Sequences & Series



Math 20 – Pre-Calculus

Chapter 1

Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Class:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |
| --- | --- |
| **Relations and Functions** | **General Outcome:**  Develop algebraic and graphical reasoning through the study of relations |
| **Specific Outcomes** | **Achievement Indicators**:  *The following set of indicators may be used to determine whether students have met the corresponding specific outcome* |
| Analyze arithmetic sequences and series to solve problems | * 1. Identify the assumption(s) made when defining an arithmetic sequence or series   2. Provide and justify an example of an arithmetic sequence   3. Derive a rule for determining the general term of an arithmetic sequence   4. Describe the relationship between arithmetic sequences and linear functions   5. Determine  in a problem that involves an arithmetic sequence   6. Derive a rule for determining the sum of  terms of an arithmetic series   7. Determine  in a problem that involves an arithmetic series   8. Solve a problem that involves an arithmetic sequence or series |
| Analyze geometric sequences and series to solve problems | * 1. Identify the assumption(s) made when defining a geometric sequence or series   2. Provide and justify an example of an geometric sequence   3. Derive a rule for determining the general term of a geometric sequence   4. Determine  in a problem that involves a geometric sequence   5. Derive a rule for determining the sum of  terms of a geometric series   6. Determine  in a problem that involves a geometric series   7. Generalize, using inductive reasoning, a rule for determining the sum of an infinite geometric series   8. Explain why a geometric series is convergent or divergent   9. Solve a problem that involves a geometric sequence or series |

**Big Ideas:**

*Students will understand …*

* Different types of sequences and series exist.
* We can use mathematics to model the pattern of the sequence or series.

By the end of the unit students should:

* Use concrete strategies to determine the pattern.
* Have an idea of where to start in breaking down the sequence/series.
* Recognize and apply patterns to familiar and unfamiliar situations (predictions).
* Know that a pattern exists.
* See patterns in life, application of patterns beyond geometric/arithmetic sequences and series.
* Make predictions based on an observed pattern.
* Determine the pattern and identify relevant elements of geometric/arithmetic sequences and series.
* Investigate or discover patterns and extend them.

1.1 Arithmetic Sequences

**Definition: *Sequence:***

* A set of numbers arranged in an order with a first term, , a second term, , a third term, , …
* A sequence is *finite* if . . . .

Example:

* A sequence is *infinite* if . . . .

Example:

**Example #1**

Complete the following sequences of numbers:

1. 5, 7, 9, \_\_\_\_\_, \_\_\_\_\_, 15 (b) 2, 6, \_\_\_\_\_, \_\_\_\_\_, 162

(c) 5, -1, -7, \_\_\_\_\_, \_\_\_\_\_, -25 (d) 80, -40, \_\_\_\_\_, \_\_\_\_\_, 5

(e) 1, 4, 9, \_\_\_\_\_, \_\_\_\_\_, 36 (f) 1, 1, 2, 3, \_\_\_\_\_, \_\_\_\_\_, 13

**Sequences**

A sequence is a set of numbers with some order.

Determine the next 3 terms in each of the sequences below.

3, 9, 15, 21, .....

5, 10, 20, 40, ....

1, 1, 2, 3, 5, 8, 13 .....

### **Arithmetic Sequence -**

In an arithmetic sequence, the number obtained by subtracting any term from the next term is a constant. This constant is called the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

**Example #2**

Determine the *common difference* for the following sequences:

1. 35, 39, 43, 47, …. *d =*

(b) 12, 2, -8, -18, … *d =*

(c)  *d =*

(d)  *d =*

**Example #3**

For the following arithmetic sequence determine .

-19, -12, -5, …

An arithmetic sequence has a first term of , and a common difference of *d*. Write out expressions for  and .













**In General:**

For an arithmetic sequence with a first term of and a common difference of *d*, the general term can be found using the formula:

**Example #4**

In the arithmetic sequence 26, 29, 32, …

1. Determine the values of  and *d*.
2. Determine a simplified expression for the general term.
3. Use your expression in (b) to determine .
4. Which term has a value of 503?

**Example #5**

(a) Determine  and  of the sequence 12, 2, -8, …

(b) How many terms are in the finite arithmetic sequence defined by

–3, 2, 7, …, 152?

Applications of Arithmetic Sequences

1. A pile of blocks is arranged in rows. The number of blocks in each row forms the arithmetic sequence 8, 14, 20, 26, . . .
2. One row contains 92 blocks. Which row is it?
3. How many blocks will be in the 40th row?
4. Hui works for a horticultural company. She plants flowers in rows. The number o flowers in each row forms an arithmetic sequence. There are 58 flowers in the eighth row and 107 flowers in the fifteenth row.
5. Determine, algebraically, the change in the number of flowers between each row.
6. Determine, using the general term for this sequence, the number of flowers in the first row.
7. Find the twelfth term for the arithmetic sequence 
8. A plumber charges $50 for making a house call. In addition to that, he/she charges $30 per hour or any portion of an hour.
9. Generate the possible charges, excluding parts, for the first 5 hours of time.
10. Sketch a graph of this sequence. Discuss the relationship between arithmetic sequences and linear functions using this example.

Homework

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1.2 Arithmetic Series

**Arithmetic Series -**

**Investigate:**

Add the following series.



**In General:**

There are three **formulae** we use to solve arithmetic series problems:



**Example #1**

Determine the sum of the first 50 terms of the arithmetic series

3 + 4.5 + 6 + …

**Example #2**

Determine  of the series 12 + 2 - 8…

**Example #3**

Determine the sum of the following series

15 + 11 + 7 + … + (-37)

**Example #4**

Determine the sum of the series –10 –13 –16 … -40

**Example #5**

In an arithmetic series, *t1* = 19 and . Find .

**Example #6**

Determine the arithmetic series which has these partial sums: .

Homework

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1.3 Geometric Sequences

### **Geometric Sequence -**

In an geometric sequence, the number obtained by dividing any term by the previous term is called the \_\_\_\_\_\_\_\_\_\_\_\_\_ .

**Example #1**

Determine the *common ratio* for the following sequences:

1. 3, 15, 75, ... *r =*

(b) 512, 256, 128, 64, ... *r =*

(c) 2, -6, 18, -54, ... *r =*

(d)  *r =*

**Example #2**

Is the sequence geometric? Explain.

a. 

b. 

1. 

**Example #3**

A geometric sequence has a first term of , and a common ratio of *r.*  Write out expressions for  and  for the sequence 













**In General:**

For an geometric sequence with a first term of  and a common ratio of *r*, the general term can be found using the formula:

**Example #4**

Find the general term **and** the 7th term of

a. 2, -6, 18, .... b. 

**Example #5**

**How many** terms are in the sequence 4, 20, 100, … , 7 812 500?

**Example #6**

In a geometric sequence,  and . Determine .

**Example #7**

If , , and  are consecutive terms in a geometric sequence, determine the exact value of each term.

**Example #8**

A ball is dropped off the roof of Ainlay, which has a height of 4.0 m. After each bounce, the ball rises to 60% of its previous height. Determine the height of the ball after the 5th bounce.

**Example #9**

A farmer in Saskatchewan wants to estimate the value of a new combine after several years of use. A new combine is worth $400 000 and it depreciates in value by 10% each year.

Determine the value of the combine at the start of the 6th year.

**Example #10**

The population Whoville increases steadily by 5% each year. The population at the beginning of 2012 was 800. Determine the population of Whoville at the beginning of 2020.

Homework

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**1.4 Geometric Series**

**Note**

A series is a sequence, with its terms added.

**Example #1**

1, 7, 49, ... is a geometric sequence

1 + 7 + 49 + ... is a geometric series

**Note**

The sum of a geometric series is

 *used when . . ..*

 *used when . . . .*

**Example #2**

Find the sum of the first 12 terms of 

**Example #3**

Find the sum of 

**Example #4**

How many terms in 5 + 15 + 45 + … are required to yeild a sum of

35 872 265?

**Example #5**

In a geometric series,  and . What is ?

**Example #5**

A ball is dropped from a height 100 m and bounces to of its previous height on each bounce. What is the total vertical distance the ball has travelled when it hits the ground for the 8th time?

Homework

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**1.5 Infinite Geometric Series**

*Infinite* ***Sequence***: a list of numbers that goes to infinity.

t1, t2, t3, …, tn where tn = 

Note: sequences can be arithmetic, geometric or other

**Example**

For the sequence defined by 

1. list the first six terms
2. What is t100?
3. What is t1000?
4. What do you notice about the terms as they get larger?

*Infinite* ***Series***

**Example:** Determine the following sums for the given series:

48+ 16 +

1. 
2. 
3. 
4. 

Conclusion

* What do you notice about the difference between the sums?
* What do you notice about the common ratio, *r,* of this series?

**Example:** Determine the following sums for the given series:

1. +12 + 48 + 192 + 768+ ….

1. 
2. 
3. 
4. 

Conclusion

* What do you notice about the difference between the sums?
* What do you notice about the common ratio, *r,* of this series?*Infinite Series:* There are two types of infinite series

1. Convergent
2. Divergent

**Examples**

(a) 16 – 12 + 9  +  Determine the sum to infinity.

(b)  Determine the sum to infinity.

(c)  Determine the sum to infinity.

1. Express as an infinite geometric series. Determine the sum of the series.

Homework

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