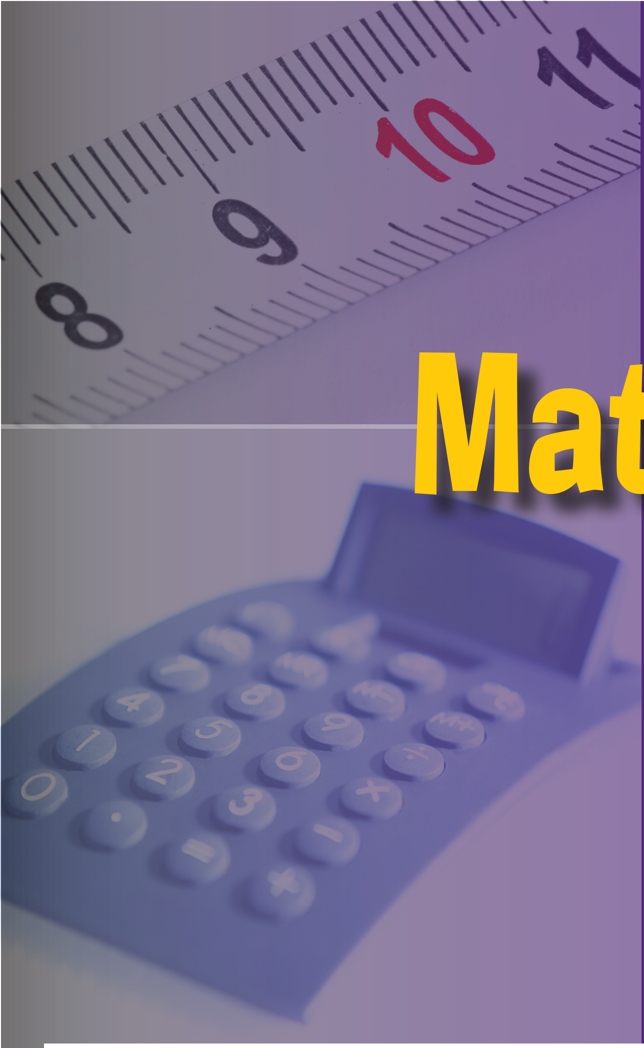
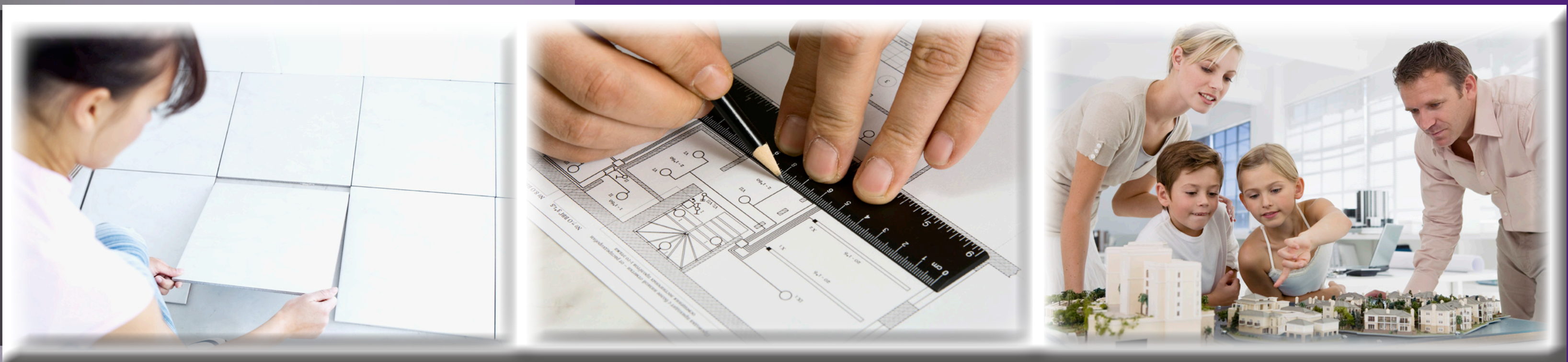
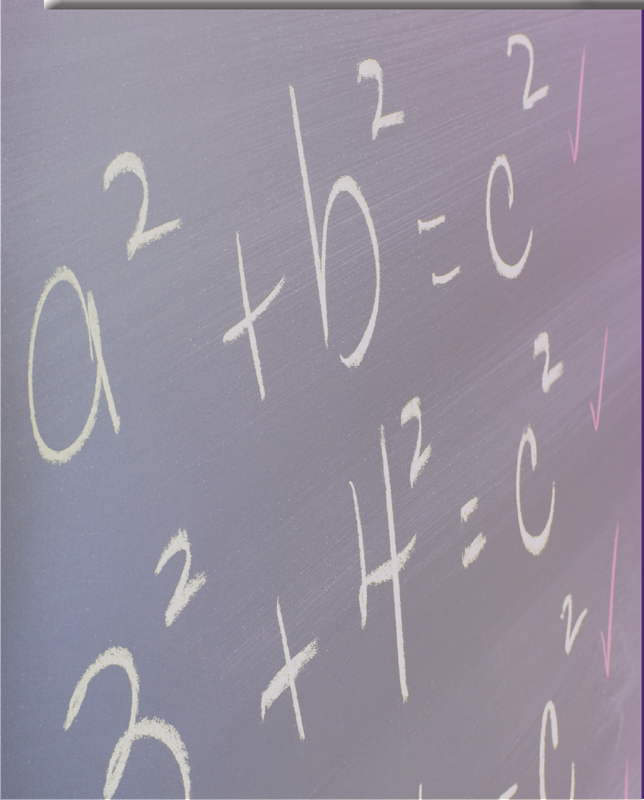


Mathematics 10-3

Unit 5
2-D and 3-D Measurements





Lesson C
Surface Area of 3-D Shapes

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Mathematics 10-3

Unit 5
2-D and 3-D Measurements

Instructions for Submitting Assignment Booklets

1. Submit Assignment Booklets regularly for correction.

2. **Submit only one Assignment Booklet at one time.** This allows your teacher to provide helpful comments that you can apply to subsequent course work and exams (if applicable).

3. **Check the following** before submitting each Assignment Booklet:

   - ☐ Are all assignments complete?
   - ☐ Have you edited your work to ensure accuracy of information and details?
   - ☐ Have you proofread your work to ensure correct grammar, spelling, and punctuation?
   - ☐ Did you complete the Assignment Booklet cover and attach the correct label?

Postal Mail

Determine sufficient postage by having the envelope weighed at a post office. (Envelopes less than two centimetres thick receive the most economical rate.)

Alternate Methods

Contact your teacher if you require an alternate method for submission of assignments.

Mathematics 10-3
Unit 5, Lesson C

Assessment

For successful completion of this course, you must do the following:

1. **Complete all questions in each Assignment Booklet to the best of your ability.** Incomplete Assignment Booklets will be returned unmarked.
2. **Achieve a Final Exam mark of at least 40%.**
3. **Achieve a final course mark of at least 50%.**

Process

- **Read the course material** and complete the practice questions as well as the assignments in this booklet.
- **Proceed carefully** through each assignment. Reflect upon your answers and prepare your written responses to communicate your thoughts effectively. Time spent in planning results in better writing.
- **Proofread your work** before submitting it for marking. Check for content, organization, paragraph construction (if applicable), grammar, spelling, and punctuation.
- **If you encounter difficulties or have any questions**, contact your course teacher at Alberta Distance Learning Centre for assistance.

Format

If you choose to **use a word processor** for your written work,

- format your work using an easy-to-read 12-point font such as Times New Roman
- include your full name and student file number as a document header
- double-space your final copy
- staple your printed work to this Assignment Booklet

Avoid plagiarism by acknowledging all sources you use. Contact your teacher if you are uncertain how to document sources.

Although you are encouraged to work collaboratively and discuss various aspects of this course with others, **all submitted work must be your own**.

Our Pledge to Students

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Lesson B
Surface Area of 3-D Shapes

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Lesson A:
Area Referents

Lesson B:
Area Problems in 2-D

Lesson C:
Surface Area of 3-D Shapes

Icons

The following icons will guide you through the course.

Let's Get Messy

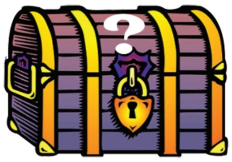
How Does It Work?

Digging Deeper

Practice Questions

Practice Solutions

Assignment



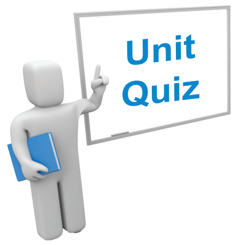
Treasure Chest

Coins

Discussions

Check It Out

How Did It Go?



Unit Quiz

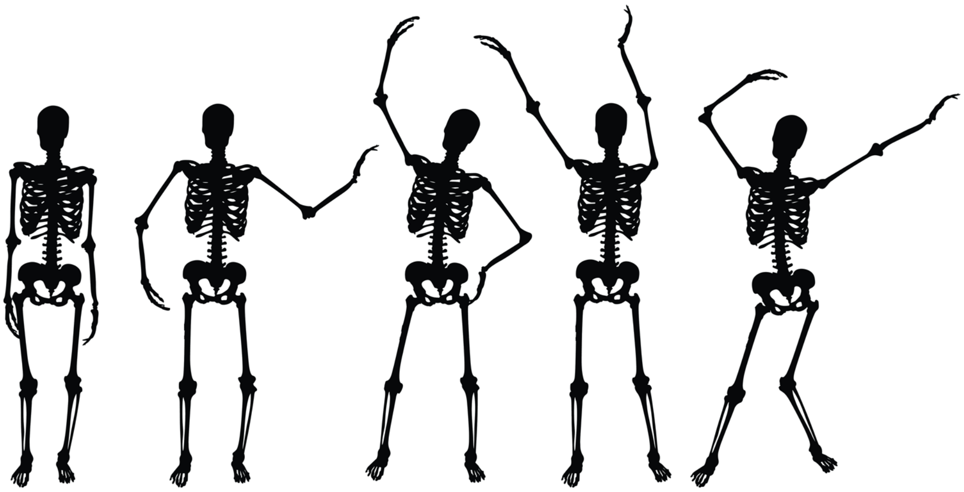
Lesson C:

Surface Area of 3-D Shapes

Lindsay and her grandparents are well underway with their greenhouse. They staked out a rectangular shape that was 8 m x 10 m, and they are now planning how to make the sides and the roof. Lindsay had thought they would use glass for the sides and the roof, but her granddad would rather use heavy plastic.

The walls will be 8 feet tall and the roof will be a half circle shape. Now comes the final task of figuring out how much plastic will be needed to cover the greenhouse.

In this lesson, you will learn how to find the surface area of three-dimensional objects as well as how 'nets' and 'skeletons' can help to do that. You will also get to explore how changing dimensions of the shapes affect perimeter and area.



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In this lesson, you will learn how to

- find surface areas of 3-D objects
- examine how changing the dimensions of rectangles will affect the area and perimeter
- determine if a solution to an area problem is correct
- recognize and use appropriate Imperial and SI units for linear measurement, area, volume, and capacity
- calculate volume

Let's Get Messy

For this activity, you will be discovering what the surface area of a sphere looks like and how an estimation of the surface area of a sphere can be determined.

You need the following items for this activity:

- geometry set
- 2 oranges (These should be large. You can use 2 grapefruits if you do not have large oranges.)
- ruler
- a few sheets of paper
- pencil

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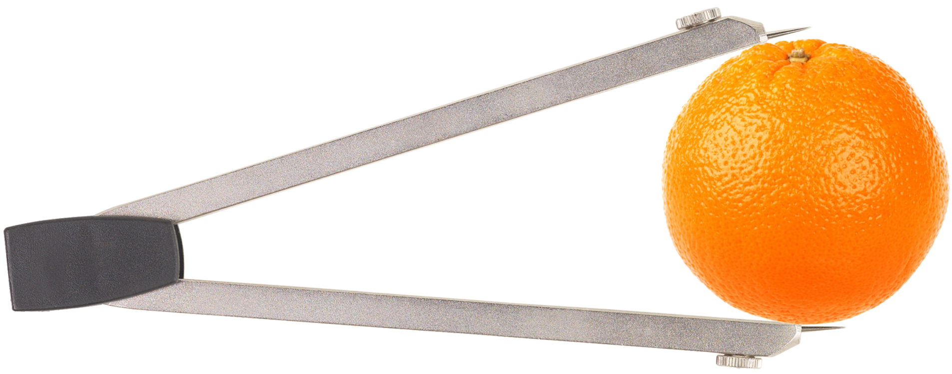
Task 1

Using your two-point compass, stretch the points to be as wide as the orange is.



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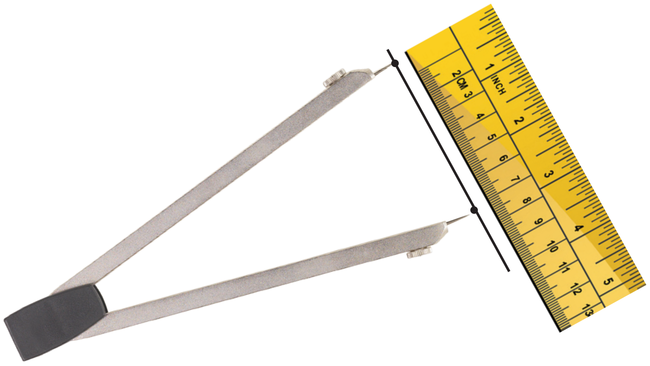
Notice the orange may be little wider from top to bottom than it is from side to side. Use the bigger width.

Task 2

Lay the ruler on the paper and draw a straight line that is longer than your compass points are apart.



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Put the compass point on one end of the line and mark on the line where the other compass point touches.

You now have a line segment that is the same length as your orange is wide. It is the diameter of the orange!

Task 3

Using the line segment that is the diameter of the orange, find the midpoint.

Measure the distance from one end of the line segment to the midpoint.

What is the distance that you measured? ______________

If the full line segment is the diameter of the orange, what is the distance to the midpoint called?

Task 4

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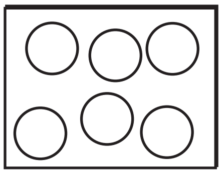
Set your pencil compass on your line segment so that the point is at the endpoint and the pencil is at the midpoint.



Task 5

Keep the compass setting and draw six circles on your paper.

Each of your 6 circles has a radius that is the same as the radius of your orange. The compass was set to be the distance from one endpoint to the midpoint of the line segment that was the diameter of the orange. The distance to the midpoint is then the radius of the orange.

Find the area of one of the circles that you drew. You saw in the Lesson 5A Get Messy activity that the area of a flat circle is found using the formula: $A = \pi \times \text{radius}^2$.

Task 6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Peel your orange. Keep the pieces to be about the size of a quarter.

Place all the pieces inside the circles. Try to cover all the space inside the circles with orange peel.

How many full circles were you able to cover with the pieces of orange peel?

You know the area of one of the circles that you drew. How much area does your orange peel cover? ________________

Now, from this Get Messy activity, you can see that the surface area of a 3-D circle or a sphere is equal to
4 X area of a circle with the same radius as the sphere.
or
Surface Area of a Sphere = 4 × pi X radius2
(or $A = 4\pi r^2$)


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Of course, we used orange peels as a referent and they are not uniform or very flexible, so the estimation could be different than the actual surface area of the orange (or sphere).

Task 7

With the second orange, use a knife carefully to peel the orange in strips. Try to keep the strips as straight as possible.

Task 8

Arrange the strips into a rectangle shape.

Use your ruler to draw a rectangle around the strips. Measure the length and the width of the rectangle.

length = _______

width = _______

Area of the rectangle = length × width = _______

Now use the radius of your orange that you determined with your compass and ruler in Tasks 1 to 4 to find the surface area of the orange using the formula:

$$\text{Surface area}_{\text{sphere}} = 4 \times \pi \times \text{radius}^2$$

Compare it to the area of the rectangle that you made with the strips of orange peel. The two areas should be about the same.

Was the area of your rectangle close to the real surface area found using the formula?

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Task 9

Eat your oranges!

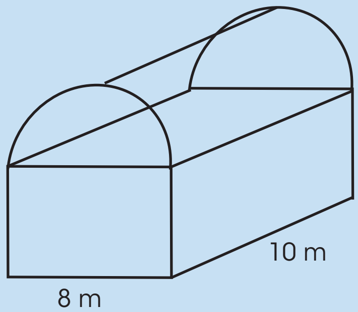
You have shown that the surface area of a sphere can be determined by using a formula that is related to the area of flat or 2-D circles.

You can also find the surface area of other 3-D objects by using formulas for the area of related flat or 2-D shapes

How Does It Work?

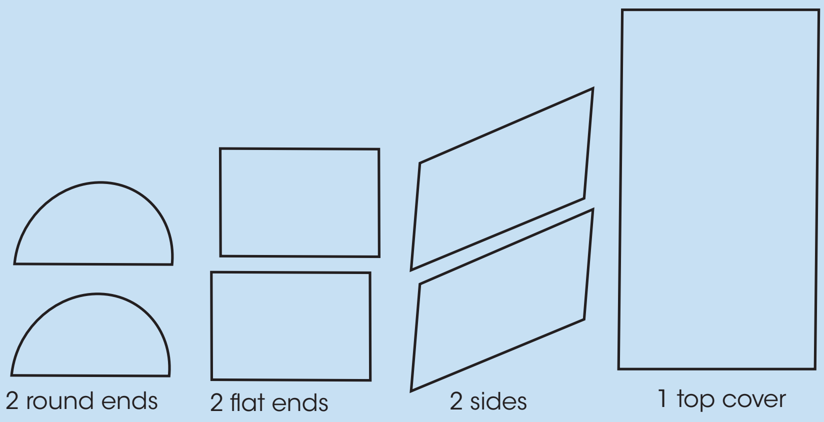
Lindsay's grandmother did some research on the plastic that they could buy to cover the green house. She found that the plastic comes in sheets that are 30 feet wide.

Lindsay was used to working in metres because she had learned about the metric SI system in school. Her grandparents were used to measuring things using the Imperial system, so strips of plastic 30 feet wide made sense to them.



However, they had to decide if the original plan of an 8 m x 10 m greenhouse would be the best size for the width of the plastic that they could buy.

First, it is helpful to picture how the greenhouse looks as a 3-D figure. Then, take each section of the greenhouse as a flat or 2-D shape to determine how much plastic is needed.

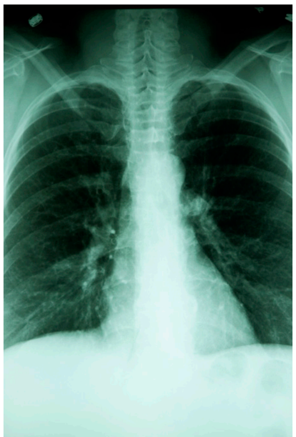


Remember the bottom rectangle is the ground and the plastic is not needed there.

skeleton: *an inner framework that gives an object shape*

Drawing a 3-D object on a 2-D surface, such as paper, is a little tricky, so we use drawings called ***skeletons*** to help get a good picture of the 3-D object.

A skeleton drawing shows the parts of the 3-D object that you cannot see. When you look at a building or other 3-D object, you can see the front and sometimes a side view, but you cannot see the back of the building. The skeleton shows what the 3-D object would look like if you could see through it to the other side.



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A skeleton of the green house might look something like this…

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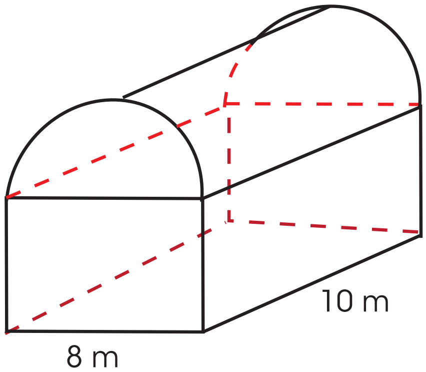

Now, the big question is this. How will the dimensions of the green house that are in metres compare to the width of the sheet of plastic that is measured in yards?

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Discussion

You can earn coins for the Mathemagical Amusement park and your bonus marks by participating in this discussion.

Remember that coins add up to make amusement park attractions and that those are bonus marks in the course! Contact your teacher to discuss the following questions.

Answer the following questions for your discussion and ask your discussion participants to comment on your answers.

1. Consider the conversion factor of 1m = 3.28 feet.

   How many feet is 8 m? ____________

   How many feet is 10 m? ____________

   ____

   ____

2. Lindsay and her grandparents decided to alter the dimensions of the greenhouse because they have to consider whether a 30-ft wide sheet of plastic will fit properly.

   Recall that the dimensions that they planned for the greenhouse were in metres and the width of the plastic is in yards.

   What dimensions for the greenhouse would you suggest?

   ____

   ____

3. Explain the choice you made for the new dimensions.

____

____

Ask two other people to comment on your suggestions.

Person 1_______________________________________

____

Person 2_______________________________________

____

4. Based on your discussion, would you stay with the dimensions you chose for their greenhouse? Explain.

____

____

5. What should the final dimensions of the greenhouse be?

____

____

6. How does changing the dimensions change the floor area of the greenhouse.

____

____

Ask two other people to comment on your suggestions.

Person 1____

____

____

____

Person 2____

____

____

____

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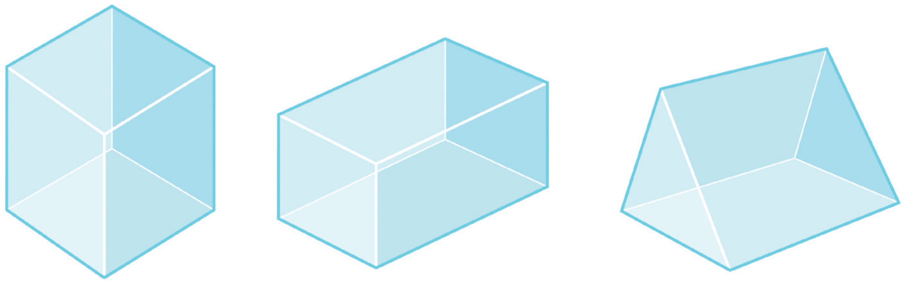
It is really helpful if you can recognize how a 3-D figure will look by its name. Here are some common 3-D objects and their names.

prisms: *always "box shapes"*

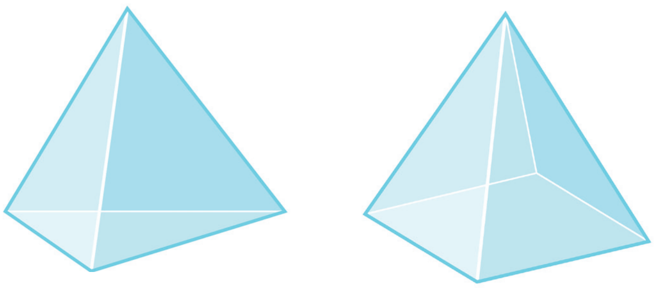
Three dimensional shapes can be placed in two categories, those with flat tops and those with pointy tops. Flat top figures include rectangular ***prisms***, cylinders, and cubes. Flat top figures have at least two bases that are opposite and parallel to each other.



Cube   Rectangular Prism   Triangular Prism

pyramids: *shapes with "pointy tops" (Think about the great Egyptian pyramids with their pointy tops)*

Pointy top figures include ***pyramids*** and cones. These figures usually have only one base.

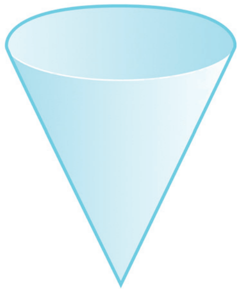


Triangular Based Pyrramid   Square Based Pyramid

A cylinder is similar to a prism. It has two bases and they are circles.

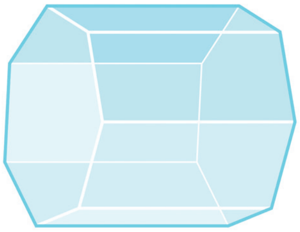
Cylinder

A cone is similar to a pyramid. Its base is a circle.



Cone

The name of the base is the starting place. Look at the shape of the base, and then decide if there is one base or two. Remember that the object may not be sitting on its base. It may be lying on its side.



Hexagonal Prism

The above 3-D shape is a hexagonal prism. The base ends are opposite each other and are connected by rectangular faces. In a prism, there is always only two faces that are considered to be the "base". In a hexagonal prism the base ends are hexagons.

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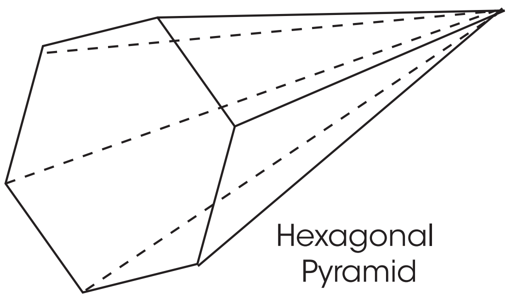
In a similar way, the base of a pyramid determines the name of the object. This pyramid is not on its base. It is laying on its side so the point is not at the top.



Hexagonal Pyramid

Example 1

Show the skeletons of the following 3-D figures.

a. rectangular prism

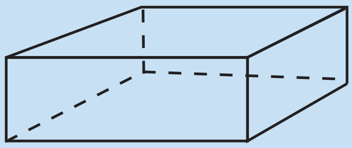
b. square pyramid

c. right cylinder

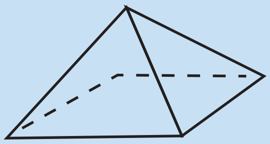
d. cone

Solution

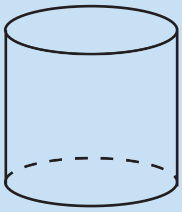
a. rectangular prism (Both bases are rectangles.)



b. square pyramid

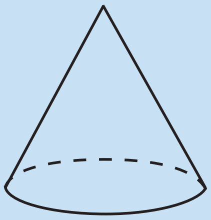


c. right cylinder (Both bases are circles.)

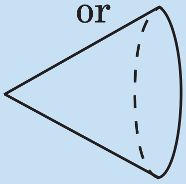


or

d. cone



or



net:
a flat version of a 3-D object

Another helpful way to picture a 3-D object is to draw a ***net***. A net is a 'flat' version of how the 3-D object would look.

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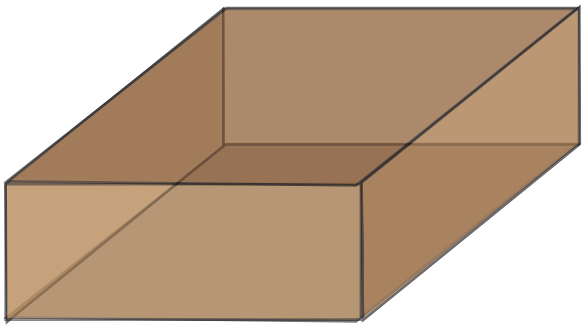
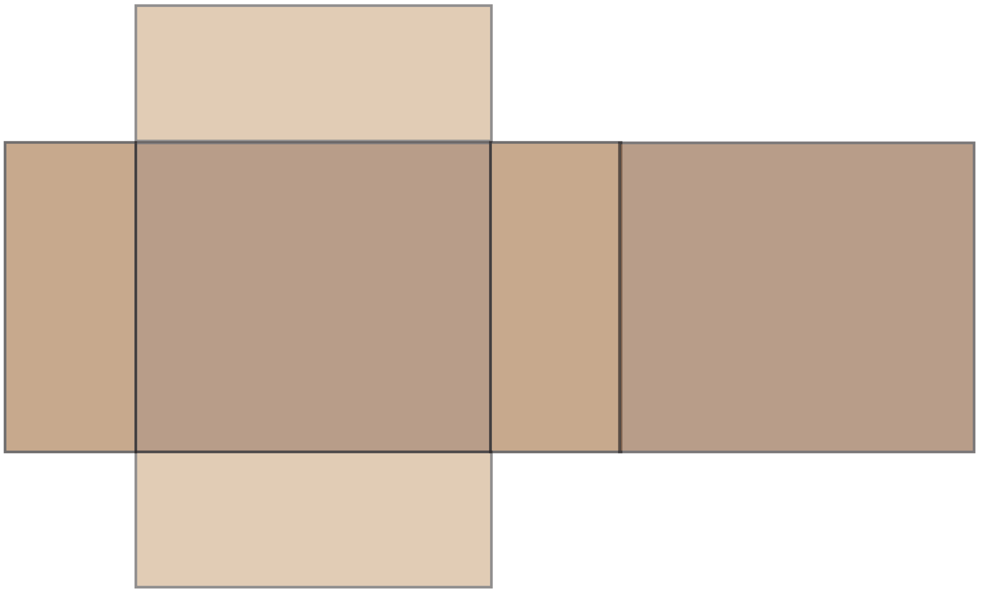
Think of how a volleyball net looks. It is a series of rectangles or squares or triangles linked together.

A fish net may be a series of small triangles linked together.

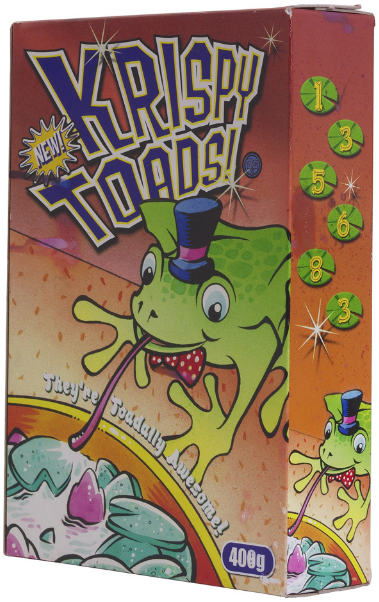
A 3-D object drawn as a net is also a series of 2-D shapes linked together. Imagine opening a box and cutting along some of the sides so that you can lay it flat.





Here is another example of a net.

Suppose you cut open a cereal box and lay it flat. What will it look like?



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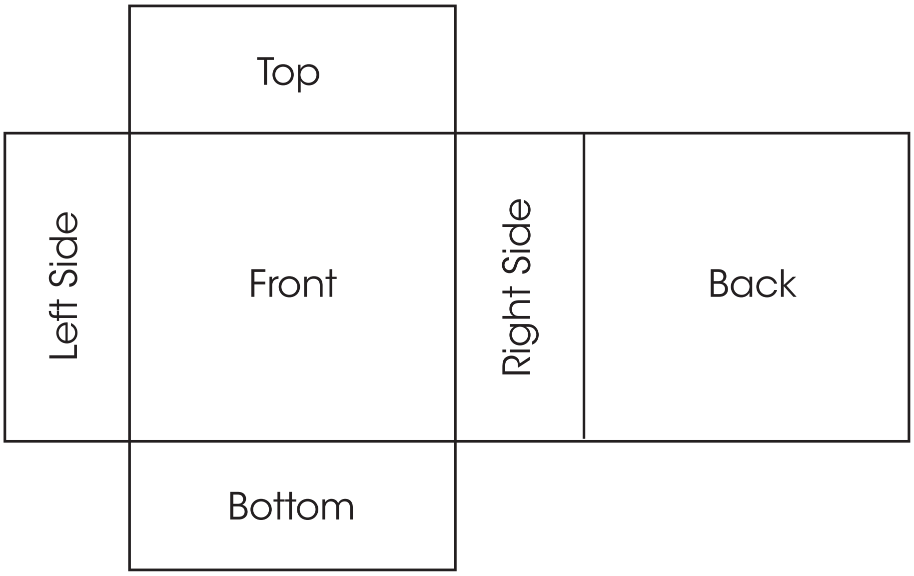
Rice Krispies are made from

a. corn
b. barley
c. rice

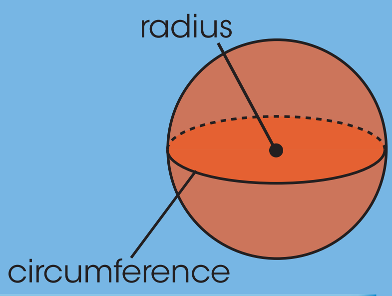
Call your instructor to give your treasure chest answer and earn coins towards bonus marks.

This view of a box is called a net.



Example 2

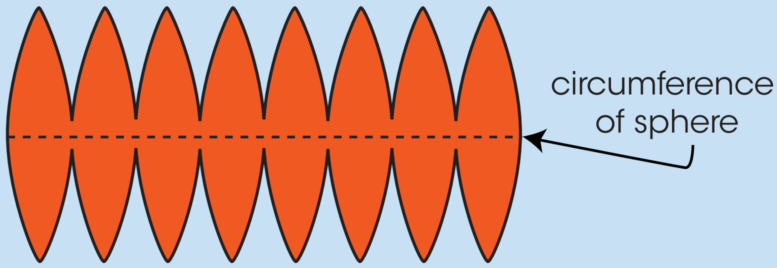
Draw a net of a sphere.



Solution

Recall when you did the Get Messy activity that you finished by peeling from the orange (or the sphere).

Imagine that you could peel uniform strips so that they were still connected. The net would then look like this.

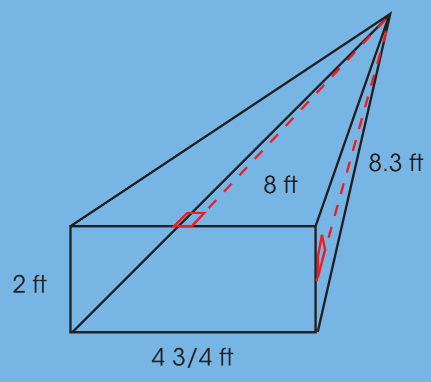


Being able to picture the net or the skeleton of a 3-D object helps when you are trying to name the object or when you are trying to do calculations about that object. Also, if you can picture the net or the skeleton of an object when you hear or read the name of that object, then you will be able to do calculations accurately for that object.

Example 3

a. What is the name of the 3-D figure shown?

b. Find the surface area of the 3-D figure shown.



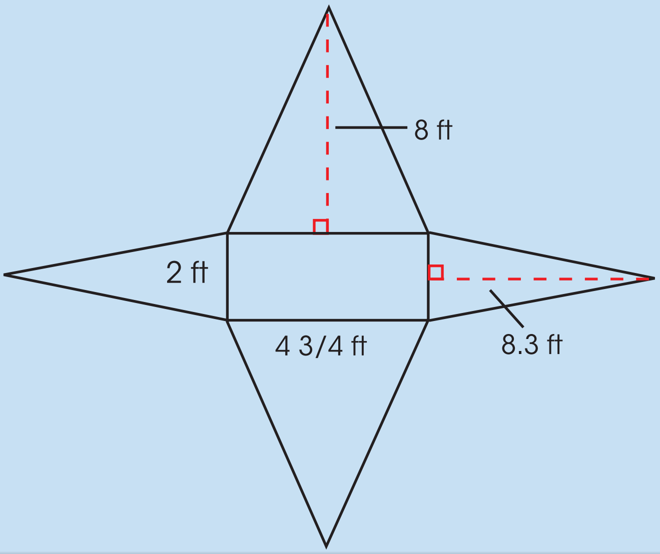
Solution a

The base of the figure is a rectangle. The sides are triangles, and they rise to a point. It is a **rectangular pyramid**. (It has a rectangular base and triangular sides that come to a point.)

Solution b

Follow the steps.

Step 1: A net is very useful to help find the total area of this object.



Step 2: Find the area of the rectangular base

Area = length × width

$= 4\,\frac{3}{4}$ ft × 2 ft

Change $4\,\frac{3}{4}$ to an improper fraction.

$4\,\frac{3}{4} = \frac{19}{4}$

$= \frac{19}{4} \times 2$ ft

$= \frac{19}{4} \times \frac{2}{1}$

Top times top and bottom times bottom!

$= \frac{38}{4}$

$= 9\,\frac{2}{4}$

$= 9\,\frac{2}{3}$ ft^2

The area of the rectangle base is 9.5 ft^2.

Step 3: Find the area of the triangles with base of $4\,\frac{3}{4}$ feet and height of 8 ft.

A = (base × height) ÷ 2

$= (4\,\frac{3}{4}$ ft × 8 ft) ÷ 2

$= (\frac{19}{4} \times 8) \div 2$

$= 38 \div 2$

$= 19 \text{ ft}^2$

There are 2 triangles that are the same size.

Area of both triangles:
$19 \text{ ft}^2 \times 2 = 38 \text{ ft}^2$

Step 4: Find the area of the triangles with the base of 2 feet and the height of 8.3 ft.

A = (base × height) ÷ 2

$= (2 \text{ ft} \times 8.3 \text{ ft}) \div 2$

$= 8.3 \text{ ft}^2$

There are 2 triangles that are the same size.

Area of both triangles:
$8.3 \text{ ft}^2 \times 2 = 16.6 \text{ ft}^2$

Step 5: Total the areas of the pieces to get the total surface area of the 3-D object.

$9.5 \text{ ft}^2 + 38 \text{ ft}^2 + 16.6 \text{ ft}^2$

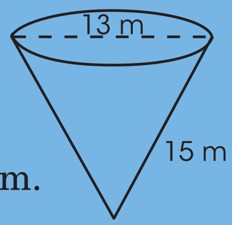
$= 64.1 \text{ ft}^2$

The total surface area is 64.1 ft^2.

Can you think of another way to cut this shape into sections so that the total area can be calculated?

Call your instructor to give your treasure chest answer and earn coins towards bonus marks.

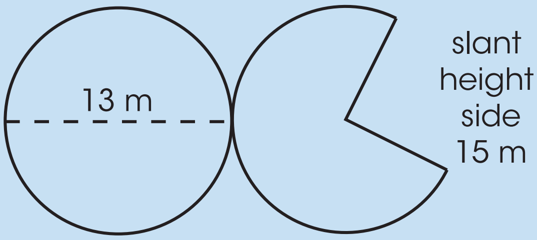
Example 4



Find the surface area of a cone with diameter of 13 m and slant height of 15 m.

Solution

Create a net



Surface area of a cone
= (π × radius^2) + (π × radius × slant height)

Remember that the surface area of a 3-D object is the area of each piece of the net.

The bottom of the cone is a circle, and that is where the π × radius^2 came from.

The section that looks like a pie with a piece missing has area = (π × radius × slant height)

In the question, the diameter is given. Use it to find the radius.

radius = $\frac{2}{3}$ × diameter

= $\frac{2}{3}$ × 13 m

= 6.5 m

Area = $3.14 \times (6.5)^2 + (3.14 \times 6.5 \times 15)$
(Remember that $6.5^2 = 6.5 \times 6.5 = 42.25$.)

$= 3.14 \times 42.25 + 306.15$

$= 132.665 + 306.15$

$= 438.815$

$= 438.82$ rounded to two decimal places because π is 3.14 when rounded to two decimal places

The surface area of the cone is 438.82 m^2.

You have seen several examples of problems involving finding area. Often, if a shape is not one that has a formula for finding its area, the shape can be cut into rectangles or triangles or circles, and then the area can be calculated.

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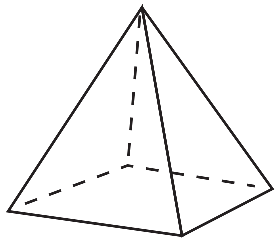
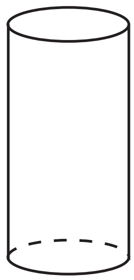
How Does It Work? Practice Questions

1. Name each of the following 3-D objects.

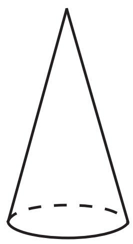
   a. ______________________________



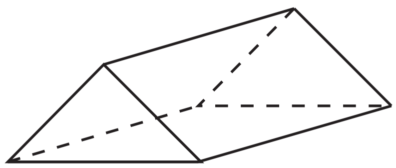
   b. ______________________________



   c. ______________________________

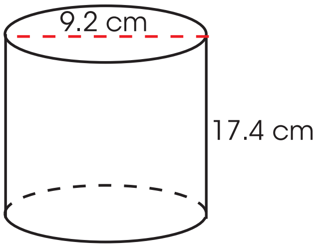


   d. ______________________________

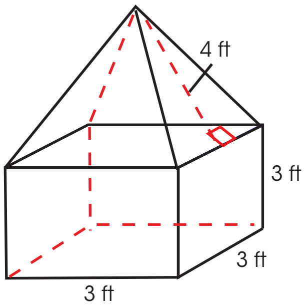


2. Find the total surface area of the following 3-D objects:

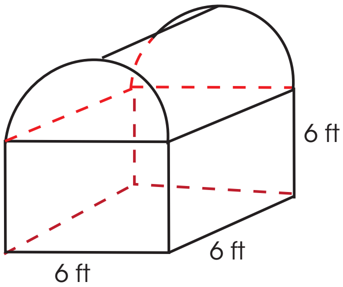
a.



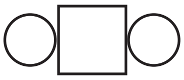
b.



c.



The diagram



represents

a. sponge bob withlarge ears
b. the net of a cylinder
c. an undetermined 3-D object

Call your instructor to give your treasure chest answer and earn coins towards bonus marks.

3. Find the surface area of a sphere with a circumference of 16.3 cm. ($SA_{sphere} = 4\pi r^2$)

4. Find the surface area of a cone with a radius of 12.3 cm and slant height of 6.4 cm. ($SA_{cone} = \pi rs = \pi r^2$)

5. The surface area of a cube is 216 sq m. Find the length of each side.

6. A classroom is 7 m long, 6.5 m wide and 4 m high. It has one door that is 3 m by 1.4 m and three windows, each measuring 2 m by 1 m. The interior walls are to be painted. The contractor charges \$5.25 per sq. m. Find the cost of the paint job.

7. The diameter of a road roller 1.4 m long is 80 cm. If it takes 600 revolutions to level a playground, find the cost of levelling the ground at \$0.75 per sq m.

Where did the name lodgepole pine tree come from?

a. It is the name of the tall straight trees that First Nations people used to build lodges or tipis.
b. Its actually "large pine tree".
c. No one knows for sure.

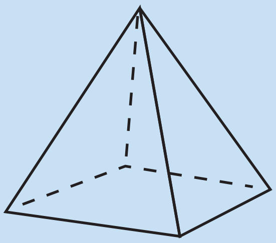
Call your instructor to give your treasure chest answer and earn coins towards bonus marks.

8. A Blackfoot tipi has diameter of 24 ½ feet. Lodgepole pine poles are used to frame the tipi. If the length of one pole from the ground to where it crosses the other poles is 35 ft, what is the total amount of material needed to wrap around the pole frame and make the tipi?

Practice Solutions

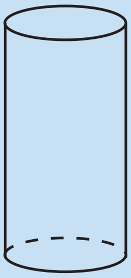
1. Name each of the following 3-D objects.

a. ______________________________



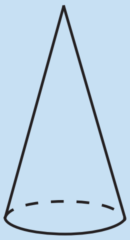
The base is square and the sides are triangular. The sides meet at a point. This is a square pyramid.

b. ______________________________



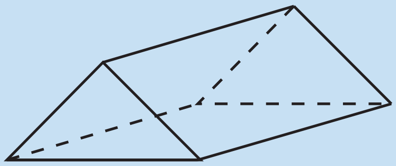
The bases are circular. This is a cylinder.

c. ______________________________



There is only one base. It is circular and the sides rise to a point. This is a cone.

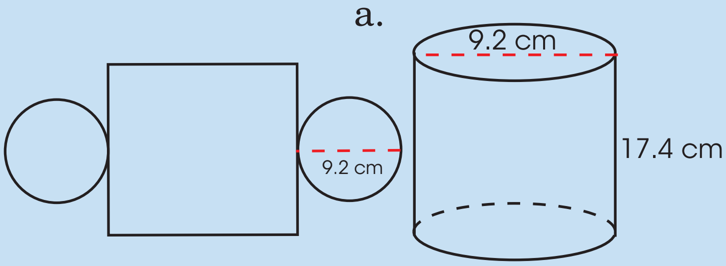
d. ______________________________



The two triangular bases are opposite each other and are connected by rectangular faces. This is a triangular prism. Note: It is not sitting on its base.

2. Find the total surface area of the following 3-d objects:

a.

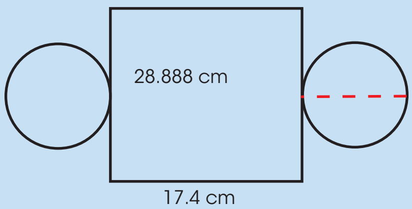


When the side of the cylinder opened, it became a flat rectangle with two circular ends.

$A_T = A_{rectangle} + A_{circle} + A_{circle}$
or
$= A_r + 2A_c$

Step 1: The $A_r = L \times W$ but we are not given the width. When the side open, it "unrolled" from the circular end. The width of the rectangle is the cirucumference of the circular end.

$C = \pi \times$ diameter
$= 3.14 \times 9.2$ cm
$= 28.888$ cm



$A_r = L \times W$
$= 17.4 \times 28.888$ cm
$= 502.6512\ cm^2$

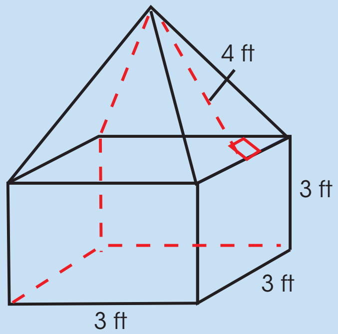
Step 2: Find the area for the circular bases.
$A_c = \pi \times radius^2$
where radius $= \frac{1}{2}$ of diameter
$= \frac{1}{2} \times 9.2$ cm

$= 4.6$ cm

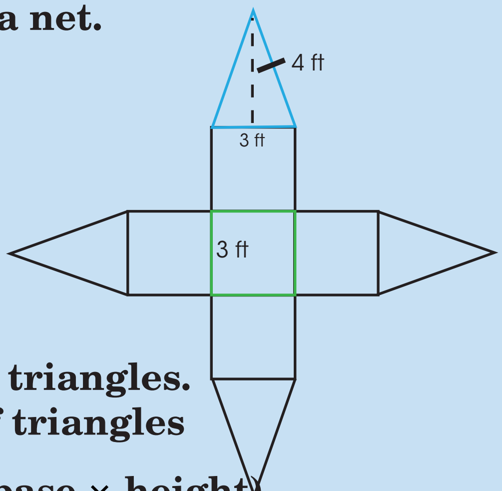
$A_c = 3.14 \times (4.6\ cm)^2$
$= 3.14 \times 21.16\ cm^2$
$= 66.4424\ cm^2$

Step 3: $= A_r + 2A_c$
$= 502.6512\ cm^2 + (2 \times 66.4424\ cm^2)$
$= 502.6512\ cm^2 + 132.8848\ cm^2$
$= 635.536\ cm^2$
$= 635.54\ cm^2$ rounded to two decimal places because $\pi = 3.14$ has two decimal places.

b.



Make a net.



There are five squares and four triangles.

A_{Total} = Area of squares + Area of triangles

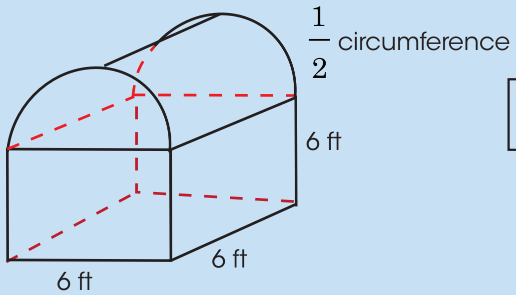
$S_A = 5 \times (\text{side} \times \text{side}) + 4 \times (\frac{1}{2} \times \text{base} \times \text{height})$

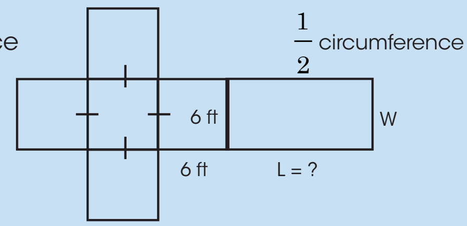
$= 5 \times (3\text{ ft} \times 3\text{ ft}) + 4 \times (\frac{1}{2} \times 3\text{ ft} \times 4\text{ ft})$

$= 45\text{ ft}^2 + 24\text{ ft}^2$

$= 69\text{ ft}^2$

c.





A_{Total} = Area of 5 squares + Area of the rectangle

$S_A = 5 \times (\text{side} \times \text{side}) + (\text{length} \times \text{width})$

The length of the rectangle is half of the circumference.

$C = \pi \times \text{diameter}$

$= 3.14 \times 6\text{ft}$

$= 18.84\text{ ft}$

$\frac{2}{3}$ **of C** $= \frac{2}{3} \times 18.84\text{ ft}$

$= 9.42\text{ ft}$

$S_A = 5 \times (6\text{ ft} \times 6\text{ ft}) + 9.42\text{ ft} \times 6\text{ ft}$

$= 180\text{ ft}^2 + 56.52\text{ ft}^2$

$= 236.52\text{ ft}^2$

3. Find the surface area of a sphere with a circumference of 16.3 cm.

$SA_{sphere} = 4\pi r^2$
$= 4 \times 3.14 \times$ radius
Use the circumference to find the radius of the sphere.
$C = \pi \times$ diameter
16.3 cm $= 3.14 \times d$ (Divide both sides by 3.14.)

$$\frac{16.3}{3.14} = \frac{\cancel{3.14} \times d}{\cancel{3.14}}$$

5.19 = diameter

$$\text{radius} = \frac{1}{2} \times \text{diameter}$$

$$= \frac{1}{2} \times 5.19 \text{ cm}$$

= 2.595 cm
$SA_{sphere} = 4 \times 3.14 \times (2.595 \text{ cm})^2$

The surface area is 84.58 cm^2

4. Find the surface area of a cone with a radius of 12.3 cm and slant height of 6.4 cm.

$SA_{cone} = \pi rs + \pi r^2$
$= (3.14 \times 12.3 \text{ cm} \times 6.4 \text{ cm}) + (3.14 \times 12.3 \text{ cm}^2)$
(Remember 12.3 cm = $12.3 \times 12.3 = 151.29$ cm)
$= 247.18 + (3.14 \times 151.29)$
$= 247.18 + 475.05$
$= 722.23 \text{ cm}^2$
The area of the cone is 722.23 cm^2

5. The surface area of a cube is 216 sq m. Find the length of each side.

6 square faces that are all the same size make the surface of the cube.

$SA_{cube} = 6 \times$ Area of one face

$216 = 6 \times$ Area of one face

$\frac{216}{6} =$ area of one face

$36 =$ Area of one face

The area of the square face is 36 m^2. Now, find the length of one side.

$36\ m^2 = \text{length}^2$

Find the square root of both sides.

$\sqrt{36\ m^2} = \sqrt{\text{length}^2}$

$\sqrt{36} = \text{length}$

$= 6\ m^2$

The length of one side of the cube is 6 m.

6. A classroom is 7 m long, 6.5 m wide and 4 m high. It has one door that is 3 m by 1.4 m and three windows, each measuring 2 m by 1 m. The interior walls are to be painted. The contractor charges $5.25 per sq. m. Find the cost of the paint job.

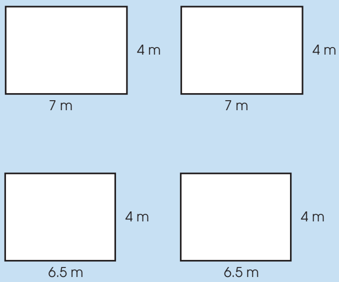
Find total surface area of walls.

$SA = 2(L \times W) + 2(L \times W)$

$= 2(7\ m \times 4\ m) + 2(6.5\ m \times 4\ m)$

$= 56\ m^2 + 52\ m^2$

$= 108\ m^2$



Subtract the area of the windows and door.

$SA_{door} = L \times W$

$= 3\ m \times 1.4\ m$

$= 4.2\ m^2$

$SA_{windows} = 3 \times L \times W$

$= 3 \times 2\ m \times 1\ m$

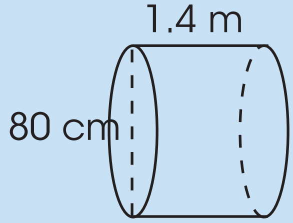
$= 6\ m^2$

= wall area - doors and windows
= 108 m² - (4.2 m² + 6 m²)
= 108 m² - 10.2 m²
= 97.8 m²
Cost = price/m² × Area
= $5.25 × 97.8 m²
= $513.45

It will cost $513.45 to paint the classroom walls.

7. A road roller is 1.4 m long and has a diameter of 80 cm. If it takes 600 revolutions to level a playground, find the cost of levelling the ground at $0.75 per sq m.

Convert diameter in cm to m.
80 cm = ? m
100 cm = 1 m
= 0.80 m



C= π × diameter
= 3.14 × 0.80 m
= 2.512 m
One revolution covers a rectangle with length of 2.512 m and width of 1.4 m.

SA = L × W
= 1.4 × 2.512 m
= 3.52 m²
One revolution covers an area of 3.51 m². It takes 600 revolutions to level the ground.
Total Area levelled
= 600 rev × 3.52 m²/rev
= 2106 m²
Cost = 2106 m² × $10.75/m²
= $1579.50
It costs $1579.50 to level the entire area.

8. A Blackfoot tipi has diameter of $24\frac{2}{3}$ feet.

Lodgepole pine poles are used to frame the tipi. If the length of one pole from the ground to where it crosses the other poles is 35 ft, what is the total amount of material needed to wrap around the pole frame and make the tipi?

Surface area of a cone is used to find the area of material needed to wrap the pole frame to make a tipi.

The bottom of the cone is the ground, so the area of the circular base does not need material.

$SA_{cone} = \pi rs + \pi r^2$
But πr^2 **= Area of the circular base (or the ground in this case)**
Remember, the base will be dirt and does not need material. The surface area calculated will be only for the body of the tipi.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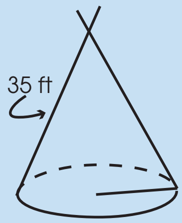
$SA_{body} = \pi rs$

$\text{diameter} = 24\frac{1}{2}\ \text{ft}$

$\text{radius} = \frac{1}{2}\ \text{diameter}$

$= \frac{1}{2} \times 24.5\ \text{ft}$

$=12.25$



$SA_{body} = 3.14 \times 12.25 \times 35\ \text{ft}$

$= 1346.275$
$= 1346.28\ \text{ft}^2$

The amount of material needed to wrap the tipi is 1346.28 ft².

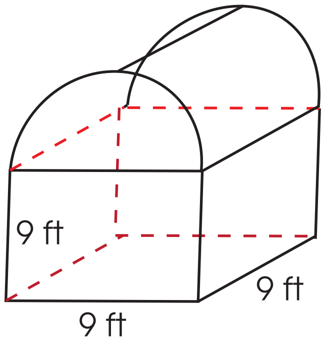
Total
44

How Does It Work? Assignment

Now it's time to show your stuff!
Put lots of details into your work.

2

1. Draw the skeleton of a triangular prism.



2. Lindsay decided that because the width of the plastic is 30 feet, the best shape for the greenhouse is a square measuring 9 m x 9 m.

   The wall height of the greenhouse will be 9 feet.

2

   a. Why did Lindsay choose the dimensions 9 m x 9 m for the greenhouse instead of 8 metres x 10 metres?

   ____

   ____

2

   b. What are the dimensions of the greenhouse in feet?

4 c. Determine an approximate length of 30-ft wide plastic needed to cover the greenhouse.

4 d. What is the surface area of the greenhouse?

2

3. The plastic costs $25.79 per linear foot for the 30-foot wide sections. To make a greenhouse properly, two layers of plastic are used. One layer is on the inside of the framing and the other layer is on the outside.

   How much will it cost to put two layers of plastic on the greenhouse that Lindsay and her grandparents are making?

2 4. Alyssa has a cube-shaped shower stall. Each edge on the cube measures 2.5 m. She wants to put tiles on the inside walls of the shower.

Each side of a square tile is 10 cm long. How many tiles will she need to cover the shower walls? (Remember: no tiles are used on the base or the top.)

6 5. A lawn roller is 150 cm long and has a diameter of 70 cm. To level a playground, it takes 750 complete revolutions. Determine the cost of levelling the playground at the rate of $2.00 per square metre.

4

6. Latisha has 50 coins Each coin has a diameter of 1.5 cm and is 0.2 cm thick. She piles them up to form a cylinder.

   What is the total surface area of a cylinder that could be made as a container for his coins?

6

7. An inflated beachball has a circumference of 24 ¼ inches.

   How much plastic material is needed to make the ball?

   Hint: Find the surface area of the ball. $SA_{sphere} = 4\ \pi r^2$

8. The slant height of a cone-shaped tent is 3.5 m and the radius of its base is 2 m.

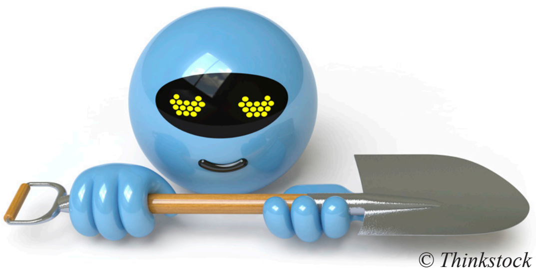
2 a. Determine the entire surface area of the tent to determine the amount of canvas needed to make the tent. (There is a bottom!) $SA_{cone} = \pi rs + \pi r^2$

2 b. How many metres of canvas will be needed if the bolt of canvas is 1.5 metres wide?

2 c. How many people can the tent accommodate if each person is allowed 2.2 m^2 of space.

2 d. How much will it cost to make the tent if canvas is \$16/m?

2 e. Kenna bought enough canvas to make the cone shaped tent and then marked the price up by 25% to sell it. How much will the customer pay Kenna for the tent?



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You are ready to start Digging Deeper!

Digging Deeper

Lindsay's greenhouse project with her grandparents is a big job. It takes not only planning and measuring but also converting and adjusting measurements until the final plan is just right.

As this course has progressed, you have gained more knowledge and understanding.

You were able to convert metres to feet, and you were able to make a reasonable plan of the floor plan of the greenhouse. You have also calculated the surface area of the greenhouse so that you would know how much plastic is needed. You have put together some skills that you learned in previous units to determine the cost of the plastic needed for the project.

Now, you will use some of your skills with perimeter and area to investigate how changes in one or more dimensions affect the area or perimeter.

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Also in this Digging Deeper section, you will use your understanding from previous lessons to work with the volume of 3-D objects.

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Example 5

a. Find the **perimeter** and **area** of a rectangle with length of 7 ft and width of 5 feet.

b. Double the length of the rectangle. What happened to the perimeter?

c. What happened to the area when the length was doubled?

Solution a

Perimeter = 2(length) + 2(width)
= 2(7) + 2(5)
= 14 + 10
= 24 ft

Area = length × width
= 7 ft × 5 ft
= 35 ft^2

Solution b

Double the length.

L = 7 ft × 2
= 14 ft
The width does not change. It is still 5 ft.

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Perimeter = 2(14) + 2(5)
= 28 + 10
= 38 ft

The first perimeter was 24 ft, but with the length doubled, the new perimeter is 38 ft.

The new perimeter is 14 ft larger than the first perimeter. The perimeter increased because the length changed.

Solution c

Area = 14 ft × 5 ft
= 70 ft^2

The new area is 70 ft^2 and the original area was 35 ft^2. The new area is double the original area.

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For Lindsay's greenhouse, the original plan was to have a rectangular floor plan. It was to be 8 m x 10 m. In the discussion of the width of the plastic sheeting, we determined that it might be better to change those dimensions. One of the examples proposed that a 9 m x 9 m greenhouse might 'fit' the plastic better.

Example 6

a. How did changing the dimensions of the greenhouse from 8 m x 10 m to 9 m x 9 m change the perimeter?

b. How did changing the dimensions of the greenhouse change the area?

Solution a

Perimeter of the greenhouse when it was 8 x 10:
Perimeter = 2(length) + 2(width)
= 2(10) + 2(8)
= 20 + 16
= 36 m

New perimeter at 9 m x 9 m:
= 2(9) + 2(9)
= 18 + 18
= 36 m

Hmmmm! The perimeter did not change!

Solution b

Area at 8 x 10:
Area = length × width
= 10 × 8
= 80 m^2

New area
= (9) × (9)
= 81 m^2

The area did change. It got larger. Although the perimeter stayed the same, the area got larger by making a square-shaped greenhouse instead of a rectangular-shaped greenhouse.

Calculating the surface area of 3-D objects determines the amount of material that will go around the object.

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3-D objects also have volume.
Recall that volume means the amount of stuff that a 3-D object can hold.

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Think of any object cut into layers. The volume of the object is the space that all the layers take when added together.

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The area covered by one layer also has a thickness, so the volume of the object is found by putting all the layers together.

Think of a tall apartment building. Each floor has a certain area. The floors are separated by walls. The thickness of the layer in a tall building is determined by how high the walls are between floors.

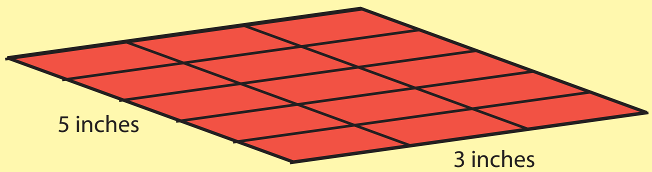
So, the volume of the building is found by determining the area of one floor and multiplying it by the number of layers in the building (or the height of the building).

Example 7

Find the volume of a rectangular prism with base measurements of 5 inches by 3 inches and a height of 7 inches.

Solution

Picture the base area:

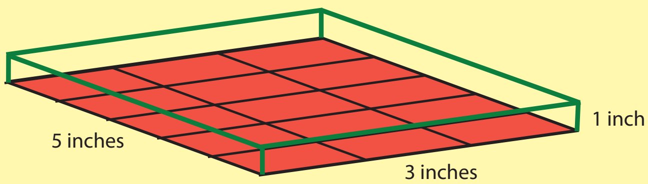


The area covered is A = length × width

$= 5\text{ in} \times 3\text{ in}$

$= 15\text{ in}^2$

Now, picture that base area with 1-inch thickness.

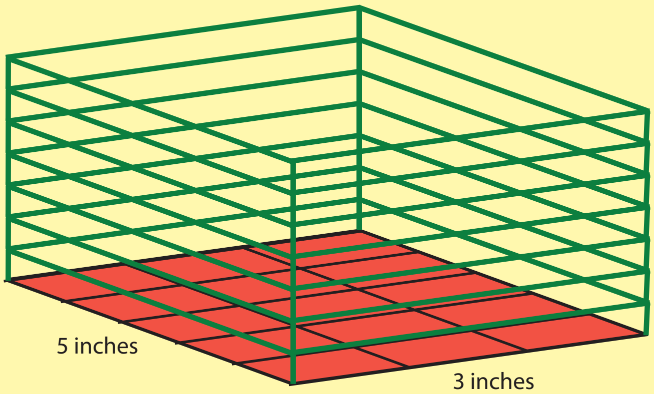


Volume of one layer = area of base × 1 inch thickness

$= 15\text{ in} \times 1\text{ in}$

$= 15\text{ in}^3$

Then, picture that there are 7 layers that are 1 inch thick.



Therefore, the volume of the whole 3-D object is

V = area of base × height

$= 15 \text{ in}^2 \times 7 \text{ in}$

$= 105 \text{ in}^3$

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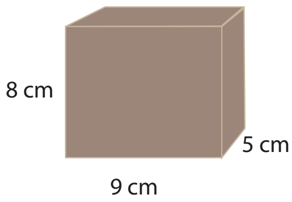
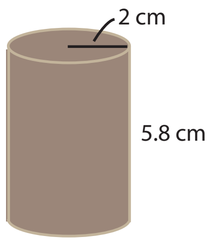
Digging Deeper
Practice Questions

1. Find the volume of the following 3-D objects.

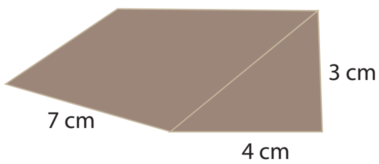
a.



b.



c.



2. A brick is 9 cm wide, 20 cm long, and 7.5 cm tall. What is the volume in cubic **inches** of a stack of 25 bricks? Remember that 1 inch = 2.54 cm. Round to the nearest inch cubed.

3. Cameron and Zach buy ice cream in two containers. Cameron's is in a box in the shape of a rectangular prism with dimensions 5 cm x 4 cm x 15 cm and Zach's is in a plastic cylinder with a base diameter of 7 cm and height 10 cm.

   a. Which container holds more ice cream - Cameron's or Zach's?

   b. How much larger is the container? (Show all your work.)

4. The edges of a rectangular prism are 5.4 cm, 7.2 cm, and 4.3 cm. Find the length of the edge of a cube with the same volume.

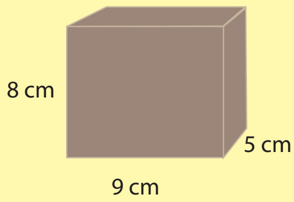
5. Match the correct unit of measure with the name.

_____	a. linear	1. area of a triangle
_____	b. acre	2. m^3
_____	c. capacity	3. yards
_____	d. volume	4. ml

Practice Solutions

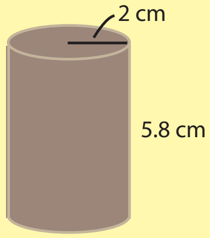
1. Find the volume of the following 3-D objects.

a.



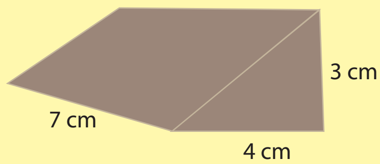
V = Area of base × height
= L × W × H
= 9 cm × 5 cm × 8 cm
= 360 cm^3

b.



V = Area of base × height
= πr^2 × height
= 3.14 × (2 cm)^2 × 5.8 cm
= 3.14 × 4 cm^2 × 5.8 cm
= 72.848 cm^3

c.



V = Area of base × height of prism
= Area of triangle × height of prism
(Note the object is not sitting on its base.)
= $(\frac{2}{3}$ × base × height) × height of prism

= $(\frac{2}{3}$ × 4 cm × 3 cm) × 7 cm

= 42 cm^3

2. A brick is 9 cm wide, 20 cm long, and 7.5 cm tall. What is the volume in cubic **inches** of a stack of 25 bricks? Remember that 1 inch = 2.54 cm. Round to the nearest inch cubed.

Step 1: Convert the centimetre dimensions to inches.

2.54 cm = 1 inch

Width

$$\frac{?\ \text{inch}}{9\ \text{cm}} = \frac{1\ \text{inch}}{2.54\ \text{cm}}$$

$$?\ \text{inch} = \frac{9 \times 1}{2.54}$$

$$= 3.54$$

The brick is 3.54 inches wide.

Length

$$\frac{?\ \text{inch}}{20\ \text{cm}} = \frac{1\ \text{inch}}{2.54\ \text{cm}}$$

$$?\ \text{inch} = \frac{20 \times 1}{2.54}$$

$$= 7.87\ \text{inches}$$

The brick is 7.87 inches long.

Height

$$\frac{?\ \text{inch}}{7.5\ \text{cm}} = \frac{1\ \text{inch}}{2.54\ \text{cm}}$$

$$?\ \text{inch} = \frac{7.5 \times 1}{2.54}$$

$$= 2.95\ \text{inches}$$

The brick is 2.95 inches tall.

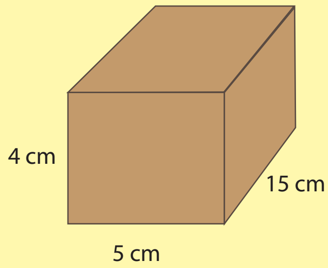
Step 2:

V = Area of base × height
= L × W × H
= 7.87 in × 3.54 in × 2.95 in
= 82.19 in^3
The volume of one brick is 82.19 in^3.

Volume of 25 bricks
= 1350 cm^3 × 25
= 33 750 cm^3

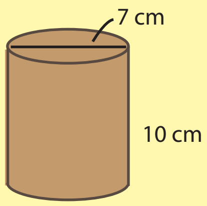
3. Cameron and Zach buy ice cream in two containers. Cameron's is in a box in the shape of a rectangular prism with dimensions 5 cm x 4 cm x 15 cm and Zach's is in a plastic cylinder with a base diameter of 7 cm and height 10 cm.

   a. Which container holds more ice cream - Cameron's or Zach's?



Cameron's container

$\mathbf{V = L \times W \times H}$
$\mathbf{= 5\ cm \times 4\ cm \times 15\ cm}$
$\mathbf{= 300\ cm^3}$



Zach's container

V = Area of base × height
$\mathbf{= \pi r^2 \times height}$
$\mathbf{r = \frac{2}{3}\ diameter}$
$\mathbf{= \frac{2}{3} \times 7\ cm}$
$\mathbf{= 3.5\ cm}$
$\mathbf{= 3.14(3.5)^2 \times 10}$
$\mathbf{= 384.65\ cm^3}$

   b. How much larger is the container? (Show all your work.)

Vz - Vc = amount of larger volume
$\mathbf{= 384.65\ cm^3 - 300\ cm^3}$
$\mathbf{= 84.65\ cm^3}$

Zach's container has the larger volume.

Zach has 84.65 cm^3 more ice cream than Cameron.

4. The edges of a rectangular prism are 5.4 cm, 7.2 cm, and 4.3 cm. Find the length of the edge of a cube with the same volume.

Step 1: Find the volume of the rectangular prism.
$V = L \times W \times H$
$= 5.4\text{ cm} \times 7.2\text{ cm} \times 4.3\text{ cm}$
$= 167.184\text{ cm}^3$

Step 2: Use the volume of the rectangular prism to find the length of one side of the cube.
$V_{cube} = 167.184\text{ cm}^3$ (the same volume as the prism)

Length of sides in a cube are equal so

$V_{cube} = \text{side} \times \text{side} \times \text{side}$

$= (\text{side})^3$

$167.184 = (\text{side})^3$

Find the cube root of each side of the equal sign.

$\sqrt{167.184} = \sqrt[3]{\text{side}}$

5.51 cm = side

The length of one side of the cube is 5.51 cm.

5. Match the correct unit of measure with the name.

3	a. linear	1. area of a triangle
1	b. acre	2. m^3
4	c. capacity	3. yards
2	d. volume	4. ml

Total
55

Digging Deeper Assignment

Now it's time to show your stuff!
Put lots of details into your work.

5

1. For each situation, name an appropriate unit of measure.

   a. Measuring the size of a suitcase to be used as a carry-on during an airplane trip

   ____

   b. Deciding how much liquid fits into a fuel tank of a tractor

   ____

   c. Calculating the area of a baseball diamond outfield

   ____

   d. Determining the amount of material that a game box requires

   ____

   e. Using three dimensions of a box to determine how much stuff would go into the box

   ____

10

2. Rebecca has 80 m of fencing. If she uses all the fencing, the perimeter of her enclosure **will always be 80 m**.

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Her enclosure will always be rectangular in shape. Rebecca is not sure what dimensions she should use to make the rectangular enclosure.

Complete the chart correctly by first adjusting the width measurement so that her enclosure will **always** use 80 m of fencing. Then, calculate each perimeter and area.

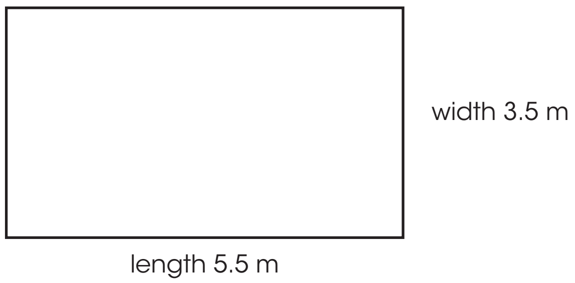
Length (m)	Width (m)	Perimeter (m) $P = 2L + 2W$	Area (m^2) $A = L \times W$
30		80	
25		80	
20		80	
15		80	
10		80	

2

3. Example 6 showed that when Lindsay adjusted the dimensions of the greenhouse from 8 m x 10 m to 9 m x 9 m, the perimeter did not change but the area did. The area became larger. Why? (Use the results from the table in Question 2 to help explain.)?

Use the following information to answer question 4.

A rectangle has a length of 5.5 m and width of 3.5 m.



2 4. a. Find the perimeter and area of the rectangle.

1 b. Double the width of the rectangle and determine the perimeter of the new rectangle.

2 c. What happened to the perimeter of the rectangle when the width was doubled?

2

d. Make a prediction. What happens to the perimeter of the rectangle if both the length and the width are doubled?

1

e. The original rectangle has a width of 3.5 m and length of 5.5 m. Double only the width and determine the area of the new rectangle.

2

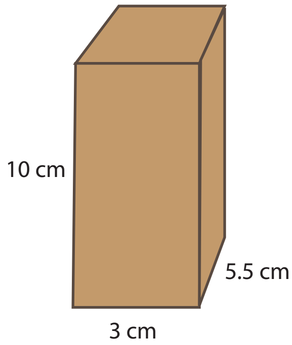
f. Make a prediction. What happens to the area if both the length and the width are doubled?

4

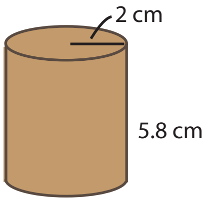
g. If the length was tripled, and the width was doubled what would happen to the area? Show an example to support your answer.

5. Find the volumes. Use the formulas given.

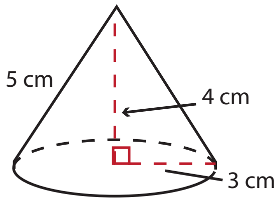
(2) a. Volume = area of the base × height



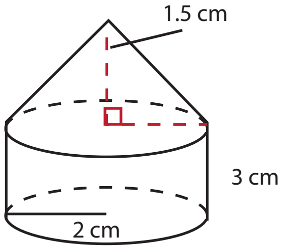
(2) b. Volume of a cylinder = area of the base × height



(2) c. Volume of a cone = $\left(\frac{1}{3}\right) \times \pi \times \text{radius}^2 \times \text{height}$

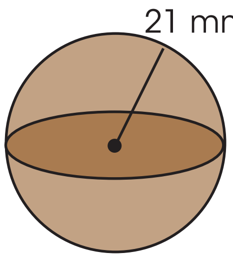


(4) d. Volume of a cone + volume of a cylinder



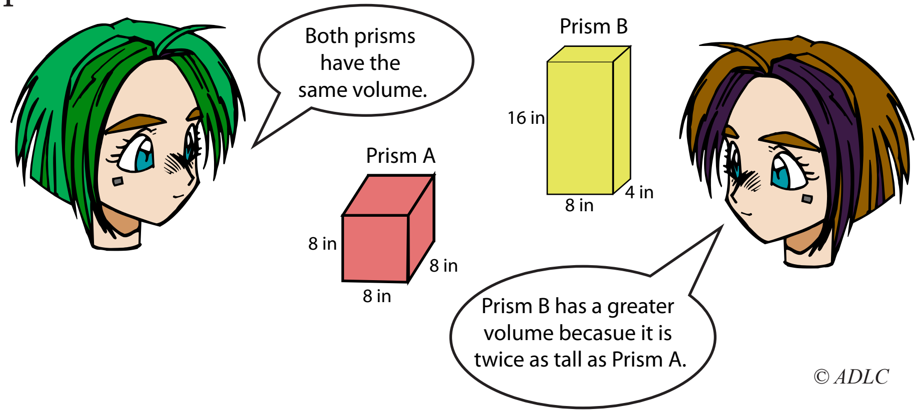
2

e. Volume of a sphere = $(\frac{4}{3}) \times \pi \times \text{radius}^3$



4

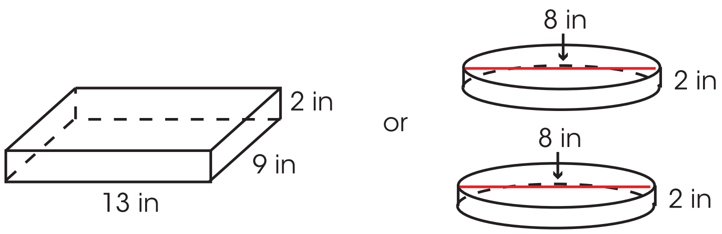
6. Nina and Alyssa are comparing the volumes of the two prisms shown.



Who is correct? Explain.

4

7. Which will hold more cake batter, the rectangular pan or the two round pans? Explain.



4

8. The three sides of a rectangular solid are 36 cm, 75 cm, and 80 cm respectively. Find the side length of a cube which will be the same volume.

Lesson Summary

In this lesson you did so many things!

You learned about 3-D objects and surface area.

You even used unit rates and measurement skills again.

You explored how changing dimensions of objects could change the perimeter and area.

You determined solutions to problems and could explain your answers.

Also, you could determine if a conclusion was correct.

Be sure to complete the checklist on the next page.

After receiving your marked lesson, you can write the Unit 5 Quiz. Then, proceed to Unit 6 Lesson A.

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How Did It Go?

Earn coins by filling in the chart below. After your teacher has looked over your checklist and talked with you about the unit, you will be able to write the unit quiz. Please call or e-mail your teacher to talk about your checklist. Your instructor will send your unit quiz with this marked lesson.

Topics	Good to Go!	Sounds familiar. I might know how.	A little fuzzy. I need to look it up.	Really not sure. I had trouble with this before.	Going to need help. I can't seem to do these.	Never heard of it before.
Perimeter						
Area						
Formulas						
Volume						
Names of 3-D objects						
Surface area of 3-D objects						
Using area to find a dimension						
Using volume to find a dimension						
Changing dimensions						
Predictions of results of changes						
Problem–solving						
Problem–solving with measurement conversions						

ALBERTA DISTANCE LEARNING CENTRE
MAT1793
Math 10-3
Unit 5: 2-D and 3-D Measurements
Lesson C: Surface Area of 3-D Shapes

Student's Questions and Comments

FOR STUDENT USE ONLY

(If label is missing or incorrect)

File Number: ____________________

FOR ADLC USE ONLY

Assigned to ____________________

Marked by ____________________

Mark: ____________________ %

Date Received: ____________________

Name

Address

City/Town Province Postal Code

Apply Assignment Label Here

Please use the correct preprinted label for this course and Assignment Booklet.

Summary

	Possible Marks	Your Marks	%
How Does It Work? Assignment	44		
Digging Deeper Assignment	55		

Teacher's Comments

____

Teacher's Signature

June 2010