

PURE MATH 10

SAPPHIRE HEIST

**ALBERTA REGIONAL PROFESSIONAL
DEVELOPMENT CONSORTIA**

Developed by AAC... everyday assessment tools for teachers

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SAPPHIRE HEIST

OVERVIEW

This task requires general triangle knowledge, trigonometric ratios, the sine law and the cosine law as well as problem solving and visualization skills. The particular focus of this assessment is the ability to determine how to find unknown values using the information given and the correct calculation of such values.

RECOMMENDED GRADE LEVEL – 10

TIME REQUIRED

- 120 minutes

GENERAL OUTCOMES

SPECIFIC OUTCOMES

No.	Description	No.	Description
	Solve problems involving triangles, including those found in 3-D and 2-D applications.	C3-5	Apply the sine and cosine laws, excluding the ambiguous case, to solve problems.
	Demonstrate an understanding of and proficiency with calculations.	C1-5	Communicate a set of instructions used to solve an arithmetic problem.

MATERIALS AND RESOURCES NEEDED

- task sheets – one per group
- rulers
- pencils and erasers
- paper
- calculators

KEYWORDS

- trigonometric ratios
- Sine Law
- Cosine Law
- angles of a triangle

INFORMATION FOR THE TEACHER

Ensure that the students have had prior experience with the trigonometric ratios, the sine and cosine laws and opposite angles. Written explanations with pictorial presentations are required to achieve the acceptable standard. Journal entries should be imaginative and reasons for a chosen path do not have to be mathematical.

DESCRIPTION OF TASK

You are a Canadian mining security team hired to help the Mabu-mabu hill tribe protect the rumoured prize sapphire of Thailand from jewel thieves. This sapphire has just been discovered in the Mabu-mabu hill tribe village in the jungle. Your trek begins at the Golden Triangle and you will meet many obstacles along your journey. The attached map has been provided to assist your team. You must determine the path that you wish to take to reach the Mabu-mabu hill tribe.

You must calculate the total distance of your trek and determine the total traveling time you will need, taking into consideration the different speeds of the modes of transportation. Calculate your angles to the nearest degree, your distances to the nearest hundredth of a kilometre and the time to the nearest hundredth of an hour. This information should be organized in a way that shows your calculations and is easy to follow. Clearly indicate which section of the trek each calculation refers to and label your map with appropriate information. Finally, write a journal entry for each section of the trek that describes your trek and the reasoning for the choices of paths so that you can find your way back from this jungle location.

TASK EXTENSIONS OR ADAPTIONS

1. Discussion as to why straight line distances are not realistic. How could this be accounted for?
2. Incorporate a look for the fastest trek.
3. Incorporate an exact time calculation: sleep, eating, travel, etc...
4. Time could be calculated in hours and minutes.

STUDENT PRODUCTS/PERFORMANCES

Task 1

- demonstration of ability to solve problems involving triangles.

Task 2

- demonstration of ability to apply the sine and cosine laws to solve problems.

Task 3

- communication of a set of instructions used to solve an arithmetic problem.

CRITERIA FOR ASSESSING STUDENT PRODUCTS/PERFORMANCES

Task 1

- recognizes which information is given and can determine other information
- recognizes which trigonometric ratio should be used
- provides accurate calculations
- executes algorithms completely and correctly
- uses appropriate mathematical notation

Task 2

- recognizes when sine and cosine laws should be used
- provides accurate calculations

Task 3

- reasoning is persuasive
- ideas are clearly organized
- explanation is coherent, justified and insightful

THINKING SKILLS

Deduction – What specific rules are operating?
What must happen because of these rules?

Error Analysis – Are there errors in
reasoning or in a process that can be described?

Abstracting – What is the relationship that
exists between the triangles? What is the
abstract pattern that lies at the heart of the
relationship?

Analyzing Perspectives – What are the
different perspectives on an issue

THINKING PROCESSES

Decision making – What important decisions
should be made?

Problem solving – What obstacles need to be
overcome?

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JOURNAL ENTRY RUBRIC

Level	Description
5 Excellent	<ul style="list-style-type: none"> • entry is persuasive and unusually well presented. • explanation and interpretation is accurate, coherent, justified and insightful. It addresses what path was taken and why.
4 Proficient	<ul style="list-style-type: none"> • explanation and interpretation may contain some minor gaps but it addresses what path was taken and why.
3 Adequate	<ul style="list-style-type: none"> • shows an adequate explanation of their journey. • addresses what path was taken but not why it was taken.
2 Developing	<ul style="list-style-type: none"> • description of trek is limited. • important ideas may be misunderstood or misapplied. The concepts and methods used are simplistic.
1 Limited	<ul style="list-style-type: none"> • no apparent effort made to include descriptions • reflects an inappropriate strategy for solving the problem.

MATHEMATICAL OPERATIONS RUBRIC

Level	Description
5 Excellent	<ul style="list-style-type: none"> • appropriate choice of trigonometric ratios and laws for a complete solution. • uses appropriate mathematical notation. • no mathematical errors.
4 Proficient	<ul style="list-style-type: none"> • appropriate choice of trigonometric ratios and laws in completing all of the necessary calculations. • uses appropriate mathematical notation. • minimal minor calculation errors appear.
3 Adequate	<ul style="list-style-type: none"> • the majority of calculations are complete and correct. • minor errors in mathematical notation occur. • reflects an appropriate strategy.
2 Developing	<ul style="list-style-type: none"> • contains major calculation errors or only a few calculations are completed. • appears to reflect an appropriate strategy, but application of strategy is unclear. • mathematical notation is improperly used several times.
1 Limited	<ul style="list-style-type: none"> • calculations are inaccurate. • proper mathematical notation is not used.

QUALITY OF COMMUNICATION RUBRIC

Level	Description
5 Excellent	<ul style="list-style-type: none"> • communication is clear and easy to follow. • an effective arrangement of ideas and details contributes to a fluent, controlled and shaped solution • solution concludes logically.
4 Proficient	<ul style="list-style-type: none"> • arrangement of ideas and details contributes to a competent and controlled solution. • solution concludes logically.
3 Adequate	<ul style="list-style-type: none"> • ideas and details are arranged in a forthright manner, coherence may falter and the communication is sometimes difficult to interpret.
2 Developing	<ul style="list-style-type: none"> • an understandable but ineffectual arrangement of ideas and details. • ideas are not clearly concluded. • gives minimal evidence of a solution process.
1 Limited	<ul style="list-style-type: none"> • a haphazard arrangement of ideas and details provides little direction for a solution. • gives no evidence of a solution process.

SAPPHIRE HEIST

You are a Canadian mining security team hired to help the Mabu-mabu hill tribe protect the rumored prize sapphire of Thailand. This sapphire has just been discovered in the Mabu-mabu hill tribe village in the jungle. Your trek begins at the Golden Triangle and you will meet many obstacles along your journey. You must determine the path that you wish to take to reach the Mabu-mabu hill tribe. This package includes a map of the Mabu-mabu region and scoring rubrics.

Information about the area

- the Mekong River flows from west to east on your map.
- The rice paddies and the dense jungle can only be hiked as the trail is extremely rigorous.
- The Pitviper snakes in the snaked valley are extremely venomous and cannot be messed with.
- You must visit the water hole to replenish your water supply.
- The thick, slippery clay must be travelled on an animal's back because man only gets stuck.
- ALL DISTANCES SHOULD BE CALCULATED AS STRAIGHT LINE DISTANCES.

COMPLETE THE FOLLOWING THREE STEPS:

- Calculate the total distance of your trek, beginning at the Golden Triangle and ending at the Mabu-mabu hill tribe. There are six triangles that you will need to make calculations for (see map). Calculate angles to the nearest degree and distances to the nearest hundredth of a kilometre.

- Calculate the total travelling time that you will need to complete your trek. Take into consideration the different speeds of the modes of transportation. Calculate time to the nearest hundredth of an hour.

Average Speeds:

- Hiking: 3 km/h
- Paddle boat: 4 km/h
- Bamboo raft: 2 km/h
- Elephant Samya: 8 km/h
- Donkey Dayou: 6 km/h
- Horse Kali: 10 km/h

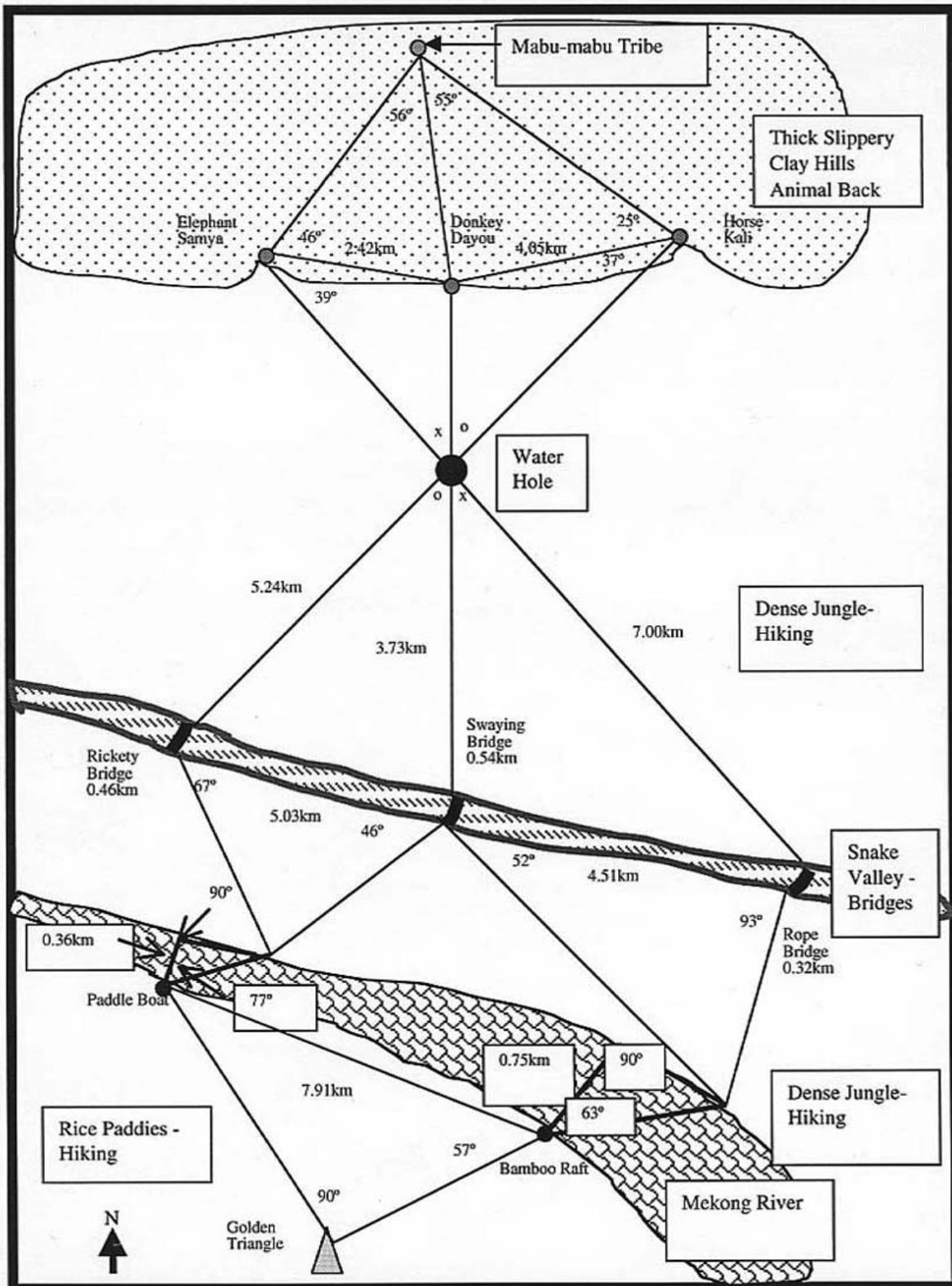
Your calculations of distance and time should be clearly and neatly organized; your work must be easy to follow with reference to the map. Be sure to indicate which section of the trek that each calculation refers to.

- Write a journal entry for each section of the trek that describes your trek and the reasoning for the choices of paths so that you can find your way back out of the jungle. Journal entries should be imaginative and reasons for a chosen path do not have to be mathematical.



HAVE FUN AND HOPE THAT YOU SURVIVE...

The Sapphire Heist



Note: The map is not drawn to scale.

STUDENT CHECKLIST

This checklist will give you an idea of how your project is coming along. Anything that “needs work” should be reviewed. You may do this checklist yourself or have some peers look at your project and review it. This must be completed before you hand in the project so that you can make improvements if needed.

ITEM DESCRIPTION	INCLUDED	NEEDS WORK
Journal		
• includes reasons for chosen paths.		
• is imaginative.		
• includes creative ideas.		
• includes a description of each section of the path.		
• is legible.		
Mathematical Operations		
• proper use of mathematical notation.		
• units of measurement are included.		
• calculations are correct.		
• appropriate laws and strategies are used.		
• distances are calculated to the nearest hundredth of a kilometer and referenced correctly.		
• angles are calculated to the nearest degree.		
• time is calculated to the nearest hundredth of an hour.		
Quality of Communication		
• each section of the trek is clearly identified (journal and calculations).		
• the ideas and calculations are easy to read and understand.		
• there is a logical conclusion to the problem.		
• ideas and details are arranged in a way that is easy to follow.		

SAPPHIRE HEIST ANSWER KEY

The Sapphire Heist	
①	<p>Rice Paddies</p> <p>Golden Δ to Paddle Boat</p> $\frac{\sin 57^\circ = x}{7.91}$ $x = 6.63 \text{ km}$ <p>time $\frac{6.63}{3} = 2.21 \text{ hours}$ 2 h 13 min.</p>
	<p>Golden Δ to Bamboo Raft</p> $\frac{\cos 57^\circ = x}{7.91}$ $x = 4.31 \text{ km}$ <p>time $\frac{4.31}{3} = 1.44 \text{ hours}$ 1 h 26 min.</p>
②	<p>Mekong River</p> <p>Bamboo Raft</p> $\frac{\cos 77^\circ = 0.36}{x}$ $x = 1.60 \text{ km}$ <p>time $= \frac{1.60}{2} = 0.80 \text{ h}$ 48 min.</p>
	<p>Paddle Boat</p> $\frac{\cos 63^\circ = 0.75}{x}$ $x = 1.65 \text{ km}$ <p>time $= \frac{1.65}{4} = 0.41 \text{ h}$ 25 min.</p>
③	<p>Dense Jungle</p> <p>Paddle Boat to Rickety Bridge</p> $\frac{\sin 67^\circ}{5.03} = \frac{\sin 46^\circ}{x}$ $x = 3.93 \text{ km}$ <p>time $= \frac{3.93}{3} = 1.31 \text{ h}$ 1 h 19 min.</p>
	<p>Bamboo Raft to Swaying Bridge</p> $\frac{\sin 35^\circ}{4.51} = \frac{\sin 93^\circ}{x}$ $x = 7.85 \text{ km}$ <p>time $= \frac{7.85}{3} = 2.62 \text{ h}$ 2 h 37 min.</p>
	<p>Paddle Boat to Swaying Bridge</p> $\frac{\sin 67^\circ}{5.03} = \frac{\sin 67^\circ}{x}$ $x = 5.03 \text{ km}$ <p>time $= \frac{5.03}{3} = 1.68 \text{ h}$ 1 h 41 min.</p>
	<p>Bamboo Raft to Rope Bridge</p> $\frac{\sin 35^\circ}{4.51} = \frac{\sin 52^\circ}{x}$ $x = 6.20 \text{ km}$ <p>time $= \frac{6.20}{3} = 2.07 \text{ h}$ 2 h 4 min.</p>

Time for

$$\text{Rickety Bridge} = \frac{0.46}{3} = 0.15 \text{ h} \\ 9 \text{ min}$$

$$\text{Swaying Bridge} = \frac{0.54}{3} = 0.18 \text{ h} \\ 11 \text{ min}$$

$$\text{Rope Bridge} = \frac{0.32}{3} = 0.11 \text{ h} \\ 6 \text{ min.}$$

Time for

$$\text{Rickety Bridge to Water Hole} = \frac{5.24}{3} = 1.75 \\ 1 \text{ h } 45$$

$$\text{Swaying Bridge to Water Hole} = \frac{3.73}{3} = 1.24 \\ 1 \text{ h } 14$$

$$\text{Rope Bridge to Water Hole} = \frac{7.00}{3} = 2.33 \\ 2 \text{ h } 20$$

Angles in triangles from Bridges to water hole

$$\cos \theta = \frac{5.03^2 - 5.24^2 - 3.73^2}{-2(5.24)(3.73)}$$

$$\theta = 66^\circ$$

$$\cos \theta = \frac{4.51^2 - 7^2 - 3.73^2}{-2(7)(3.73)}$$

$$\theta = 35^\circ$$

Water Hole to Animals

Water Hole to Elephant Samya

$$\frac{\sin 35^\circ}{2.42} = \frac{\sin 106^\circ}{x}$$

$$x = 4.06 \text{ km}$$

$$\text{Time } \frac{4.06}{3} = 1.35 \text{ h}$$

1h 21 min

Water Hole to Donkey Dayou

$$\frac{\sin 35^\circ}{2.42} = \frac{\sin 39^\circ}{x}$$

$$x = 2.66 \text{ km}$$

$$\text{Time } \frac{2.66}{3} = 0.89 \text{ h}$$

53 min

$$\text{or } \frac{\sin 66^\circ}{4.05} = \frac{\sin 37^\circ}{x}$$

$$x = 2.67 \text{ km}$$

$$\text{Time } \frac{2.67}{3} = 0.89 \text{ h}$$

53 min

Water Hole to Horsekali

$$\frac{\sin 66^\circ}{4.05} = \frac{\sin 77^\circ}{x}$$

$$x = 4.32 \text{ km}$$

$$\text{Time} = \frac{4.32}{3} = 1.44 \text{ h}$$

1h 26 min

Thick Slippery Clay Hills

Elephant Samya to Mabu-Mabu Tribe

$$\frac{\sin 56^\circ}{2.42} = \frac{\sin 78^\circ}{x}$$

$$x = 2.86 \text{ km}$$

$$\text{Time} = \frac{2.86}{8} = 0.4 \text{ h}$$

$$= 21 \text{ min}$$

Donkey Dayou to Mabu-Mabu Tribe

$$\frac{\sin 56^\circ}{2.42} = \frac{\sin 46^\circ}{x} \quad \text{or} \quad \frac{\sin 25^\circ}{x} = \frac{\sin 55^\circ}{4.05}$$

$$x = 2.10 \text{ km}$$

$$\text{Time} = \frac{2.10}{6} = 0.35 \text{ h}$$

$$= 21 \text{ min}$$

$$x = 2.09 \text{ km}$$

$$\text{Time} = \frac{2.09}{6} = 0.35 \text{ h}$$

$$= 21 \text{ min.}$$

Horse Kali to Mabu-mabu Tribe

$$\frac{\sin 55^\circ}{4.05} = \frac{\sin 100^\circ}{x}$$

$$x = 4.87 \text{ km}$$

$$\text{Time} = \frac{4.87}{10} = 0.5 \text{ h}$$

$$= 29 \text{ min.}$$

