Mathematics 10-3 Unit 5 2-D and 3-D Measurements



Lesson B Area Problems in 2-D





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Mathematics 10-3

Unit 5 2-D and 3-D Measurements



Unit 1: Money Math

Unit 2: Personal Finances

Unit 3: Linear Measurements

Unit 4: Measurement Systems

Unit 58 2+D and 3-D Measurements

Unit 6: Lines, Angles, and Shapes

Unit 7:

Pythagorean Theorem and Right Triangles

> Unit 8: Introduction to Trigonometry

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- 1. Submit Assignment Booklets regularly for correction.
- 2. **Submit only one Assignment Booklet at one time.** This allows your teacher to provide helpful comments that you can apply to subsequent course work and exams (if applicable).
- 3. Check the following before submitting each Assignment Booklet:
 - \Box Are all assignments complete?
 - □ Have you edited your work to ensure accuracy of information and details?
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Mathematics 10-3 Unit 5, Lesson B

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- 2. Achieve a Final Exam mark of at least 40%.
- 3. Achieve a final course mark of at least 50%.

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- **Read the course material** and complete the practice questions as well as the assignments in this booklet.
- **Proceed carefully** through each assignment. Reflect upon your answers and prepare your written responses to communicate your thoughts effectively. Time spent in planning results in better writing.
- **Proofread your work** before submitting it for marking. Check for content, organization, paragraph construction (if applicable), grammar, spelling, and punctuation.
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- double-space your final copy
- staple your printed work to this Assignment Booklet

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Lesson A: Area

Referents

Lesson B8 Area Problems In 2-D

Lesson C: Surface Area of 3-D Shapes

lcons

The following icons will guide you through the course.



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Lesson B:

Area Problems in 2-D

Lindsay and her grandparents are deciding how the inside of the greenhouse will be set up.

They have decided that the greenhouse will be rectangular and the dimensions will be 8 m by 10 m. Along with hanging pots of flowers and strawberries, they will also have a section of dirt on the ground as well as some tables with small pallets for starting seeds such as tomatoes and cucumbers. Of course, some space is needed between the tables and sections for paths.







Lindsay has a great idea! She suggested that two corners be used for the dirt sections and the hanging baskets can go over those sections. She thinks that there is room for three rows of tables in the greenhouse as well.

Here are the things that you will learn in this lesson.

- solve area problems
- convert an area measurement in an SI area unit to another SI area unit
- convert an area measurement in an Imperial area unit to another Imperial area unit





Let's Get Messy

By participating in this Get Messy activity, you can earn coins towards your Mathemagical Kingdom bonus marks!

You need the following items for this activity:

- print-outs of shapes
- grid paper
- scissors
- glue



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Unit 5: 2-D and 3-D Measurements

Lesson B: Area Problems in 2-D 3

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Task 1

Cut out the rectangle for Task 1. Place it on the grid paper so that the edges of the shape line up with the grid lines (do not glue it) and record the following:

length _____

width _	
---------	--

What is the area of the rectangle?

What is the formula that will determine the area of the

rectangle?_____

Task 2

Draw a diagonal in the rectangle using a ruler. Cut along the diagonal.

Into what two shapes did you cut the rectangle?

Are the areas of the two shapes equal? _____ Explain.

The two shapes you made were triangles.



The length of the rectangle is the base of the triangle.

The width of the rectangle is the height of the triangle.

Notice that the base and the height of the triangle meet at 90 degrees.



The upside down triangle has the same base and the same height as the bottom triangle. That means the triangles are equal in size and must have the same area.

Glue the two triangles onto the grid paper so that the base and height match the grid lines. Use the grid squares to determine the area of each triangle.

What is the area of one triangle? _____

Is it half of the area of a rectangle? _____

Explain what formula could be used to determine the area of any triangle.



Task 3

Use the second rectangle. Cut it out and glue it to your grid paper.

What is the area of the rectangle below in grid units?



Recall in Unit 3 that you made a rectangle from straws, and then you made a parallelogram from the rectangle by 'squishing' your rectangle to one side.

On your grid paper rectangle, draw a parallelogram by 'squishing' it one grid unit to the right.



Is the area of the rectangle different from the

area of the parallelogram? _____ Explain.

Task 4

Cut out the parallelogram for Task 4. Draw a vertical line to make a triangle on one corner.



Cut along the vertical line that you drew. Glue both pieces on your grid paper so that they fit together to make a rectangle.

Tasks 3 and 4 show that the area of a parallelogram is related to the area of a rectangle. Use Task 3 and Task 4 to develop the formula for the area of a parallelogram.

Task 5

Cut out the trapezoid for Task 5. Glue it to the grid paper.

Estimate the area of the trapezoid by counting the squares it covers.



Task 6

Cut out the trapezoid for Task 6. Draw two vertical lines to make two triangles and one rectangle. Cut along the lines. Glue the two triangles and the rectangle on the grid paper. What is the area of each shape?

Triangle 1 area = _____

Rectangle area = _____

Triangle 2 area = _____

Find the total area of the trapezoid by adding the areas of each section.

Area of the trapezoid

= Area of triangle 1 + Area of rectangle + Area of triangle 2

= _____

Task 7

Cut out the trapezoid for Task 7. Notice that it is the same size as the one for Task 6. Think about this: if the base length and the top length are known, is that enough information to calculate the area of the trapezoid?

Yes _____ or No _____



Would knowing the base, the top, and the height be enough information to find the area of the trapezoid?

Yes _____ or No _____



Draw a line from one corner to the opposite corner.



Now, the trapezoid is cut into only two shapes—two triangles that are different sizes. Triangle 1 has a base measurement of 'b' and triangle 2 has a base measurement of 'a'. Both triangle 1 and 2 have the same height, which is the height of the trapezoid.



Determine the area of triangle 1 _____

Determine the area of triangle 2 _____

What is the area of the trapezoid? _____

Did your answer match the answer you got when you found the area of three shapes? _____

If not, where do you need to make changes?

Task 8

Cut out the circle. Cut along the dotted lines to make wedges. Rearrange the wedges to make a rectangle.



Notice that the distance around the circle is cut into sections and each edge of the section becomes part of either the top or the bottom of the rectangle. Half the edges are at the top and half the edges are at the bottom. The width of the rectangle is the **radius** of the circle.

Recall that the circumference of the circle is the distance around the edge of the circle. Half the circumference is along the length of the rectangle.





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How Does It Work?

In the 8 m x 10 m greenhouse, the floor will be 1 m by 1 m sidewalk slabs. However, now there will be two triangular corner sections that are not covered because the dirt will be used to grow watermelon and cantaloupe as well as pumpkins. These plants need lots of space as well as a long, warm growing season, so they will grow in the greenhouse the whole summer.



The two triangles of dirt are to be the same size and will be located on either side of the door at the front of the greenhouse.

Example 1

One triangle will be 3 m along the base and 5 m along the height. Recall that the sidewalk slabs are 1 m x 1 m.

- a. How many slabs are needed for the greenhouse if the whole floor is covered with slabs?
- b. How many slabs will not be needed if two triangles are left as dirt and not covered with sidewalk slabs?

Solution a

Area = length x width

 $= 8 m \times 10 m$ = 80 m²

Each slab is 1 m^2 , so 80 slabs are needed to cover the whole floor.

Solution b

Step 1: Construct a picture of the greenhouse sections.





8.5 m

6 m

Example 3

Calculate the area of the shape shown in the diagram.

Do you recognize this shape? It is similar to the shape of the key that most schools now use for basketball.

Solution

The diagram shows a composite shape.

Step 1: Cut the shape into common shapes.

One method is to cut the key into four shapes. You should find a semicircle, a rectangle, and two equal-sized triangles.



To find the area of the original shape, find the area of each section and add the areas.

Step 2: Find the area of the semi-circle.

Area of a circle = $\pi \times \text{radius}^2$ Because the section is a semi-circle, and a semi-circle is half of a circle, find the area of the circle and then divide by 2.



Composite shape means

a. the shape of Willow Creek Composite High School.
b. the shape of a compost box.
c. a shape made

c. a shape mad of other common shapes

Call your instructor to give your treasure chest answer and earn coins towards bonus marks.







3 ft

4 ft 6 in



Can you think of another way to cut this shape into sections so that the total area can be calculated?

Call your instructor to give your treasure chest answer and earn coins towards bonus marks.

Example 4

Calculate the area of the regular hexagon.

Solution

Step 1: Cut the original diagram into sections.



Because the shape is a regular hexagon, the lengths of the sides are equal.

Each of the six triangles is the same size.

Find the area of one triangle, and then multiply by 6 to get the total area.

Step 2: Before any calculations can be done, all the units of measurement must be the same. The base of the triangle measures 4 feet 6 inches, which is a mixture of units.

> Because the height of the triangle is given in feet, it would be best to change 4 ft 6 in to feet only.

Recall that there are 12 inches in 1 foot.

```
4 ft + 6 inches
= 4 ft + 6/12 ft
= 4 ft + <sup>1</sup>/<sub>2</sub> ft
= 4 <sup>1</sup>/<sub>2</sub> feet or 4.5 ft
```





Example 5

An enclosure for gophers at the Calgary Zoo is rectangular and has a rectangular border around it. The entire area set aside for the gophers is 18.7 m long and 11.4 m wide. The border is included in the area and is there so people are not able to reach in to try to touch the animals. It is 125 cm wide all around the enclosure.

- a. What is the area of the enclosure that the gophers are in?
- b. What is the area of the border?




You have seen several examples of problems involving finding area. Often, if a shape is not one that has a formula for finding its area, the shape can be cut into rectangles or triangles or circles, and then the area can be calculated.









Here are some practice questions for you!

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How Does It Work? Practice Questions

1. In Example 1, the floor of Lindsay's greenhouse is 8 m by 10 m. Two triangles were left as soil. What is the area covered by cement slabs?



2. Find the area of the following shapes:



3. Find the area of the shape below. Tick marks show lengths that are the same size.



4. Ed wishes to buy a plot of land. The first plot he views is a square and has a side measure of 68 m. The second plot of land is rectangular and has a length of 160 m. Both plots of land have the same area.

What is the width of the rectangular plot of land?

5. A square picture has a midpoint that is 9 cm away from each of the sides. What is the area of the picture?

6. The town of Milk River is planning a water fountain for its public park. It is to be rectangular with dimensions 22.5 ft x 18 ft. In the middle of the fountain will be a circular area filled with plants and a statue of the founding mayor. The radius of the circular planter is 7 ft.

What is the area of the fountain that will be filled with water?





1. In Example 1, the floor of Lindsay's greenhouse is 8 m by 10 m. Two triangles were left as soil. What is the area covered by cement slabs?

Practice Solutions



Find the total area of the green house, and then subtract the area of the triangles that will be used for planting directly into the soil.

Total area = length × width = 10 m × 8 m = 80 m²

Area of triangles of soil

In example 1, the area not covered by cement slabs is 15 m^2 .

Area of the greenhouse floor that is left = total area – area of dirt triangles. = 80 m² – 15 m² = 65 m²

The area of the greenhouse floor that is covered by cement slabs is 65 m^2 .

2. Find the area of the following shapes:





- d. Area of a parallelogram = base × height = 3.5 cm × 5 cm = 17.5 cm²
- 3. Find the area of the shape below. Tick marks show lengths that are the same size.



Two sides have one tick mark each. They are equal in length and they also add to be the same as the side that measures 4.25 m.

Cutting the diagram into two rectangles helps to calculate the area of the entire shape.

Rectangle 1 is 4.25 m long. Now, determine its width based on the double tick marks.

The sides with double tick marks are the same length. Those two sides combine to be the same length as the 6.5m side.

The width of rectangle 1 is $\frac{6.5 \text{ m}}{2} = 3.25 \text{ m}.$ 4.25 m



The total area of the shape is 20.71875 m²

4. Ed wishes to buy a plot of land. The first plot he views is a square that has a side measure of 68 m. The second plot of land is rectangular and has a length of 160 m. Both plots of land have the same area. What is the width of the rectangular plot of land?

Step 1: Find the area of the square plot of land.



Step 2: The area of the rectangular plot of land is the same as the square plot. Use the area of the rectangular plot to find the width of the rectangular plot of land.



The rectangular plot of land has a width of 28.9 m.

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5. A square picture has a midpoint that is 9 cm away from each of the sides. What is the area of the picture?

The midpoint of the picture is the same distance from each side of the picture. The midpoint is 9 cm from each side.

The distance from one side of the picture to the other side is 9 cm + 9 cm = 18 cm.

Therefore, the length of one side of the picture is 18 cm.

Area of a square = length × length

= 18 cm × 18 cm = 324 cm²

The area of the picture is 324 cm².

6. The town of Milk River is planning a water fountain for its public park. It is to be rectangular with dimensions 22.5 ft x 18 ft. In the middle of the fountain will be a circular area filled with plants and a statue of the founding mayor. The radius of the circular planter is 7 ft.

What is the area of the fountain that will be filled with water?

Step 1: Find the area of the rectangular area.

Area of rectangle = length × width = 22.5 ft × 18 ft = 405 ft²

The area of the rectangle is 405 ft².

Step 2: Find the area of the circular plant holder in the middle of the fountain.

Area of a circle = π × radius² = 3.14 × (7 ft)² = 153.86 ft²

The area of the circular plant holder is 153.86 ft².

Step 3: Because the circular plant holder will not be filled with water (it will be filled with plants), subtract the area of the circle from the area of the rectangle to get the actual area of the fountain that will be filled with water.

> Area of the fountain to be filled with water = rectangular area - circular area.

= 405 ft² - 153.86 ft² = 251.14 ft²

The area to be filled with water is 251.14 ft².

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Total 43

(2)

How Does It Work? Assignment

Now it's time to show your stuff! Put lots of details into your work.

1. Show on the grid paper, what the greenhouse will look like if two triangles measuring 5 m along the base and 5 m along the height are in each of the two corners.

Remember that the dimension across the green house is 8 m, and the greenhouse is 10 m long. $1 \text{ sq} = 1 \text{ m}^2$

2

2. Show on grid paper, how many tables that measure 1.5 m x 3 m could be arranged and still have 1 m paths between the tables.

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Image: Second				

2

ADLC Math 10-3

4. Sidney makes quilts. A piece of material that goes into the pattern of the quilt is shown below.

a. Calculate the area of one of these pieces of fabric.

2

6

 b. Sidney has a piece of fabric left from another project. It is rectangular and measures 24 inches by 29 inches. How many pieces of the quilting pattern can Sidney make from the leftover material? (2)

2

(2)

(2)

5. At the NWMP historical Museum in Fort Macleod, Alberta, a replica of the inside grounds of the police outpost fort has been made. In the grounds of the fort is a rectangular plot of grass with a border of gravel. The dimensions of the entire area are 35 ft long and 21.5 ft wide.

The border of gravel is a uniform 3 feet wide all the way around the grass.

a. What is the total area including the gravel border?

b. What is the area of the grass only?

c. What is the area of the gravel?

d. If gravel for the present depth costs \$10.59/ft², what is the cost of filling the border with another layer of gravel?

6. Vern needs 6.5 m^2 of cloth to make a dress for a client. He has a square of material that has that area, but the shape of the pattern pieces for the dress requires a piece of cloth that is rectangular. He has another piece of material that is rectangular and has the same area as the square piece of material. It is 3 m long. Vern needs it also to be at least 2 m wide.

Will the rectangular piece of material be wide enough?

(3)

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(4)

2

(2)

- 7. Sheet metal can be used to make washers. A square piece of sheet metal measures 10 cm on one side.
- a. If the washers have an outside diameter of 2 cm, how many washers can be cut from a piece of sheet metal?

b. If the washers have an inside diameter of 1.25 cm, what is the area of one washer?

c. How much sheet metal is recycled (not used to make washers) in one square piece?

You are ready to start Digging Deeper!

How many Stanley cups did Bryan Trottier win during his NHL career?

a. 4 b. 5 c. 6

Call your instructor to give your treasure chest answer and earn coins towards bonus marks.

Digging Deeper

A rich part of Canadian culture and history is the game of hockey. Many Canadians have become famous hockey players in the National Hockey League.

Bryan Trottier was inducted to the NHL Hockey Hall of fame in 1997. Considered to be one of the hockey greats, Bryan was born in 1956 in Val Marie, Saskatchewan. He played centre for two NHL teams, the Pittsburgh Penguins and the New York Islanders, over 18 seasons. Trottier's father was Métis because his grandmother on his father's side was Cree/Chippewa.

A hockey arena is a very exciting place to be if you like to play or watch hockey.

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Did you realize that a great deal of math is involved in designing and building the arena for pro and even backyard athletes?

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In Unit 4, you learned how to convert measurement units within one system to other appropriate units of measure within that system. Inches can be converted to feet in the Imperial system, or centimetres can be converted to metres in the metric system.

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Area measurements can also be converted within a measurement system. Perhaps one area measurement is in square centimetres and another is in square metres. Before any calculations are made, the units have to be common or the same.

Unit 5: 2-D and 3-D Measurements

85 ft

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The distance between the blue lines in a hockey arena is 60 ft.

- a. What is the **area** between the two blue lines?
- b. What are the **dimensions** in **yards** of the rectangle formed between the two blue lines and what is the **area** in **square yards**?

30 ft

200 ft

30 ft

Solution a

The area is calculated by finding the area of a rectangle. The distance across the arena ice is the length of the rectangle and the distance between the two blue lines is the width of the rectangle.

Area = length \times width

The length is 85 ft and the width is 60 ft.

Area = $85 \text{ ft} \times 60 \text{ ft}$

 $= 5100 \text{ ft}^2$

Area between the two blue lines is 5100 ft²

Step 1: To convert the dimensions of the rectangle formed by the two blue lines, you must remember that 1 yard = 3 feet.

85 feet $\times \frac{1 \text{ yard}}{3 \text{ feet}}$

 $= (85 \div 3)$ yards

= 28.33 yards

The **length** of the rectangle is 28.33 yards.

 $60 \text{ feet} \times \frac{1 \text{ yard}}{3 \text{ feet}} = 20 \text{ yards}$

The **width** of the rectangle is 20 yards.

The dimensions are 28.33 yds by 20 yds.

Step 2: The area in yards can be found by using the same formula, but use the dimensions in yards.

Area = length \times width

= 28.33 yards × 20 yards

 $= 566.67 \text{ yards}^2$

The area is 566.67 yards².

Example 7

A flat or tray used in a green house can be a square tray of 1 m^2 . The starter pots are 4 cm square pots.

How many starter pots can fit into one greenhouse flat?

Solution

Step 1: Change the units to be the same.

A 1 m^2 flat means that it is square in shape and the length of each side is one metre. One metre has 100 cm.

Therefore, the side measure of the flat is 100 cm.

Step 2: Find the area of the flat in cm².

Area = length × width = 100 cm × 100 cm = 10,000 cm²

Step 3: Find the area of one starter pot.

Area = length × width = $4 \text{ cm} \times 4 \text{ cm}$ = 16 cm^2

The greenhouse effect is a global warming issue believed to be caused by too much

a. CO₂ b. H₂O c. MSG

Call your instructor to give your treasure chest answer and earn coins towards bonus marks.

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Number of pots = total area \div area of one starter pot = 10 000 cm² \div 16 cm² = 625

One flat can hold 625 starter pots.

Example 8

The end view of a cattle shelter is a trapezoid in shape. The bottom of this end wall measures 16 ft 5 inches. The shortest side of the wall to the roof is 7 ft. The tallest side of 7 ft the wall to the roof is 9 ft 8 inches. Find the area of the end wall.

9 ft 8 in

16 ft 5 in

Solution

There are several ways to find the area of a trapezoid. You can cut into two triangles or cut it into a triangle and a rectangle. First, make all the units to be the same.

Step 1: Change all the units to be in the same Imperial measurement unit.

The best way to convert is to put the measurements all in feet.

= 16 ft + 5 inches

 $= 16 \text{ ft} + \frac{5}{12} \text{ feet}$

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Example 9

The face-off circle in an NHL hockey arena is 30 feet in diameter.

Find the area of the face off circle in

- a. feet squared
- b. yards squared

Solution a

Area of a circle = $\pi \times \text{radius}^2 (A = \pi r^2)$

The diameter of the circle is 30 feet.

The radius is half the diameter.

radius =
$$\frac{30}{2}$$

= 15 ft
Area = 3.14 × (15)²
= 3.14 × (225)
= 706.5 ft²

Unit 5: 2-D and 3-D Measurements

$$706.5 \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = \frac{? \text{ yd}^2}{706.5 \text{ ft}^2} \times 706.5$$
$$\frac{706.5}{9} = ? \text{ yd}^2$$
$$78.5 = ? \text{ yd}^2$$
Either way, the result is that the area of the face off circle is 78.5 yd^2.

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Example 10

- a. Convert 67 328 cm^2 to m^2 .
- b. Convert 1423 mm² to cm².
- c. Convert 1.2 km^2 to m^2 .

Solution a

Use your knowledge of area and of metric unit conversion to convert \mbox{cm}^2 to \mbox{m}^2

 $1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^2$ $1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$ And 1 m = 100 cm $100 \text{ cm} \times 100 \text{ cm} = 10,000 \text{ cm}^2$ It follows, then, that $10,000 \text{ cm}^2 = 1 \text{ m}^2$.

Now, set up a ratio.

10,000 cm² = 1 m² 67,328 cm² = ? m²

 $\frac{1 \text{ m}^2}{10 \text{ } 000 \text{ m}^2} = \frac{? \text{ m}^2}{67 \text{ } 328 \text{ cm}^2}$

Multiply both sides by 67 328

$$67 \ 328 \times \frac{1 \ \text{m}^2}{10 \ 000 \ \text{m}^2} = \frac{? \ \text{m}^2}{67 \ 328 \ \text{cm}^2} \times 67 \ 328$$
$$\frac{67 \ 328}{10 \ 000} = ? \ \text{m}^2$$
$$6.7328 \ \text{m}^2 = ? \ \text{m}^2$$
$$= 6.7328 \ \text{m}^2$$
$$\text{So, } 67328 \ \text{cm}^2 = 6.7328 \ \text{m}^2$$

Solution b

Use your knowledge of area and of metric unit conversion to convert mm^2 to cm^2 .

 $1 \text{ mm} \times 1 \text{ mm} = 1 \text{ mm}^2$ $1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^2$ And 1 cm = 10 mm $10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$ It follows that 100 mm² = 1 cm².

Now, set up a ratio.

 $100 \text{ mm}^2 = 1 \text{ cm}^2$

 $1423 \text{ mm}^2 = ? \text{ cm}^2$

 $\frac{1 \text{ cm}^2}{100 \text{ mm}^2} = \frac{? \text{ cm}^2}{1423 \text{ mm}^2}$

Multiply both sides of the equal sign by 1423 $\rm mm^2.$

$$1423 \times \frac{1 \text{ cm}^2}{100 \text{ mm}^2} = \frac{? \text{ cm}^2}{1423 \text{ mm}^2} \times 1423$$
$$\frac{1423}{100} = ? \text{ cm}^2$$
$$= 14.23 \text{ cm}^2$$
$$1423 \text{ mm}^2 = 14.23 \text{ cm}^2$$

Solution c

Use your knowledge of area and of metric unit conversion to convert km^2 to $m^2. \label{eq:metric}$

 $1 \text{ km} \times 1 \text{ km} = 1 \text{ km}^2$ $1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$

And 1000 m = 1 km

 $1000 \text{ m} \times 1000 \text{ m} = 1\ 000\ 000\ \text{m}^2$

It follows that 1 000 000 $m^2 = 1 \text{ km}^2$.

Now, set up a ratio.

 $1\ 000\ 000\ m^2 = 1\ km^2$

? $m^2 = 1.2 \text{ km}^2$

 $\frac{?\,m^2}{1.2\,\,km^2} = \frac{1\,\,000\,\,000\,\,m^2}{1\,\,km^2}$

Multiply both sides by 1.2 km².

? $m^2 = 1\ 000000 \times 1.2$

 $= 120 \ 000 \ m^2$

 1.2 km^2 = 120 000 m²

Although you have skills in conversion and knowledge of ratios, there are other ways to convert one SI area unit to another.

In Unit four, you saw a pattern in the names of the metric units. Using the prefix in the name of the measurement unit helps to know how big it is compared to other metric measurement units.

Name of Metric Capacity Unit	Abbreviation	
millimetre	mm	
centimetre	cm	
metre	m	
kilometre	km	

Following the pattern of naming metric units helps to change from one SI area unit to another.

Recall that in the Imperial system, the names do not have a pattern to help with conversion.

A chart definitely helps to convert one Imperial area measurement to another.

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Digging Deeper Practice Questions

- 1. Do the following conversions
 - a. 134 246 cm² = _____ m²
 - b. $6.2 \text{ m}^2 = ___cm^2$
 - c. $45 \text{ yd}^2 = __ft^2$
 - d. 72 in² = _____ft²
- 2. Determine the following areas.
 - a. Determine the area of the circle in m^2 .

b. The pentagon has a perimeter of 7.5 km. Determine the area in $\rm km^2.$

c. Determine the area in ft^2 .

- 3. Find the area of a triangle that has a base of 9 inches and a height of 1 $\frac{1}{2}$ feet.
- 4. The area of a circle is 114 in^2 .
 - a. What is the diameter of the circle in feet?

b. What is the circumference of the circle in feet?

5. A standard stop sign is 30 inches across from the midpoint of one side to the midpoint of the side opposite. The perimeter is 100 inches.

What is the area of a stop sign in feet²?

6. The square shown below has an area of 144 inches².

a. What is the length of one side?

b. What is the perimeter in inches?

c. What is the perimeter in feet?

7. A standard baseball diamond is 90 feet between bases. What is the area in yards² inside the diamond shape formed by the base paths?


Practice Solutions

- 1. Do the following conversions
 - a. $134\ 246\ cm^2 = 13.4246\ m^2$
 - b. $6.2 \text{ m}^2 = 62 000 \text{ cm}^2$
 - c. $45 \text{ yd}^2 = 2025 \text{ ft}^2$
 - d. $72 \text{ in}^2 = ___6 \text{ ft}^2$
- 2. Determine the following areas.
 - a. Determine the area of the circle in m^2 .



To find the area of a circle the formula is $A_c = \pi \times radius^2$

Determine the radius from the given diameter.

Radius =
$$\frac{1}{2}$$
 of diameter.

$$\mathbf{r} = \frac{1}{2} \times (50 \text{ cm})$$

Method 1: Now that the radius has been calculated, note that the question asks for the area of the circle in metres squared. The radius is in centimetres.

It might be best to convert the radius units to metres now, and then calculate the area. Use your conversion chart skills to convert 50 centimetres to metres.

1 m = 100 cm? m = 50 cm $\frac{1 m}{100 cm} = \frac{? m}{50 cm}$ Multiply both sides by 50 cm.

 $=\frac{50 \text{ m}}{100 \text{ cm}}$ = 50 cmThe radius in metres is 0.50

Finally, the area can be calculated.

 $A_{C} = \pi \times radius^{2}$ = 3.14 × (0.50m)² = 3.14 × (0.25m)² = 0.785 m²

The area of the circle is 0.785 m².

Method 2: Use the radius measurement in centimetres to calculate the area. Then, convert to metres squared at the end.

 $A_{c} = \pi \times radius^{2}$ = 3.14 × (50 cm)² = 3.14 × (2500 cm²) = 7850 cm²

Now, convert the answer that is in square centimetres to metres squared.

 $1 m^2 = 10 000 cm^2$

To convert cm² to m², divide by 10 000.

 $\frac{7850 \text{ cm}^2}{10\ 000 \text{ cm}^2} = 0.7850 \text{ cm}^2$

b. The pentagon has a perimeter of 7.5 km. Determine the area in km^2 .



Using the diagram, make 5 triangles that are the same size. Calculate the area of one triangle, and then multiply by 5 to get the area of the regular pentagon.

Area of one triangle = ½ × base × height

The perimeter is known. Because this is a regular pentagon, the length of one side can be calculated by dividing the perimeter by 5.

Length of one side = $\frac{7.5 \text{ km}}{5}$ Length of one side = 1.5 km

1.5 km and 1032 m are related linear SI measurements.
But kilometre and metre are not the same units. To do the area calculations, the units must be the same.
1.5 km = 1500 m.

The area of one triangle uses the length of one side as the base of the triangle and the distance from the midpoint of the pentagon to the midpoint of the side as the height of the triangle.

Area_T = ¹/₂ × base × height = ¹/₂ × 1500 m × 1032 m = 774 000 m²

The question required the answer to be given in square kilometres.

Convert 774 000 m² to km².

 $1 \text{ km}^{2} = 1 \ 000 \ 000 \ \text{m}^{2}$? km² = 774 000 m² $\frac{1 \text{ km}^{2}}{1 \ 000 \ 000 \ \text{m}^{2}} = \frac{? \text{ km}^{2}}{774 \ \text{m}^{2}}$ Multiply both sides by 774 000 m² ? km² = 0.774 km² c. Determine the area in ft^2 .



Section the diagram into two rectangles and use the 'tick marks' to determine the dimensions of each rectangle.

Area of Rectangle 1 = length × width = 9 inches × 4 inches



Area of Rectangle 2 = length × width = 3 inches × 1 foot

Convert 3 inches to feet.

$$= \frac{3}{12} \text{ ft} \times 1 \text{ ft}$$
$$= \frac{3}{12} \text{ ft} \times \frac{1}{1} \text{ ft}$$
$$= \frac{3}{12} \text{ ft}^2$$
$$= \frac{1}{4} \text{ ft}^2$$

Total area of the shape = area of Rectangle 1 + area of Rectangle 2

= $\frac{1}{4}$ ft² + $\frac{1}{4}$ ft² multiply top to top and bottom to bottom.

= $\frac{2}{4}$ ft² reduce the fraction to lowest terms. = $\frac{1}{2}$ ft²

The area of the composite shape is $\frac{1}{2}$ ft².

3. Find the area of a triangle that has a base of 9 inches and a height of 1 $\frac{1}{2}$ feet.

Area of a triangle = $\frac{1}{2} \times base \times height$

= $\frac{1}{2} \times 9$ inches $\times 1 \frac{1}{2}$ ft

Notice that the units of measure are not the same. Either 9 inches must be converted to feet or 1 ½ feet must be converted to inches. Because the expected units are not specified in the question, either is acceptable.

Method 1: Convert all to inches.

1 ¹/₂ feet converted to inches:

1 foot = 12 inches ¹/₂ foot = 6 inches 1 ¹/₂ feet = 12 inches + 6 inches = 18 inches.

Now, the area can be calculated. Area = $\frac{1}{2} \times 9$ inches $\times 18$ inches = 81 inches² Method 2: Convert all units to feet.

9 inches =
$$\frac{9}{12}$$
 feet
= $\frac{3}{4}$ feet as a reduced fraction
Now, the area can be calculated.
Area = $\frac{1}{2} \times \frac{3}{4}$ ft $\times 1$ $\frac{1}{2}$ ft
To do math with fractions, 1 $\frac{1}{2}$ ft should be written
as an improper fraction.
1 $\frac{1}{2}$ ft = $\frac{3}{2}$ ft
Area = $\frac{1}{2} \times \frac{3}{4}$ ft $\times \frac{3}{2}$ ft
= $\frac{1 \times 3 \times 3}{2 \times 4 \times 2}$ tops \times tops and bottoms \times bottoms.
= $\frac{9}{16}$ ft²

The area of the triangle in square feet is $\frac{9}{16}$ ft².

- 4. The area of a circle is 114 in^2 .
 - a. What is the diameter of the circle in feet?

Given the area of a circle, the radius of the circle can be found by rearranging the area of a circle formula.

Area of a circle = pi × radius² Because you know the area and need to find the radius, you want a formula to determine the radius. Therefore, you 'undo' whatever is happening to the radius in the area formula. $A_{c} = 3.14 \times (radius)^{2}$ First, divide both sides by 3.14 (opposite of multiply)

 $A_{c}/3.14 = radius^{2}$

Now, apply square root to both sides (opposite of squaring).

$$\sqrt{\frac{A_{\rm C}}{3.14}} = \sqrt{\rm radius^2}$$

Now, you have a formula for radius of a circle when given the area.

$$r = \sqrt{\frac{A_{C}}{3.14}}$$

= 114 inches²/3.14
= 36.31 inches²

= 6.03 inches rounded to two decimal places. The number of decimal places required is not specified.
Round to two decimal places because the value of pi was rounded to two decimal places.

You were asked for the diameter and diameter = 2 \times radius

= 2 × 6.03 inches

= 12.05 inches rounded to two decimal places

Recall that there are 12 inches in 1 foot, and the question asked for the diameter to be expressed in feet. Convert 12.05 inches to feet.

12.05 inches × 1 foot/ 12 inches

= 12.05/12 = 12.05 ÷ 12 = 1.00 ft

12.05 inches is very close to 1 foot.

The diameter of the circle is approximately 1.00 ft rounded to two decimal places.

b. What is the circumference of the circle in feet?

```
Circumference = π × diameter
= 3.14 × 1.00 ft
= 3.14 ft
The circumference is about 3.14 ft.
```

5. A standard stop sign is 30 inches across from the midpoint of one side to the midpoint of the side opposite. The perimeter is 100 inches.

What is the area of a stop sign in feet²?



Stop signs are regular octagons or shapes with 8 sides equal in length. A regular octagon can be cut into 8 triangles of equal area.

Find the area of one triangle and multiply by 8 to get the area of the octagon or stop sign. To find the area of one triangle, the side length of the octagon must be used as the base of the triangle. The perimeter of the stop sign is 100 inches. Use the perimeter to determine one side length. Perimeter = sum of the sides

Perimeter = 8 × side length 100 inches = 8 × side length 100 inches÷8 = side length 12.5 inches = side length

One side of the stop sign is 12.5 inches long.

The height of one triangle is also needed to find the area of one triangle. From the midpoint of the stop sign to the midpoint of one of the side lengths is the height of one triangle.

The distance from one side to the other is 30 inches. From the midpoint of the sign to one side is half of 30 inches.

$$\frac{1}{2} \text{ of } 30 \text{ inches}$$
$$= \frac{1}{2} \times 30 \text{ inches}$$
$$= 15 \text{ inches}$$

The height of one triangle is 15 inches. Now, that you know the base and height of one triangle, the area can be calculated.

A_T =
$$rac{1}{2}$$
 × base × height
= $rac{1}{2}$ × 12.5 inches × 15 inches
= 93.75 inches²

The area of one triangle is 93.75 inches² The total area of the octagon or stop sign = 8 × 93.75 inches² = 750 inches²

The last step is to express the answer in square feet as specified in the question.

Determine a conversion factor for square inches to square feet.

1 foot \times 1 foot = 1 foot² 12 inches \times 12 inches = 144 inches² 1 foot² = 144 inches²

Set up a ratio. 1 ft² = 144 in² ? ft² = 750 in²

 $\frac{1 \text{ ft}^2}{144 \text{ in}^2} = \frac{? \text{ ft}^2}{750 \text{ in}^2}$ Multiply both sides by 750 in² $\frac{750}{144} = ? \text{ ft}^2$ $= 5.2 \text{ ft}^2$

The area of a standard stop sign is about 5 ft².

6. The square shown below has an area of 144 inches².



a. What is the length of one side?

Area of a square = side x side Area of a square = side²

To find the length of one side, find the square root of both sides of the equal sign to 'undo' what is being done and get 'side' by itself.

```
\sqrt{144 \text{ inches}^2} = \text{side}
= 12 inches
```

b. What is the perimeter in inches?

The perimeter of a square = side + side + side + side, and because the side lengths are equal in a square, the short cut for perimeter of a square is

Perimeter = 4 x side = 4 x 12 inches = 48 inches. The perimeter is 48 inches. c. What is the perimeter in feet?

To convert the perimeter to feet, recall that 12 inches are in 1 foot.

```
48 inches = ? feet

12 inches = 1 foot

\frac{? \text{feet}}{48 \text{ inches}} = \frac{1 \text{ foot}}{12 \text{ inches}}
Multiply both sides by 48 inches

? feet = \frac{48}{12}

? feet = 4

The perimeter is 4 feet.
```

7. A standard baseball diamond is 90 feet between bases. What is the area in yards² inside the diamond shape formed by the base paths?

A diagram will help to solve this question.

90 feet between bases means that the side length is 90 feet.

Determine the area of the square formed by the base lines in square feet first. Then, convert the area to square yards.

Area = side x side

= side² = 90 ft × 90 ft = 8100 ft²

Now, change square feet to square yards.

1 yard = 3 feet 1 yard × 1 yard = 1 yard² 3 feet × 3 feet = 9 feet² 1 yard² = 9 feet²



Now, set up a ratio.

$$\frac{? yd^2}{8100 ft^2} = \frac{1 yd^2}{9 ft^2}$$

Multiply both sides by 8100 ft².

 $9 ext{yd}^2 = \frac{8100}{9}$

 $= 900 yd^{2}$

The area of the square made by the base lines of a baseball diamond is 900 yd².



Total 46

(5)

Digging Deeper Assignment

Now it's time to show your stuff! Put lots of details into your work.

- 1. Do the following conversions
 - a. 1.7 km^2 = _____ m²
 - b. 2.5 km^2 = _____ ha
 - c. $2 \text{ miles}^2 = ____ \text{feet}^2$
 - d. 1 acre = _____ miles²
 - e. 1 acre = _____ feet²

The area of the triangle shown is 8.2 inches².



(2)

2. Find the height of the triangle.

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3. The area that a lawnmower blade covers is a 1.32665 m^2 circle. What is the length of the blade of the lawnmower in metres?



4. Lindsay's grandma found some tables at Canadian Tire that would be great for the greenhouse. They measure 85 cm wide and 3.5 m long.

What is the area of one tabletop?

5. Lisa has made an embroidery design on a piece of square cloth that has an area of 100 in². She has enough ribbon to go all the way around the outside edge of the piece of square cloth.



- (4)
- a. How long is the ribbon? Answer in feet.



b. If she uses the ribbon to go around the embroidery design in a circle that touches the edges of the cloth, what would the diameter of the circle be? Answer in inches.



c. What would the area of the circle be that the ribbon would make around the embroidery design?

d. After putting the ribbon around the design in a circle, Lisa trimmed off the corners of the square piece of cloth. How much material was trimmed off? Answer in square inches.

- 6. The central circle of a dreamcatcher has an area of 1 ft². Clayton plans to make three more circles in the dreamcatcher that will be $\frac{1}{3}$ of the area of the central circle.
 - a. What is the radius of the large central circle in inches?



b. What is the diameter of the large central circle?

c. What is the circumference of the large circle?

(2)

2

(2)

(1)

(1)

(2)

2

(2)

2

d. What is the radius of each of the smaller circles?

e. What is the diameter of each of the smaller circles?

f. What is the circumference of each of the smaller circles?

g. Clayton uses rawhide wrapped on sturdy wire to make the circles in his dreamcatchers. How much wire will he need to make this dreamcatcher?

1 h. The wire costs \$2.50/foot. What is the cost for the wire?

7. The goal crease in an NHL hockey arena is a half circle with a diameter of 8 ft.



a. Find the area of the goal crease in feet squared.

b. Find the area of the goal crease in yards squared.

c. Find the area of the goal crease in yards squared using a different method than you did in part b.

2

2

2

(1)

- 8. Convert 734 216 m^2 to km^2 .
- 2
- 9. Explain a rule that could be made using base 10 as a factor of conversion when converting metres squared to kilometres squared.

- 2
- 10. Explain a rule that could be made using base 10 as a factor of conversion when converting any SI area unit to another SI area unit.

- 2
- 11. Complete the following chart.

Imperial Linear Unit	Imperial Area Unit

Lesson Summary

In this lesson, you learned that the two main systems that we work with, metric and Imperial, developed in different ways.

The Imperial system uses fractions because a fraction indicates how many parts the unit was sectioned to make smaller units.

You also learned that the metric system or SI system uses decimals because the decimal values are related to groups of 10. Because the system has groups of 10 small units making the bigger units, multiplying and dividing by 10 moves the decimal place.

Please attach your return address label to the back of this booklet and send Unit 5 Lesson B to be marked.

You are now ready to proceed Unit 5 Lesson C.



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Unit 5: 2-D and 3-D Measurements Lesson B: Area Problems in 2-D



