Rational Expressions & Equations/

Reciprocals



Math 20 – Pre-Calculus

Chapter 6

Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Class:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |
| --- | --- |
| **Algebra and Number** | **General Outcome:**  Develop algebraic reasoning and number sense. |
| **Specific Outcomes** | **Achievement Indicators**:  *The following set of indicators may be used to determine whether students have met the corresponding specific outcome* |
| Determine equivalent forms of rational expressions  (limited to numerators and denominators that are  monomials, binomials or trinomials). | 4.1 Compare the strategies for writing equivalent forms of rational expressions to the strategies  for writing equivalent forms of rational numbers.  4.2 Explain why a given value is non-permissible for a given rational expression.  4.3 Determine the non-permissible values for a rational expression.  4.4 Determine a rational expression that is equivalent to a given rational expression by  multiplying the numerator and denominator by the same factor (limited to a monomial or a  binomial), and state the non-permissible values of the equivalent rational expression.  4.5 Simplify a rational expression.  4.6 Explain why the non-permissible values of a given rational expression and its simplified form  are the same.  4.7 Identify and correct errors in a simplification of a rational expression, and explain the   1. reasoning. |
| Perform operations on rational expressions (limited to  numerators and denominators that are monomials,  binomials or trinomials). | 5.1 Compare the strategies for performing a given operation on rational expressions to the  strategies for performing the same operation on rational numbers.  5.2 Determine the non-permissible values when performing operations on rational expressions.  5.3 Determine, in simplified form, the sum or difference of rational expressions with the same  denominator.  5.4 Determine, in simplified form, the sum or difference of rational expressions in which the  denominators are not the same and which may or may not contain common factors.  5.5 Determine, in simplified form, the product or quotient of rational expressions.  5.6 Simplify an expression that involves two or more operations on rational expressions. |
| Solve problems that involve rational equations  (limited to numerators and denominators that are  monomials, binomials or trinomials). | *(It is intended that the rational equations be those that can be simplified to linear and*  *quadratic equations.)*  6.1 Determine the non-permissible values for the variable in a rational equation.  6.2 Determine the solution to a rational equation algebraically, and explain the process used to  solve the equation.  6.3 Explain why a value obtained in solving a rational equation may not be a solution of the  equation.  6.4 Solve problems by modelling a situation using a rational equation. |

**Big Ideas:**

*By the end of this unit you should be able to . . .*

* **FACTOR** rational expressions in order to simplify them

* Determine for which value(s) of *x* an expression is undefined (will usually involve factoring)
* Add and subtract rational expressions which usually involves finding common denominators and making equivalent expressions
* Multiply and divide rational expressions which usually involves factoring, simplifying and then multiplying
* Solve rational EQUATIONS
* Create a system of rational equations from a word problem and solve

6.1 Rational Expressions

Review of Fraction Basics

Fractions and “0”

Simplifying Rational Expressions / Non-Permissible Values

Rational Expression –

Non-permissible values –

Example: For each rational expression, determine all the non-permissible values.

a)  b)  c) 

Rational expressions can be simplified by:

Example: Simplify each rational expression. State the value(s) of any restrictions.

a)  b)  c) 

e)  f)  g) 

**Homework**

**Pg. 317 (#1-4, 6-8)**

6.2 Multiplying and Dividing Rational Expressions

Review of Steps to Multiplying or Dividing Fractions

Simplify each of the following.

(a)  (b)  (c) 

(d)  (e)  (f) 

(g) 

To Multiply Rational Expressions

Example: Multiply and write your solution in simplest form. Identify all non-permissible           values.

a)  b) 

c)  d) 

To Divide Rational Expressions

The Non-Permissible Values in Division of Rational Expressions

For example, in:



the non-permissible values are 

Example: Divide and write your solution in simplest form. Identify all non-permissible           values.

a)  b)  c) 

Example: Simplify the following expression and identify any non-permissible values



**Homework**

**pg. 327 (#1,2, 4, 7,8,12)**

6.3 Adding and Subtracting Rational Expressions

Review of Steps to Adding or Subtracting Fractions

Simplify. Express all answers in simplest form.

(a)  (b)  (c) 

(d)  (e)  (f) 

To Add or Subtract Rational Expressions

Case 1: Denominators are the same Case 2: Denominators are different

Example: Determine each sum or difference. Identify all non-permissible values.

a)  b) 

Example: **Identify the lowest common denominator** for each group of rational                 expressions.

a)  b)  c) 

Example: Determine each sum or difference. Identify all non-permissible values..

a)  b) 

c)  d) 

**Homework**

**pg. 336 (#1,3-10)**

6.4 Rational Equations

Rational Equation –

Steps to Solving a Rational Equation:

Example: Solve each of the following equations. Identify all non-permissible values.

(a)  (b) 

(c)  (d) 

Example: Colin rows his boat 24 km downstream and back to where he began. When the                 average speed of the current is 2 km/h, Colin can complete the journey in 9 hrs.                 What is Colin’s average rowing speed in still water?

|  |  |  |
| --- | --- | --- |
|  | Downstream | Upstream |
| Distance (km) |  |  |
| Average speed (km/h) |  |  |
| Time (h) |  |  |

Example: Katie mows the lawn in 40 min. When Will and Katie work together, they can                 mow the lawn in 24 min. How long would it take Will to mow the lawn on his                 own?

|  |  |  |  |
| --- | --- | --- | --- |
| Time | Fraction of lawn mowed by Katie | Fraction of lawn moved by Will | Fraction of lawn mowed working together |
| 1 |  |  |  |
| 2 |  |  |  |
| 24 |  |  |  |

**Homework**

**pg. 348 (#1-4, 12-14)**

7.4 Reciprocal Functions

**INVESTIGATION:** For each given value of *f(x),* determine the corresponding value

of  :

|  |  |
| --- | --- |
|  |  |
| -10 000 |  |
| -10 |  |
| -1 |  |
| -0.0001 |  |
| 0 |  |
| 0.0001 |  |
| 1 |  |
| 10 |  |
| 10000 |  |

**Observations** . . . .

**IN GENERAL:**

For any function, , the graph of the reciprocal function, , can be found using the following general rules:

* Where the value of the **original function is zero**, the value of the reciprocal function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, and a vertical asymptote exists.
* Where the value of the **original function is a very small positive** number, the value of the reciprocal function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* Where the value of the **original function is a very small negative** number, the value of the reciprocal function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* Where the value of the **original function is a very large positive** number, the value of the reciprocal function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* Where the value of the **original function is a very large negative** number, the value of the reciprocal function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* Where the value of the **original function is 1**, the value of the reciprocal function is \_\_\_\_\_\_\_.
* Where the value of the **original function is –1**, the value of the reciprocal function is \_\_\_\_\_\_\_.
* These points, where the original function and reciprocal function have the same unchanged value, are called

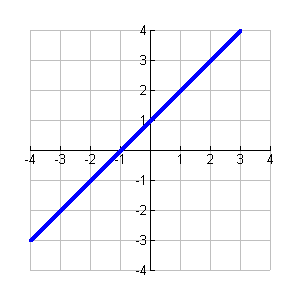
Reciprocal function –

Vertical Asymptote –

Horizontal Asymptote –

Invariant Point(s) –

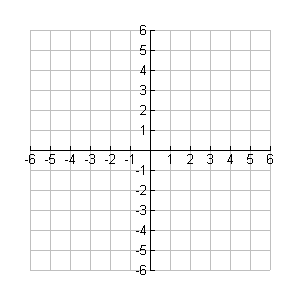
Example: The graph of  is shown below. Use this graph to draw the graph of                 .



Example: A function is defined by the expression 

(a) Find the equation of its reciprocal function, *.*

(b) Sketch the graph of  Then use the graph of y = f(x) to sketch the graph         of  on the same grid.

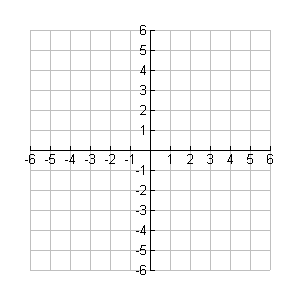


1. For the reciprocal function, , determine:

i. Domain: iii. Vertical Asymptote:

ii. Range: iv. Horizontal Asymptote:

Example: A function is defined by the equation  Use the graph of this function to sketch the graph of  then analyze this reciprocal function by stating the domain, range, zeros, asymptotes, and the coordinates of any max/min points.



1. Domain:
2. Range:
3. Zeros
4. max/mins:
5. Vertical Asymptote:
6. Horizontal Asymptote:

**Homework:**

**pg. 403 (#1-9)**