penny.

Area of a circle = $\pi \times \text{radius}^2$ where $\pi = 3.14$ rounded to two decimal places.

The diameter of the penny is 20 mm and the radius is half of the diameter.

$$\begin{aligned} \text{radius} &= 1/2 \text{ of diameter} \\ &= 1/2 \times 20 \text{ mm} \\ &= 10 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Area of the penny} &= 3.14 \times 10^2 \\ &= 3.14 \times 100 \\ &= 314 \text{ mm}^2 \end{aligned}$$

Step 5: Find the approximate area of the \$5 bill using the area of one penny.

$$\begin{aligned} \text{Approximate area of \$5 bill} &= \text{area of one penny} \times \text{number of pennies} \\ &= \frac{314 \text{ mm}^2}{\text{penny}} \times 28 \text{ pennies.} \\ &= 8792 \text{ mm}^2 \end{aligned}$$

The approximate area of the \$5 bill based on using pennies as area referents is about 8792 mm^2 .



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Hey! Did you notice that you used a unit rate
 $\left(\frac{314 \text{ mm}^2}{\text{penny}}\right)$ in this example using area referents?



Check it Out

Have some fun playing
Lesson 5A - Match the Referent.



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Now it is your turn. The "How Does It Work?" practice questions on the next page will give you some practice working with area referents.

You will also learn more about determining the area of diagrams on grid paper.



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How Does It Work?

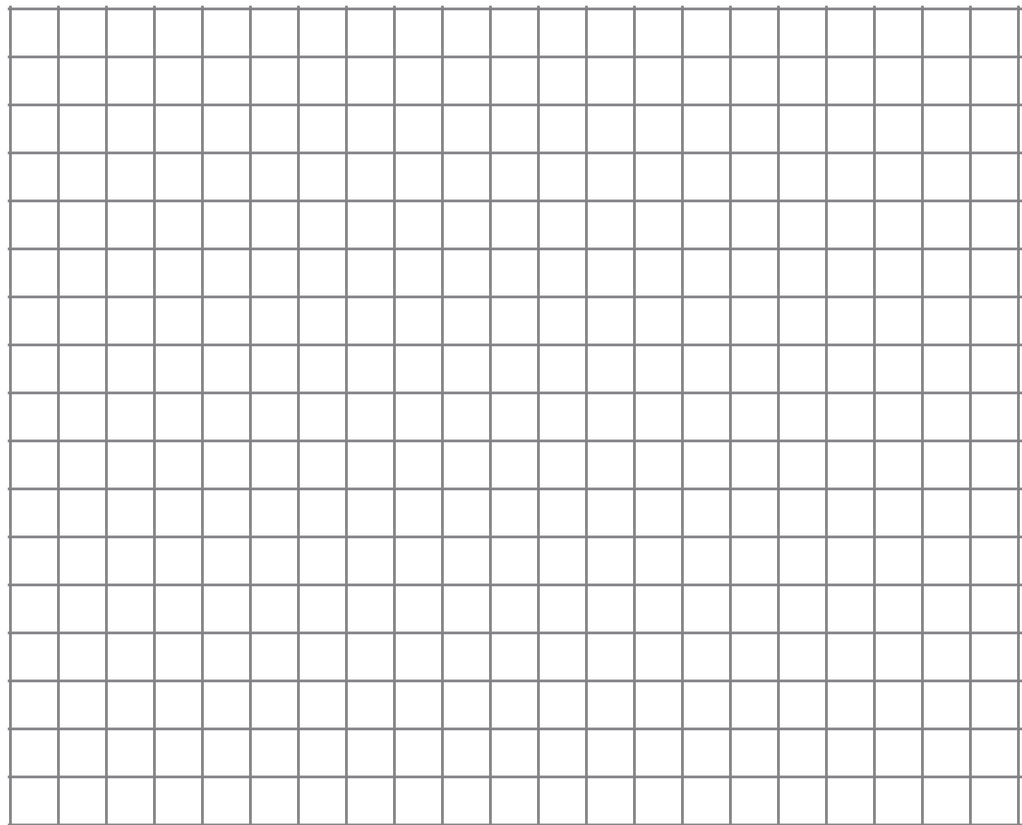
Practice Questions

Earn coins towards Mathematical Amusement park Bonus marks by doing these practiced questions.



To earn full coins, your answers will have to be different than the solutions given.

1. Show two different rectangles on the grid paper with an area of 18 units^2 each.



2. Luke wants to put ceramic tile in his bathroom. He does not have a measuring tape, but he knows that the floor tiles he wants to buy are 1 ft long by 1 ft wide or have an area of 1 ft².

List three things that he could use as area referents that would help him to estimate how many floor tiles he would need for his bathroom.

a. _____

b. _____

c. _____

3. A sheet of loose-leaf paper is 8 ½ inches wide and 11 inches long. For what would a sheet of loose-leaf paper be a good area referent?

4. For what three objects could a quarter be used as an area referent?

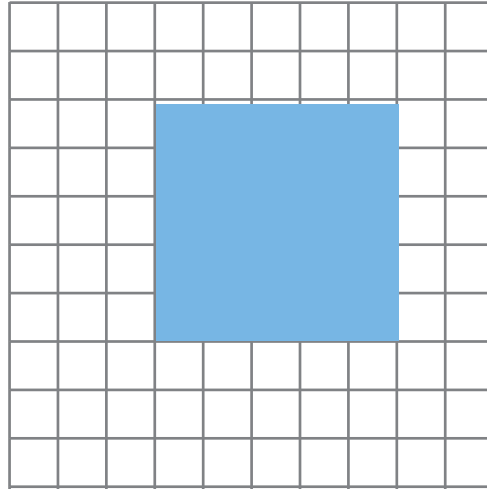
a. _____

b. _____

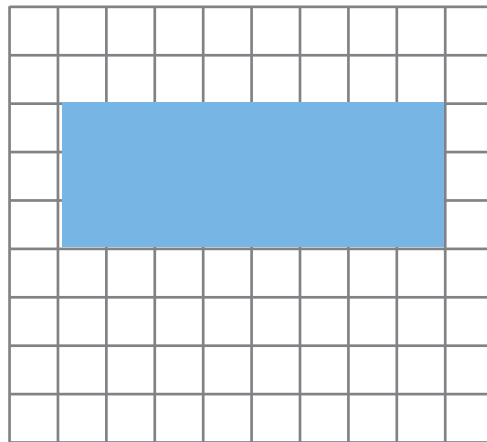
c. _____

5. Use the diagram to determine the area measurements of each shape.

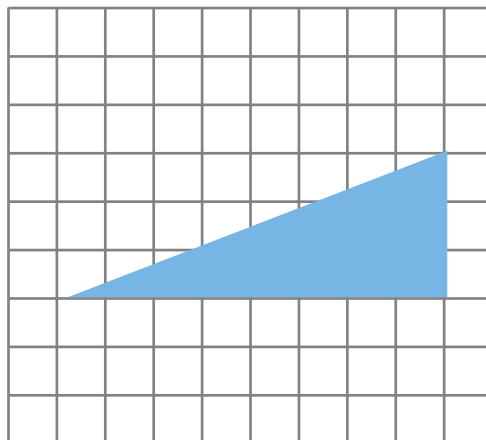
a. 1 square = 1 inch²



b. 1 square = 1 cm²



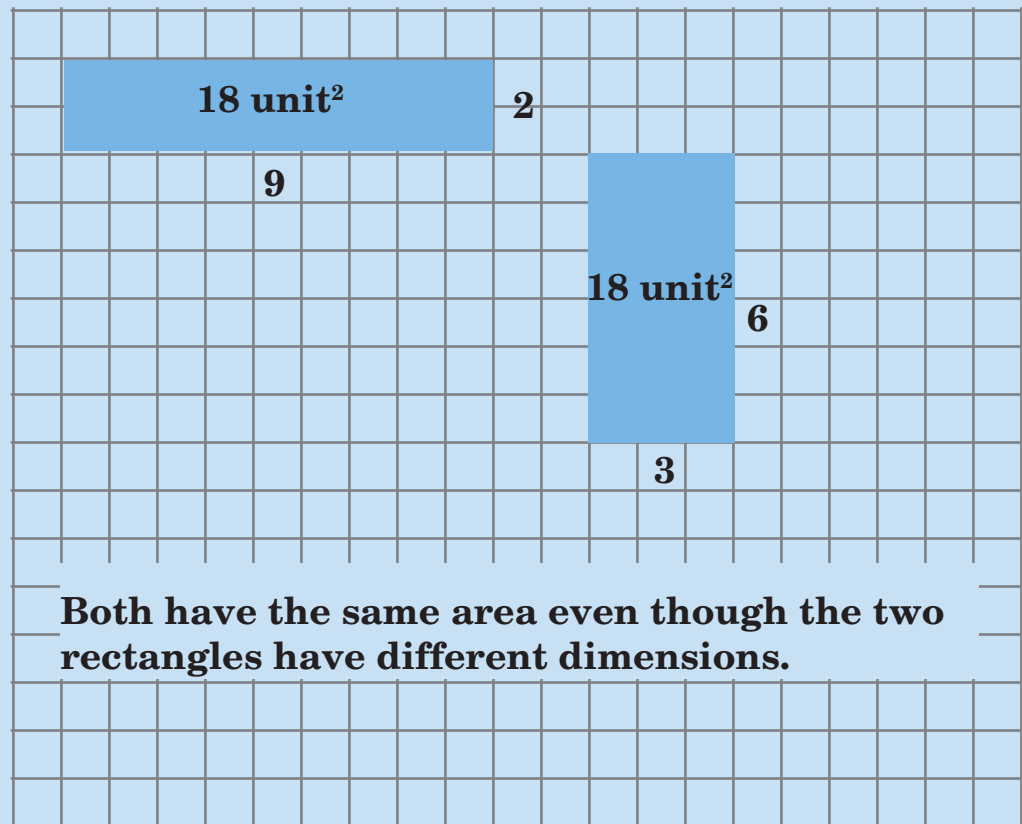
c. 1 square = 1 m²





Practice Solutions

1. Show two different rectangles on the grid paper with an area of 18 units^2 each.

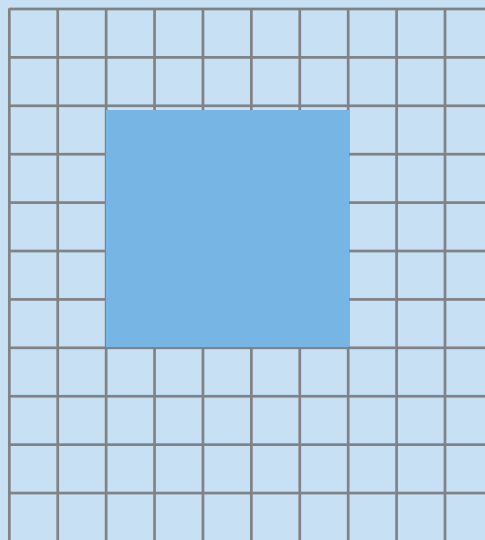


2. Luke wants to put ceramic tile in his bathroom. He does not have a measuring tape, but he knows that the floor tiles he wants to buy are 1 ft long by 1 ft wide or have an area of 1 ft^2 . List three things that he could use as area referents that would help him to estimate how many floor tiles he would need for his bathroom.

Here are some suggestions:

- a. **a paper towel or napkin**
- b. **the front cover of a 3-ring binder for loose-leaf paper**
- c. **a laptop**

3. A sheet of loose-leaf paper is $8\frac{1}{2}$ inches wide and 11 inches long. For what would a sheet of loose-leaf paper be a good area referent?
- **The area inside a large picture frame, or the amount of glass needed for the frame**
 - **The amount of counter-top granite needed to cover kitchen counters**
 - **The amount of wall tile needed for the interior of a shower stall**
4. For what three objects could a quarter be used as an area referent?
- a. **The material needed to make a cover for a cell phone**
 - b. **The size of sticky notes to pick up at the store**
 - c. **The size of an animal's paw print**
5. Use the diagram to determine the area measurements of each shape.
- a. $1 \text{ square} = 1 \text{ inch}^2$

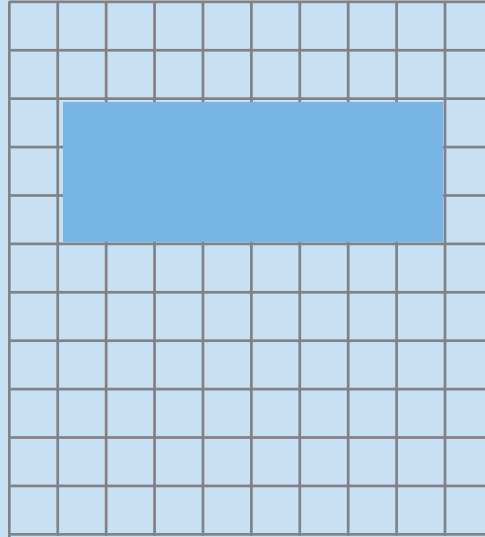


Area = 25 squares

1 square = 1 inch²

A = 25 inches²

b. 1 square = 1 cm²

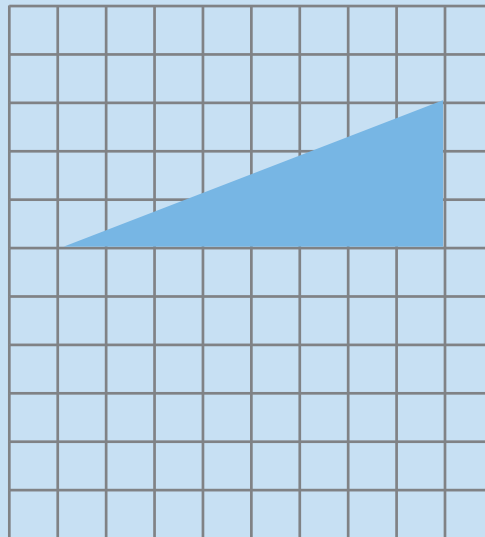


Area = 24 squares

1 square = 1 cm²

A = 24 cm²

c. 1 square = 1 m²



Area = $\frac{1}{2}$ of the

rectangle area

= $\frac{1}{2}$ of 24 squares

= $\frac{1}{2} \times 24$

= 12 squares

1 square = 1 m²

A = 12 m²



Total
41

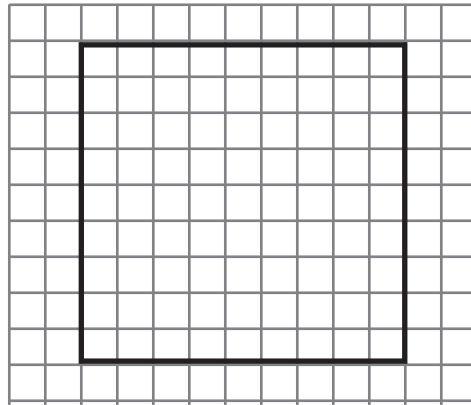
How Does It Work? Assignment

Now it's time to show your stuff!
Put lots of details into your work.

1. Use the diagrams to determine the area measurements of each shape.

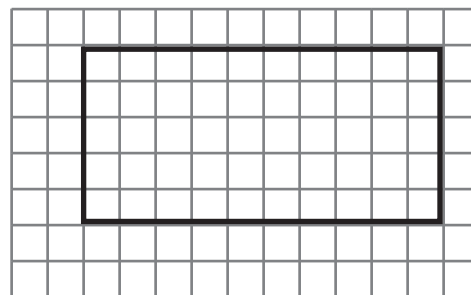
②

a.



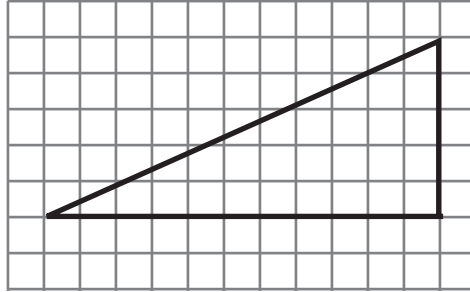
②

b. (1 square = 1 cm²)



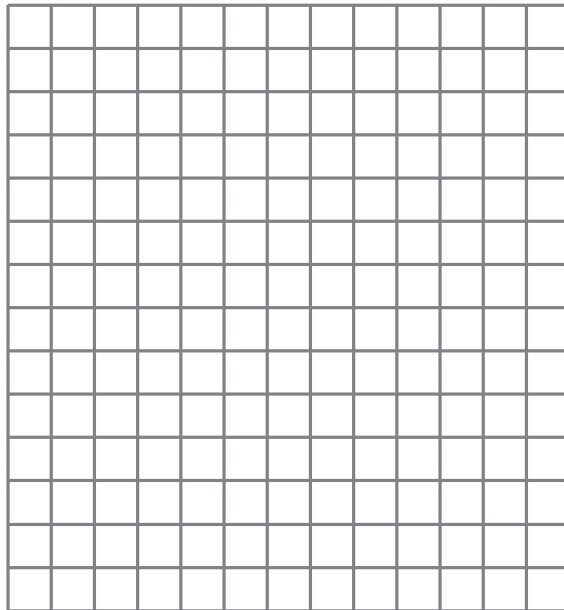
c. (1 square = 1 inch²)

②



②

2. a. Draw a rectangle that has a length of 4 inches and a width of 3 inches.
(1 square = 1 inch²)

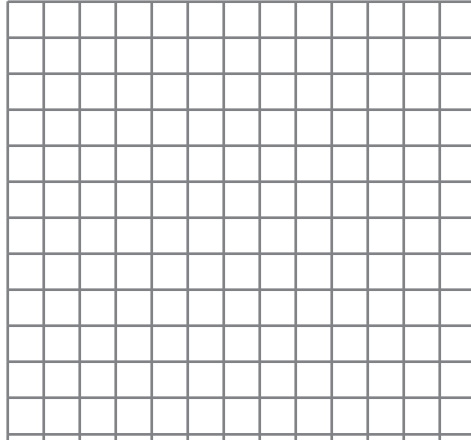


②

b. What is the area of your rectangle?

②

3. a. Draw a rectangle with a base of 12 cm and a height of 6 cm. (1 square = 1 cm²)



②

- b. What is its area?

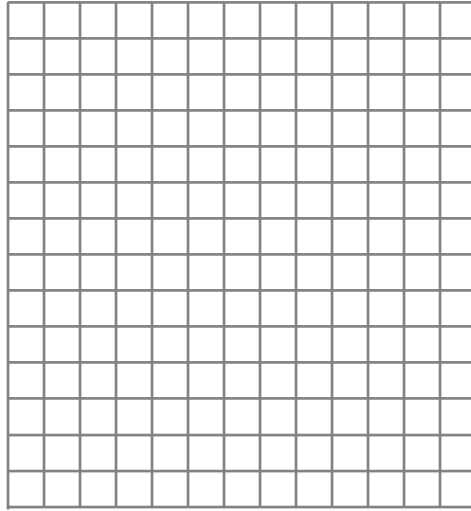
②

- c. What is the area of a triangle with a base of 12 cm and a height of 6 cm?

②

- d. How did you decide what the area should be?

- ① 4. a. Draw a rectangle with area of 24 cm^2 .



- ① b. What is the length of your rectangle?

- ① c. What is the width of your rectangle?

- ② d. How many different rectangles would be drawn using whole numbers for the lengths of the sides of the rectangle?

- ② e. What are the dimensions of the different rectangles that could be made?

length _____ width _____

length _____ width _____

length _____ width _____

④

5. Pretend that you are a teacher. Explain to your students why a group of rectangles can have the same area but different dimensions. You may wish to use a diagram to help explain.

6. Name a referent that could be used to estimate the following areas. Remember to use area referents.

②

a. Size of a window in a house

②

b. Area of a back yard

②

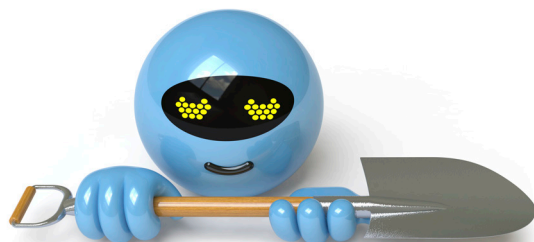
c. Size of glass in a picture frame

- ② 7. Explain how the area of a triangle is related to the area of a rectangle with similar dimensions.

8. Refer to Example 3. The approximate area of a \$5 bill was found using pennies.

- ② a. Is the area in the solution larger or smaller than the actual area of the \$5 bill?

- ② b. Can you think of a more accurate way to use pennies to determine the area of a \$5 bill? Explain.



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You are ready to start Digging Deeper!



Digging Deeper

Not every referent fits perfectly into the area of a shape. You have seen an example where the area of a room could be measured closely using a ceiling tile as a referent. You also saw an example where an area could be estimated closely by using a referent that did not fit exactly such as when a penny was used as a referent.

Referents may not fit exactly, but they can give good estimates of the size or area of objects. Also, not every shape is a perfect rectangle or a square. Some shapes are odd or irregular.



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Doug's back yard is not a perfect rectangle. He lives on a corner lot and his back yard is called *pie-shaped*. When using a referent to estimate the area of odd-shaped objects, you will most likely find that the referent will not be a perfect fit.

Estimating the area of shapes that do not fit exactly into a regular shape can get a little tricky sometimes!

Here you will learn how to estimate areas of irregular shapes using area referents. You will also investigate a little more closely when it is best to use Imperial area units of measure or SI area units of measure.

Example 4

Doug plans to sod his backyard this summer. Suggest a good referent for Doug to use to estimate how much sod he will need for his backyard. Remember it is pie-shaped.



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Solution

Several referents would be good to estimate the amount of sod needed for his back yard. Here are a few:

- sidewalk slabs
- doormat
- car or vehicle
- lawnmower

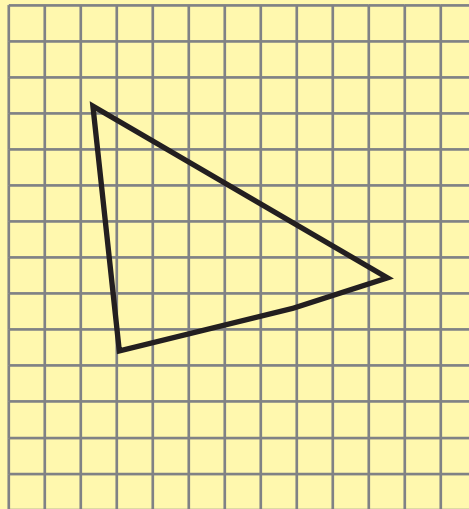


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If Doug has a sidewalk slab that is square like Lindsay used, or rectangular like Wendy used with her ceiling tiles, he could picture how many slabs will cover his yard although the square shapes of the slabs would not fit perfectly into the pie shape of his lot.

Because it is pie-shaped, a rectangular referent will not fit exactly. Another good choice for an area referent for Doug's back yard would be the welcome mat.

It would be light weight and easily moved along the dimensions of the yard. The sides of the mat also bend. A flexible referent is helpful when the referent does not fit exactly.

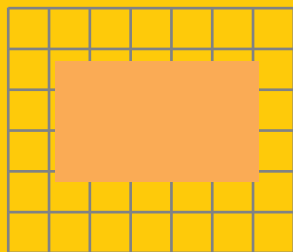


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Example 5

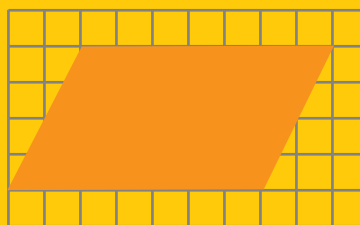
Estimate the area of the following shapes.

a.



© ADLC

b.



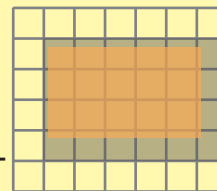
© ADLC

Solution a

Step 1: Over estimate the area.

To over-estimate the amount of area, count each square on the grid paper that is used partially as if it is one full square unit of area.

The amount of area that the actual shape covers is not as big as your estimation; you have over-estimated the area.

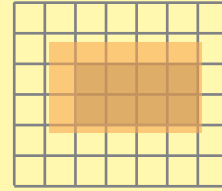


The over-estimated area is for a 6 unit by 4 unit rectangle.

$$\begin{aligned} A &= 6 \times 4 \\ &= 24 \text{ unit}^2 \end{aligned}$$

Step 2: Now, under-estimate. Using a similar technique, do not count the squares on the grid paper that are not used fully in the area of the irregular shape.

This time, the area estimation that you have made is smaller than the actual amount of area covered by the shape.



This is an under-estimation.

The underestimated area is for a rectangle that is 4 units by 2 units.

$$\begin{aligned} A &= 4 \times 2 \\ &= 8 \text{ units}^2 \end{aligned}$$

Which estimation do you think is closest to the real value of the area of the rectangle?

Actual area is in between the two. But to get a closer estimation, take the average of the two estimations.

Average area estimation

$$\begin{aligned} &= \frac{\text{larger area} + \text{smaller area}}{2} \\ &= \frac{24 \text{ unit}^2 + 12 \text{ unit}^2}{2} \\ &= \frac{36}{2} \\ &= 18 \text{ unit}^2 \end{aligned}$$

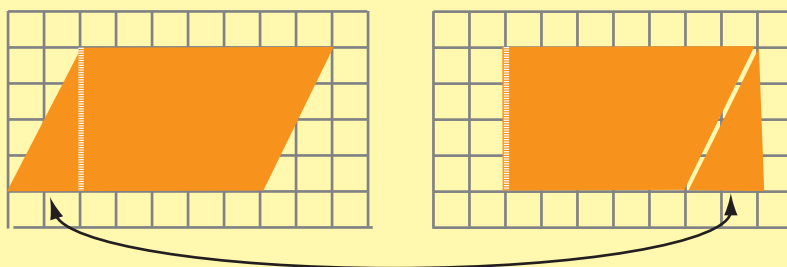
The rectangle has an area of about 18 square units.

Solution b

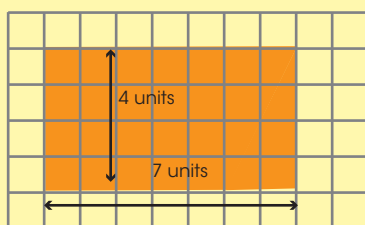
A parallelogram is very similar to a rectangle. It has a length and a width.

The sides of the parallelogram do not make the width of the parallelogram. In a rectangle, the sides of the rectangle do make the width of the rectangle.

Recall that the area of the rectangle can be found by counting the number of squares that make the length of the rectangle and then multiplying by the number of rows that make the width of the rectangle. Is it the same for a parallelogram?



This corner is 'cut' off and flipped and then moved over to the other end.



© ADLC

Now it looks like a rectangle!

A parallelogram is really a squished rectangle or an altered rectangle. The area is the same as a rectangle with the same length and width although the area is not covering a block of space.

irregular:
not uniform in
shape or size
(Dimensions
may or may
not match
one another.
Opposite of
regular shapes
that have
matching
dimensions.)

Over-estimation and under-estimation are both ways to estimate the area of a shape that does not quite fit exactly into a grid paper diagram, or “**irregular shapes**”. Picturing parts of the shape moving to take up area in another spot on the grid paper is another way.

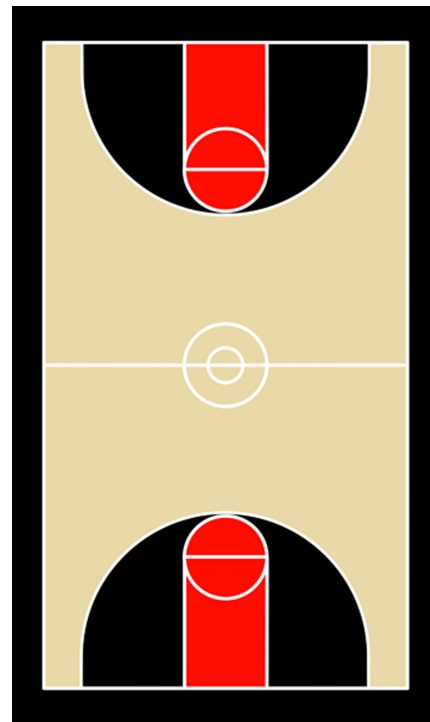
Of course, estimation is just that—an estimation. It is not exact. However, it should be fairly accurate. If material is being purchased to cover a certain area and an under-estimation is used, then the amount of material bought would not be enough. If an over-estimation is used, there will certainly be enough material to do the job, but it may be too much, and that would be a waste of material and money.



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Over-estimation and under-estimation are good ways to estimate, especially if both are used and then an average of the two estimations is taken.

In the game of basketball, a certain area is marked off and called the 'key'. Players on offense, or those who are trying to score a basket, cannot remain in the key for more than three seconds. It would be too easy to stand under the basket in the key and wait for a pass to come and then shoot and score. The players trying to score must move in and out of the key.



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The key in this diagram is the rectangle with a half circle or a semi-circle attached (it is red).

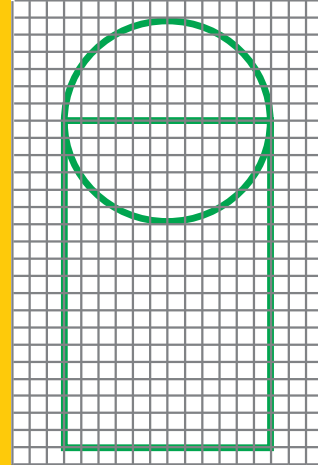


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Example 6

Estimate the area covered by the basketball key diagrammed on the grid paper.

Scale: 1 square = 1 ft².



Solution

The rectangle of the key is marked clearly on the grid lines. Across the base of the key is 12 squares and the bottom of the key to the top of the key is 19 squares. It is an exact fit.

The area of only the rectangle part of the key is

$$12 \text{ squares} \times 19 \text{ squares} = 228 \text{ squares.}$$

The diagram states that 1 square represents 1 ft².

Now, the challenge is to find the area of the semi-circle at the top of the key.

First, over-estimate.

You can recognize that the diameter of the semicircle is the same as the width of the key. The semicircle is 12 squares across. From the foul-shot line to the top of the semi-circle is the radius of the semi-circle. Therefore, the semicircle is 6 rows high.



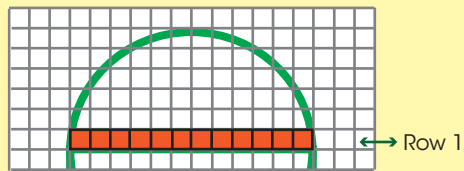
Treasure Chest

Basketball is a sport that was invented by James Naismith (a gym teacher) in 1891. Mr. Naismith was from

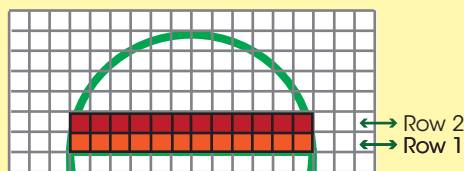
- Barrhead, Alberta
- Almonte, Ontario
- Montreal, Quebec

Call your instructor to give your treasure chest answer and earn coins towards bonus marks.

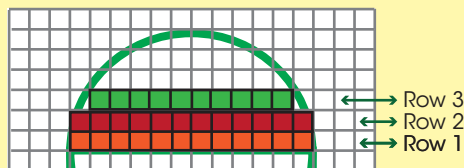
The first row has 12 squares.



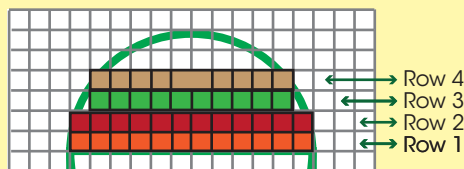
The second row has almost 12 squares.



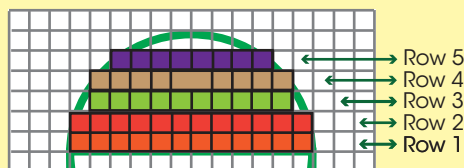
The third row has almost 10 squares.



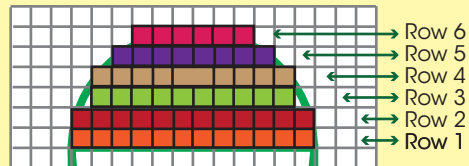
The fourth row has almost 10 squares.



The fifth row has almost 8 squares.



And the top row has almost 6 squares.

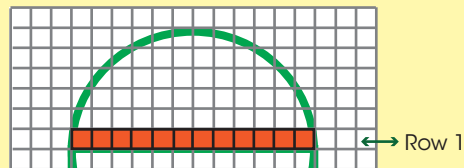


Together, the semi-circle uses almost
 $12 + 12 + 10 + 10 + 8 + 6 = 58$ squares.

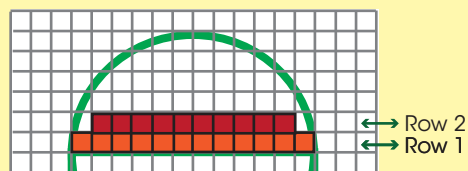
But, that is too big for the actual area of the semi-circle.

Now, under-estimate.

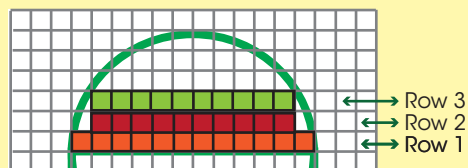
The first row has 12 squares.



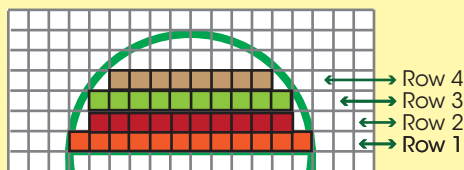
The second row has 10 squares.



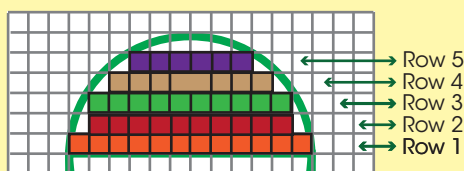
The third row has 10 squares.



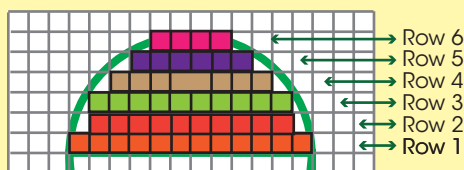
The fourth row has 8 squares.



The fifth row has 6 squares.



And the sixth row has 4 squares.



The total squares in the under-estimation is
 $12 + 10 + 10 + 8 + 6 + 4 = 50$ squares.

Obviously, the true area of the semi-circle is between 58 and 50 squares. To get a closer estimation, determine the average of the over-estimation and the under-estimation.

Do you remember how to calculate an average?
 Add all the values together, and then divide by the number of values used.

In this case, the two values to be added are the over-estimation and the under-estimation.

$$\begin{aligned}
 &= \frac{58 \text{ squares} + 50 \text{ squares}}{2} \\
 &= \frac{108}{2} \\
 &= 54
 \end{aligned}$$

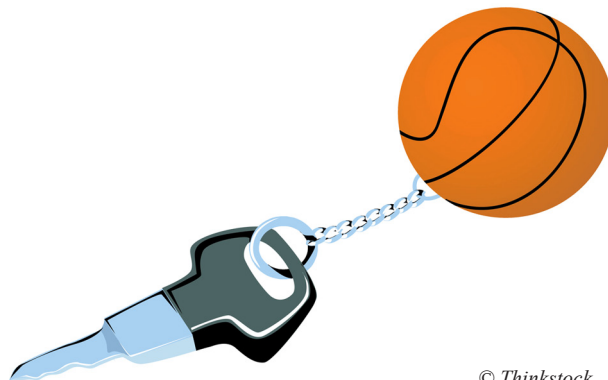
The average of the estimations is 54 squares.

The estimation of the area of the key, according to the diagram given, is area of the rectangle + estimated area of the semicircle.

$$228 \text{ squares} + 54 \text{ squares} = 282 \text{ squares}$$

Because 1 square = 1 ft², 282 squares = 282 ft².

Remember that the semicircle area was an estimation of the actual area. The area of the key is approximately 282 ft².



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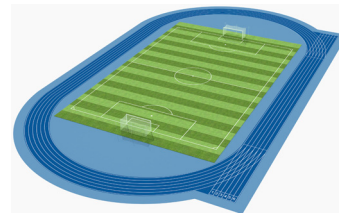


Check it Out

Tip from your teacher
Lesson 5A - *Fits Just Right.*

Think back to Unit 4. You learned about the Imperial and metric measurement systems. You also learned that certain units of measure are appropriate for measuring certain things.

- Feet and metres are both appropriate for measuring the length of a soccer field.



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- Litres and cups are appropriate for measuring the amount of water a person should drink in one day.

Area measurements have appropriate units as well.

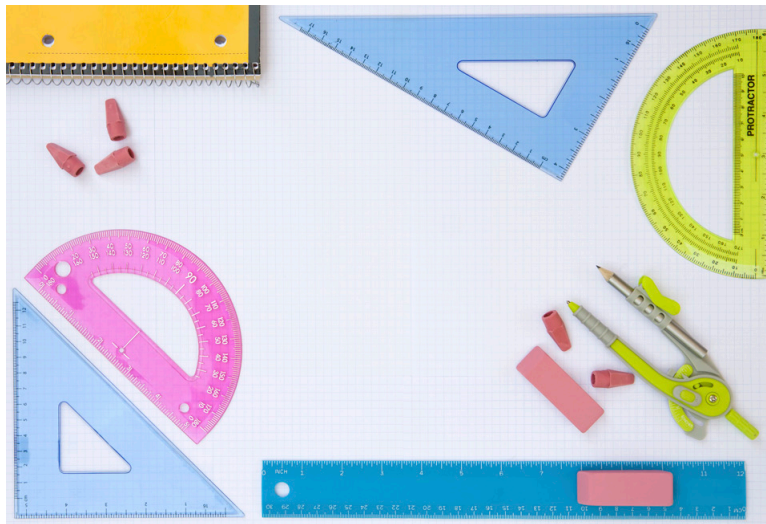
- Square feet and square metres are appropriate measures for the floor area of a house.
- Acres is an Imperial area unit that measures how much land a farmer has. In the metric system, land is measured in hectares.



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Also, think back to Unit 1 when you learned about unit rates and unit pricing. Estimating the amount of material needed to cover things can be used to estimate the cost of material needed when a unit price is known.

In this Digging Deeper section, you have learned how to estimate areas based on diagrams on grid paper. You have also learned some appropriate units of area measure in both the SI (metric) system and in the Imperial system.



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Some practice questions will help tie all these ideas together.

Complete the practice questions before you progress to the Digging Deeper assignment.



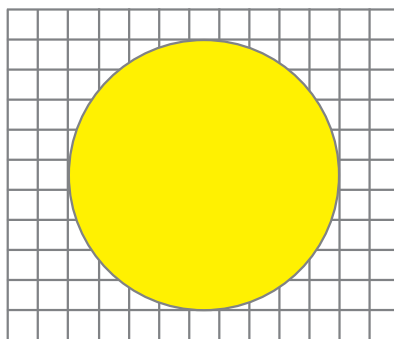
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Digging Deeper

Practice Questions

1. Estimate the area of the circle shown on grid paper.



1 square = 1 cm²

2. Name an object that can be measured in cm².

3. Name an object that can be measured in square feet.

4. Acres are units of area measure. Which system does 'acres' belong to, Imperial or metric (SI)?

5. Name an object that can be measured in acres.

6. A hectare is $10,000 \text{ m}^2$. Name 4 things that are measured appropriately in hectares.



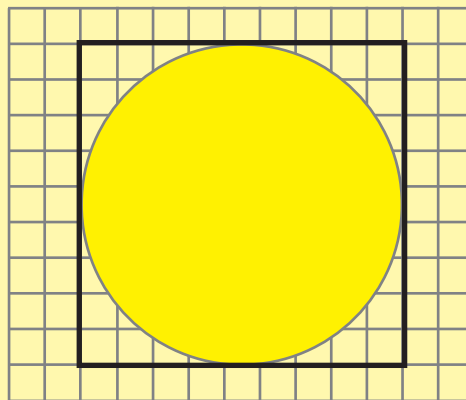
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Practice Solutions

1. Estimate the area of the circle shown on grid paper.
The same method of over-estimation and under-estimation will be shown but in steps and not explained in detail.

Step 1: Make a square around the circle.

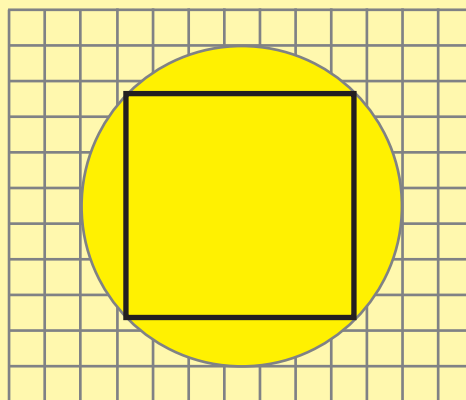


1 square = 1 cm^2

The square is too big, but its sides touch the edge of the circle.

Step 2: Determine the area of the big square. The big square is 9 squares by 9 squares = 81 squares.

Step 3: Make a square inside the circle.



1 square = 1 cm^2

The square is too small, but the corners touch the edge of the circle.

Step 4: Determine area of small square. The small square is 6 squares by 6 squares = 36 squares.

Step 5: Find the average

$$\begin{aligned}
 &= \frac{\text{large area} + \text{small area}}{2} \\
 &= \frac{81 + 36}{2} \\
 &= \frac{117}{2} \\
 &= 58.5 \text{ squares}
 \end{aligned}$$

The approximate area of the circle is 58.5 squares cm²

2. Name an object that can be measured in cm².

Many objects can be measured in cm². An item that is fairly small, such as a sheet of loose-leaf paper, could be one.

3. Name an object that can be measured in square feet.

Objects that can be measured in square feet are carpet or other floor covering.

4. Acres are unit of area measure. Which system does ‘acres’ belong to, Imperial or metric (SI)?

Acres are an area measurement from the Imperial system. Land in Canada was measured in acres and is still often referred to in acres because road distances and other divisions in the country were based on land area measurement in the Imperial system. Roads and land divisions are not easily changed.

5. Name an object that can be measured in acres.

Amount of land consumed by forest fires, farmland, or town lots could be measured in acres.

6. A hectare is $10,000 \text{ m}^2$. Name 4 things that are measured appropriately in hectares.

A football field is longer than 100 m if the end zones are included. Also, the width of the football field is nearly 100 m if the sideline areas are included. $10,000 \text{ m}^2$ could have come from multiplying $100 \text{ m} \times 100 \text{ m} = 10,000 \text{ m}^2$.

The following can all be measured in hectares:

- **football field**
- **a playground**
- **farmland**
- **a parking lot**



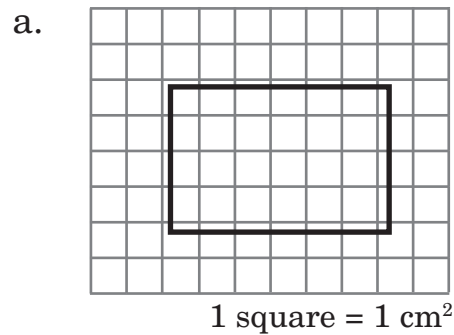
Total
44

Digging Deeper Assignment

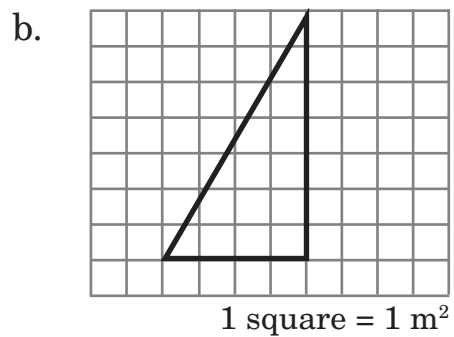
Now it's time to show your stuff!
Put lots of details into your work.

1. Estimate the area of the following shapes.

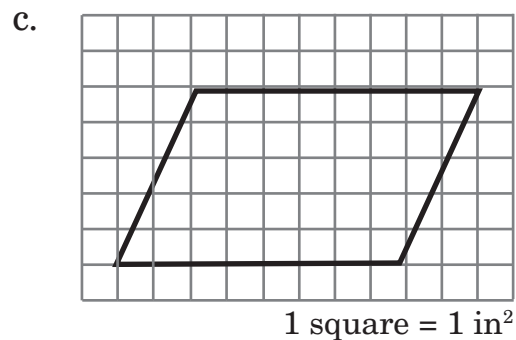
②



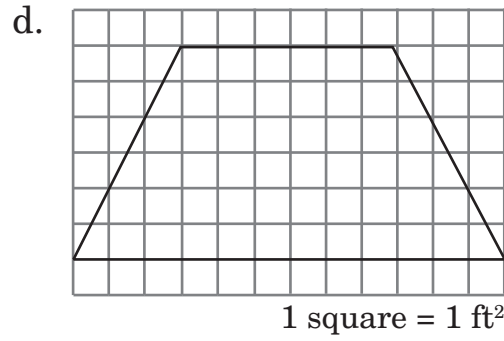
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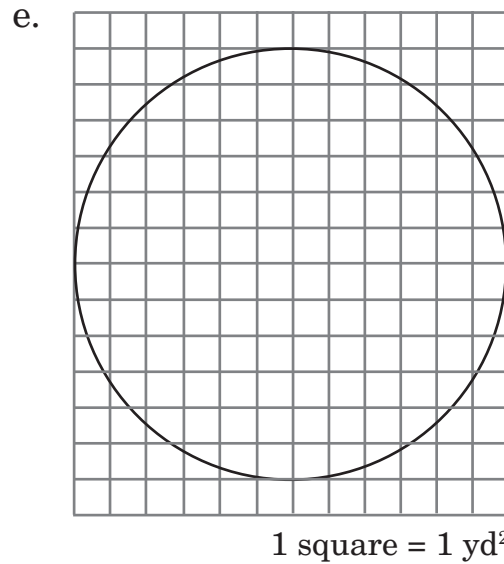
②



②



②



①

2. Name an object that can be measured in square feet.

④

3. Fill in the following chart with the appropriate related Imperial area measurement units.

Metric Area Unit	Imperial Area Unit
m²	
cm²	
hectare	
km²	

⑧

4. The chart below has a list of objects. Complete the chart with the appropriate metric area unit and the appropriate Imperial area unit.

Objects	Metric Area Unit	Imperial Area Unit
paper label on a can of soup		
a playground		
a gym wall		
top of a thumb tack		

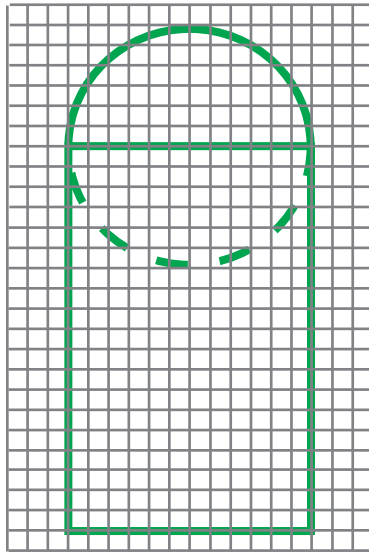
①

5. a. Why do many Canadians still use the Imperial unit acres when speaking of large areas of land?

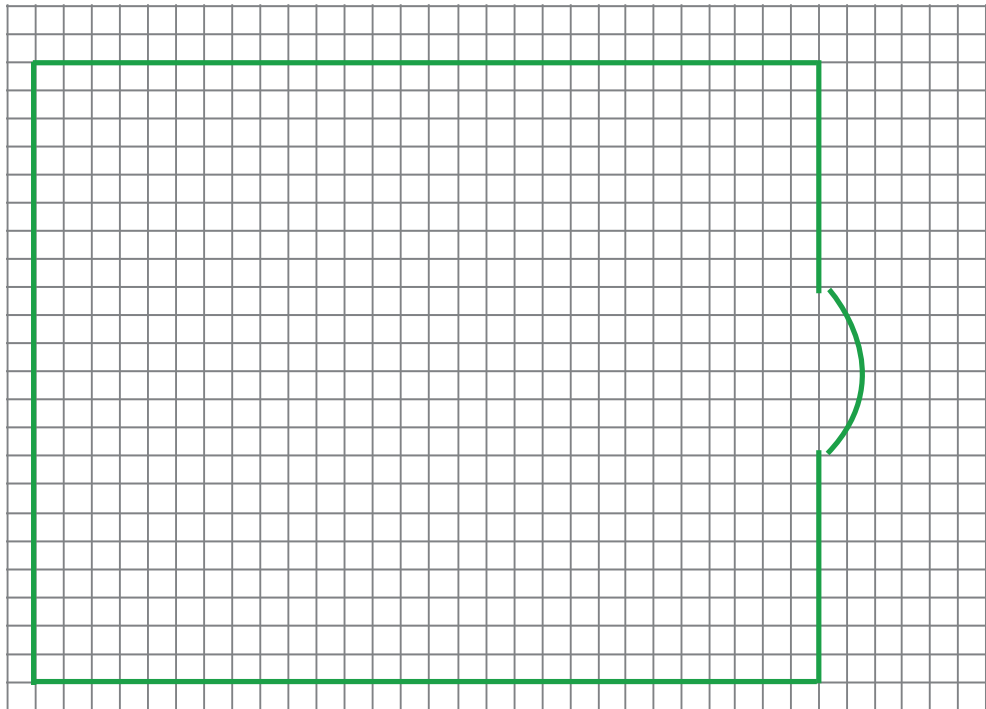
②

- b. Name another case where an Imperial unit of area measurement is still commonly used in Canada. Explain why the related metric unit has not replaced it.

- ② 6. Estimate the area of the basketball key shown in the diagram below. (1 square = 1 ft²)



7. The floor surface for Nicole's family room is shown in the diagram below. (1 square = 1 ft²)



④

a. Explain the steps you would follow to estimate the area of Nicole's family room.

②

b. Decide if your estimation is accurate. Would your estimation be too big or too small?

②

c. What adjustments to your area estimation could you do to make the estimation more accurate?

②

d. If the carpet that Nicole likes is \$14.99/ft², about how much will carpet for her family room cost?

- ② 8. Remember that in Lesson 1A you learned about unit pricing. The price per square foot of carpet is a unit price. Laura's family room is 620 ft^2 . It cost \$8363.80 to buy exactly enough carpet for the room.

What is the unit price for the carpet that Laura bought?

- ② 9. Explain how a placemat could be used to estimate the amount of granite needed to make a kitchen countertop.

- ② 10. Think about using a placemat for an area referent of a countertop. Is a placemat a good area referent to estimate the area of a countertop. Explain.

Lesson Summary

You have accomplished much in this lesson!

You used your knowledge of referents from previous units and applied it to area estimations.

Also, your understanding of metric units of measure helped you use area referents to make close estimations of either metric or Imperial areas.

Please attach your return address label to the back of this booklet and send Unit 5 Lesson A to be marked.

You are now ready to proceed Unit 5 Lesson B.



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