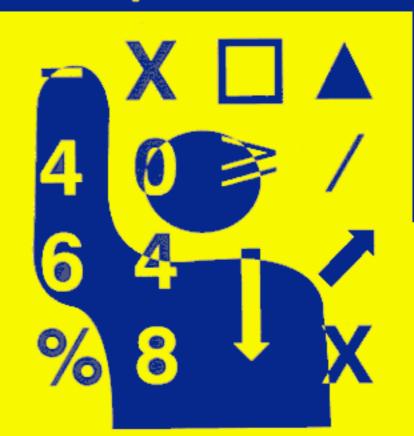


10B Probability

Help Booklet



Support for Primary Teachers in Mathematics

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ESSO

CIMT School of Education University of Exeter



Mathematics Enhancement Programme

Help Module 10

PROBABILITY

Part B

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PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

1. ALGEBRA 6. HANDLING DATA

2. DECIMALS 7. MENSURATION

8. EQUATIONS 8. NUMBERS IN CONTEXT

4. FRACTIONS 9. PERCENTAGES

5. GEOMETRY 10. PROBABILITY

Notes for overall guidance:

• Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.

- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirely. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B, Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks. Note that:

M *marks* are for method;

A marks are for accuracy (awarded only following

a correct M mark);

B marks are independent, stand-alone marks.

We hope that you find this module helpful. Comments should be sent to:

Professor D. N. Burghes CIMT, School of Education University of Exeter EXETER EX1 2LU

The full range of Help Modules can be found at

ACTIVITIES

Activity	10.1	Misconceptions
Activity	10.2	Evens and Odds
Activity	10.3	Experimental Probability
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Activity	10.9	Birthdays
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Misconceptions

Misconceptions about probability may include the following incorrect assumptions:

- All events are equally likely
- Later events may be affected by or compensate for earlier ones
- When determining probability from statistical data, sample size is irrelevant
- Results of games of skill are unaffected by the nature of the participants
- 'Lucky/Unlucky' numbers, etc. can influence random events
- In random events involving selection, results are dependent on numbers rather than ratios
- If events are random then the results of a series of independent events are equally likely, e.g. HH is as likely as HT
- When considering spinners, probability is determined by number of sections rather than size of angles.

This activity is intended to provide an opportunity to look at common misconceptions. The statements given are all incorrect. Read them through and check that you realise why each one is a misconception.

Misconceptions

1.

I've spun an *unbiased* coin 3 times and got 3 heads. It is more likely to be tails than heads if I spin it again.

2.

Aytown Rovers play Betown United. Aytown can win, lose or draw, so the probability that

Aytown will win is $\frac{1}{3}$.

3.

There are 3 red beads and 5 blue beads in a bag. I pick a bead at random. The probability that it is red is $\frac{3}{5}$.

4.

I roll two dice and add the results. The probability of getting a total of 6 is $\frac{1}{12}$ because there are 12 different possibilities and 6 is one of them.

5.

It is harder to throw a six than a three with a die.

6.

Tomorrow it will either rain or not rain, so the probability that it will rain is 0.5.

7.

Mr Brown has to have a major operation. 90% of the people who have this operation make a complete recovery. There is a 90% chance that Mr Brown will make a complete recovery if he has this operation.

8.

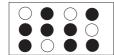
If six fair dice are thrown at the same time, I am less likely to get 1, 1, 1, 1, 1, 1 than 1, 2, 3, 4, 5, 6.

Misconceptions

9.

There are more black balls in box A than in box B. If you chooses 1 ball from each box you are more likely to choose a black ball from A than from B.

A





В

10.

I spin two coins. The probability of getting heads and tails is $\frac{1}{3}$ because I can get

Heads and Heads, Heads and Tails or Heads and Tails.

11.

John buys 2 raffle tickets. If he chooses two tickets from different places in the book he is more likely to win than if he chooses two consecutive tickets.

12.







Each spinner has two sections – one black and one white. The probability of getting black is 50% for each spinner.

13.

13 is an unlucky number so you are less likely to win a raffle with ticket number 13 than with a different number.

14.

My Grandad smoked 20 cigarettes a day for 60 years and lived to be 90, so smoking can't be bad for you.

15.

It is not worth buying a national lottery card with numbers 1, 2, 3, 4, 5, 6, on it as this is less likely to occur than other combinations.

16.

I have thrown an unbiased dice 12 times and not yet got a six. The probability of getting a 6 on my

next throw is more than $\frac{1}{6}$.

Evens and Odds

This is a simple game, where you throw a dice which controls the position of your counter on a 3×3 board.

FINISH	
	START

Place your counter at the START square. Throw a dice.

If you get an EVEN number, you move your counter one square upwards.

If you get an ODD number, you move your counter one square left.

If your counter moves off any side of the board, you lose!

If your counter reaches the FINISH square, you have won.

Play the game a few times and see if you win.

How many 'odds' and how many 'evens' do you need to get to win?

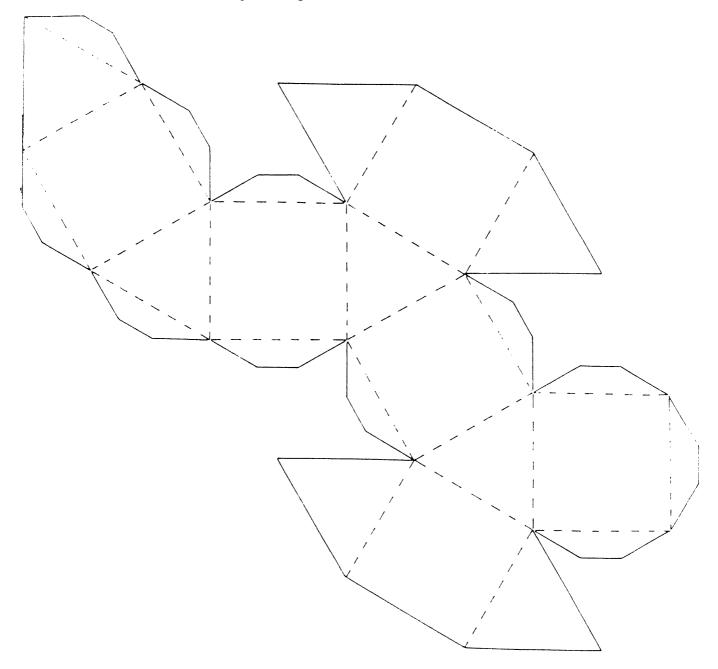
What is the probability of winning?

Extension

Analyse the same game on a 4×4 , 5×5 , ..., board.

Experimental Probability

The net of a cuboctahedron is given below. It consists of 6 squares and 8 triangles. Make this 3-dimensional object using card.



If this object is thrown, what do you think will be the probability of it landing on

- (i) one of its square faces
- (ii) one of its triangular faces?

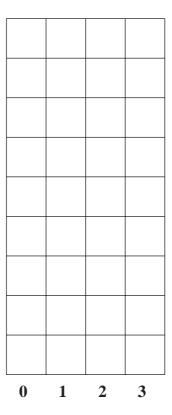
Throw the object (at least 100 times) and estimate these probabilities.

How close are they to your original estimates?

Tossing Three Coins

Toss three coins on to the table.

Record the number of heads uppermost by shading a square in the correct column. Repeat the experiment 20 times.



Problems

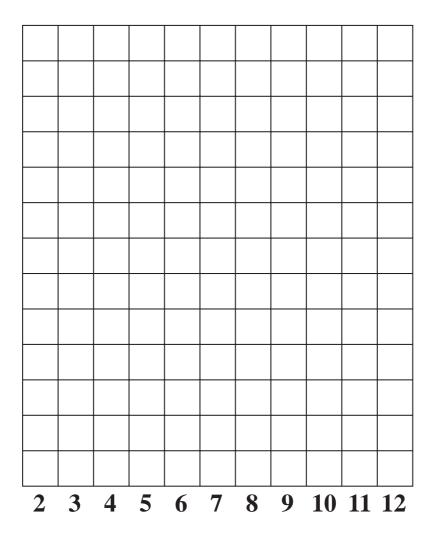
- 1. Find the probability of obtaining 0, 1, 2 or 3 Heads.
- 2. Multiply your theoretical probabilities by 20 to give theoretical frequencies.
- 3. Compare the theoretical and observed frequencies.

Throwing Two Dice

Throw the two dice.

Record the total by shading a square in the correct column.

Repeat the experiment 40 times.



Problems

- 1. Find the probability of obtaining 2, 3,, ..., 11, 12.
- 2. Multiply your theoretical probabilities by 40 to give theoretical frequencies.
- 3. Compare the theoretical and observed frequencies.

A Russian Fable

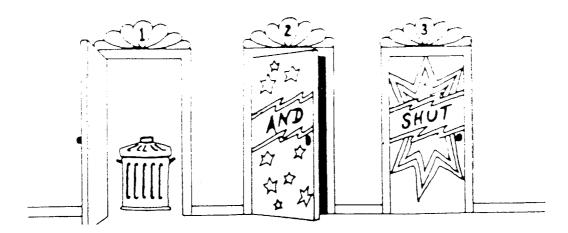
This is the method traditionally used in some Russian villages to see which of the girls in the village are to be married next year! You take three blades of grass, folded in two, and hold them in your hand so that the six ends are hanging down. A young girl ties the ends together in pairs. If, on release, a large loop is formed, the girl will be married next year.

- 1. What are the possible outcomes for this experiment in terms of small, medium and large loops?
- 2. By labelling the six ends (say *a* and A for the two ends of one blade of grass), consider all the possible outcomes and hence find the probability of getting the large loop.
- 3. Test your predicted probabilities by using small lengths of string and getting the class to work in pairs, recording their answers. Collect all the data together and use it to work out the experimental probabilities. Compare these to the theoretical values found in question 2.
- 4. If a Russian village has 30 young girls and they all go through this ritual, how many do you estimate will be predicted to marry next year?

Extension

What happens if either 4 or 5 blades of grass (string) are used? What is the probability of now obtaining one large loop?

Open and Shut Case



In a Game Show in America, the contestant is offered a choice of one of three doors to open. Behind one of these doors is the star prize, a *car*,— but behind the other two doors are *dustbins*!

Once the contestant has chosen say Door 2, the host, who already knows what is behind each door, opens one of the doors, say Door 1, to reveal a dustbin.

He then asks the contestant,

"Do you want to stick with your original choice (Door 2) or switch to the other closed door (Door 3)?"

1. Is it to your advantage to change your choice from Door 2 to Door 3?

It is easy to provide an argument for either policy.

- ARGUMENT 1 When one door is opened, there is an equal chance of the car being behind either of the other two doors, so there is no need to change.
- ARGUMENT 2 There is a 2 in 3 chance of being wrong initially. If you were wrong and changed, you would now be right, so the probability is reversed and you will now be right 2 out of 3 times.
- 2. Simulate this Game Show by playing it with a partner. One of you is the contestant and the other the Game Show host. You will need to play the game at least 20 times in order to gain insight into the solution

If this simulation does not convince you, then try using a *computer program* to simulate the situation 10 000 or 20 000 times.

Extension

Suppose there are now four doors with a star prize behind one door and dustbins behind each of the other doors.

Again the contestants are offered the chance of changing their choices.

Should they change, and if they do, what is now their probability of winning?

Fruit Machines

A fruit machine with 3 DIALS and 20 SYMBOLS (not all different) on each dial is illustrated opposite. Each dial can stop on any one of its 20 symbols, and each of the 20 symbols on a dial is equally likely to occur.

So, for example, the *Grapes* on DIAL 1 are likely to occur on average 7 times out of 20.

You inset 10p, press a button and the three dials spin round. You then press more buttons to stop each dial at random.

The three symbols highlighted determine how much, if anything, is won.

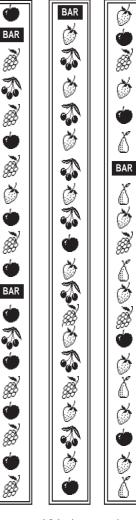
Payout

For example, Suppose the machine makes the payouts shown opposite.

C	ombination	(in 10p's)
3	BARS BAR	40
	STRAWBERRIES 💸	5
3	GRAPES 🖓	5
3	APPLES 🍅	5
2	BARS	20
2	CHERRIES 🔊	5

1. Copy and complete the frequency chart below for each dial.

Symbol	Dial 1	Dial 2	Dial 3
BAR	2	1	1
STRAWBERRY	1	8	
GRAPE	7		
APPLE			
CHERRY			
PEAR 💍			



We want to find the probability of each of the combinations above to see if it is worth playing. We first consider the 3 BARS combination.

- 2. (a) In how many ways can you obtain 3 BARS?
 - (b) How many possible combinations (including repeats) are there?
 - (c) What is the probability of obtaining 3 BARS?

We can find the probability of the other winning combinations in the same way.

3. Find the probabilities of obtaining all the other winning combinations. Your expected winnings in 10 pences are

 $40 \times (probability of 3 BARS) + 5 \times (probability of 3 STRAWBERRIES) +....$

but you must take off your initial payment of 10 pence.

4. What is the expected gain or loss for each go?

Extension

Design your own fruit machine, work out the probabilities of certain combinations, assign payouts and check whether the player expects to gain or lose money.

Birthdays

First try this experiment:

find out the birthdays of as many of your family as possible. Do any of them have birthdays on the same day of the year?



Now try the same experiment with all the pupils in your class or a group of friends. We will see how likely it is that two members of a group have the same birthday.

Consider each member of a group, one by one. The first person will have their birthday on a particular day.

- 1. What is the probability of the second person having a different birthday from the first?
- 2. What is the probability of the third person having a birthday different from both the first and second person?
- 3. What is the probability that at least two of the first three people have the same birthday?

This solves the problem of a group of three people. As expected, it is not likely that any 2 out of 3 people will have the same birthday.

- 4. Repeat the problem above for 4 people. What is the probability that at least 2 of them have the same birthday?
- 5. Using either a computer programme or a calculator, solve the problem for a group of n people, where n = 10, 20, 30, etc.
- 6. What is the probability that 2 pupils in your class have the same birthday?

Extension

How many people are needed in the group to be 95% sure that there will be at least two with the same birthday?

Notes and solutions are only given where appropriate.

10.2 3 'odds' and 3 'evens' in the first six throws of the dice. This has a probability of

$$\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3 \times 20 = 0.3125$$

since there are 20 distinct ways of arranging three 'evens' and three 'odds'.

- 10.4 1. No. of Heads 0 1 2 3Probability $\frac{1}{8} \frac{3}{8} \frac{3}{8} \frac{1}{8}$
 - 2. No. of Heads 0 1 2 3

 Theoretical frequency 2.5 7.5 7.5 2.5

10.5 1. To find probabilities, first draw up table of outcomes.

		2nd Dice						
		1	2	3	4	5	6	
	1	2	3	4	5	6	7	
	2	3	4	5	6	7	8	
1st Dice	3	4	5	6	7	8	9	
	4	5	6	7	8	9	10	
	5	6	7	8	9	0	11	
	6	7	8	9	10	11	12	

Total	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	<u>4</u> 36	<u>5</u> 36	<u>6</u> 36	<u>5</u> 36	<u>4</u> 36	$\frac{3}{36}$	$\frac{2}{36}$	<u>1</u> 36

- **10.6** 1. 3 small loops; 1 small loop and 1 medium loop; 1 large loop.
 - 2. Probabilities $\frac{1}{15}$, $\frac{6}{15}$, $\frac{8}{15}$
 - 4. About 16!

ACTIVITIES 10.1 - 10.9

Notes for Solutions

- Yes, you should change choice; or, at least toss a coin to show which of the two doors you now go for if you stay with your original choice, your original chance of winning, (1/3), will not change!
 (You might need to write a computer simulation, as suggested, to argue this!)
- **10.8** 2. (a) 2 (b) 800 (c) $\frac{1}{4000}$

3.	Symbols	Number of ways	Probability
	3 STRAWBERRIES	56	$\frac{56}{8000}$ (1 × 8 × 7 = 56)
	3 GRAPES	42	$\frac{42}{8000}$
	3 APPLES	64	$\frac{64}{8000}$
	3 BARS	94	$\frac{94}{8000} \left(2 \times 1 \times 19 + 2 \times 19 \times 1 + 18 \times 1 \times 1 = 94\right)$
	2 CHERRIES	280	$\frac{280}{8000} (2 \times 7 \times 20 = 280)$

- 4. On average, you will lose almost 5p per go.
- **10.9** 1. $\frac{364}{365}$ 2. $\frac{363}{365}$ 3. $1 \frac{364}{365} \cdot \frac{363}{365} \approx 0.008$ 4. 0.0164
 - 5. $n = 10 \Rightarrow p = 0.117$; $n = 30 \Rightarrow p = 0.706$; $n = 10 \Rightarrow p = 0.117$

TESTS

- 10.1 Mental Practice
- 10.2 Mental Practice
- 10.3 Revision
 Answers

Test 10.1 Mental Practice

Answer these questions as quickly as you can, but without the use of a calculator.

- 1. If I throw a fair dice 30 times, how many SIXES would I expect to get?
- 2. The probability of a train being late is 0.2. What is the probability of it not being late?
- 3. A coin is tossed two times. Give all the possible outcomes.
- 4. A biased coin has a probability of $\frac{2}{3}$ of obtaining heads when thrown. What is the probability of obtaining tails when the coin is tossed once?
- 5. In a raffle, 100 tickets are sold. If you have bought 5 tickets, what is the probability of you winning the first prize?
- 6. When you throw a fair dice, what is the probability of obtaining the number 2?
- 7. A bag contains 6 RED balls and 4 BLUE balls.

One ball is taken out at random.

What is the probability of it being a BLUE ball?

- 8. When you throw a fair dice, what is the probability of obtaining an even number?
- 9. A bag contains 20 discs, numbered 1 to 20. A disc is selected at random. What is the probability that the number on it is divisible by 3?
- 10. A fair coin is tossed twice, What is the probability of obtaining two HEADS?

Test 10.2 Mental Practice

Answer these questions as quickly as you can, but without the use of a calculator.

- If I toss a fair coin 50 times, how many HEADS would I expect to get?
- 2. The probability of it raining tomorrow is 0.3.

What is the probability of it not raining tomorrow?

3. A dice is thrown once, and a coin is tossed once.

List all the possible outcomes.

- 4. A biased coin has a probability of 0.6 of obtaining TAILS when tossed. What is the probability of obtaining HEADS when the coin is tossed once?
- 5. In a raffle, 500 tickets are sold. If you have bought 10 tickets, what is the probability of you winning the first prize?
- 6. When you throw a fair dice, what is the probability of obtaining the number 6?
- 7. A bag contains 6 RED balls, 7 GREEN balls and 2 YELLOW balls.

One ball is picked at random.

What is the probability of it not being RED?

- 8. When you throw a fair dice, what is the probability of obtaining a number which is odd?
- 9. A bag contains 50 discs, marked 1 to 50. A disc is selected at random. What is the probability of obtaining a number which is divisible by 10?
- 10. A fair dice is thrown twice. What is the probability of obtaining two SIXES?

40 minutes are allowed

1. Express the probability of the following events as

Certain

Probable

Possible

Unlikely

Impossible

- (a) Tomorrow is a Saturday.
- (b) The first day of each year is a Sunday.
- (c) There are 24 hours in each day.
- (d) It will snow next week.
- (e) You will live for more than 50 years.

(5 marks)

- 2. If I throw a fair die 90 times, how many times should I expect to get
 - (a) an odd number,

(b) a '2'? (3 marks)

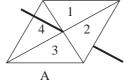
3. With a biased coin, the probability of obtaining HEADS is $\frac{2}{3}$.

What is the probability of obtaining TAILS?

(1 mark)

4. Gary spins two spinners A and B.

List all possible combinations of scores.

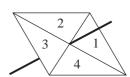


4 1 2 B

(2 marks) (LON)

5. A spinner, with its edges numbered one to four, is biased.

For one spin, the probability of scoring 1 is 0.2, the probability of scoring 3 is 0.15 and the probability of scoring 4 is 0.3.



(a) Calculate the probability of scoring 2 with one spin.

(1 mark)

- (b) (i) Explain why the spinner is described as biased.
 - (ii) If the spinner were fair, what could you say about the probabilities of each of the four possible scores?

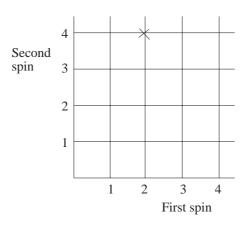
(2 marks)

- (c) The spinner is spun twice.
 - Represent all the possible outcomes by crosses on the diagram below.

The outcome

Second spin 4 First spin 2

has been marked for you.



- (ii) Copy the diagram and draw a ring round those crosses for which the total of the two spins is 6. (3 marks) (MEG)
- A bag contains 40 marbles, 25 green ones and 15 red ones. A marble is picked at random 6. from the bag.

What is the probability of picking a red marble?

(2 marks)

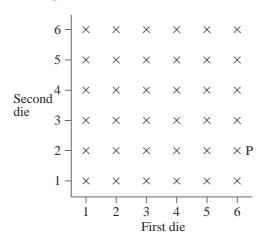
- 7. Sangita is playing 'heads or tails' with her friend. She spins a fair coin four times and gets four heads.
 - What is the probability that she gets a tail with her next spin of the coin? (a) (1 mark)
 - If Sangita spins the coin 600 times in succession, approximately how (b) many times should she expect to get a tail? (1 mark) (NEAB)
- 8. In a raffle 100 tickets are sold. Only one prize can be won.
 - (a) Nicola buys one ticket. What is the probability that she wins the prize?

(1 mark)

(1 mark)

- (b) Dee buys five tickets. What is the probability that she wins the prize?
- Keith buys some tickets. The probability that he wins the prize is $\frac{3}{20}$. (c)
 - (i) What is the probability that he does not win the prize?
 - (ii) How many tickets did he buy? (3 marks)

9. The diagram shows all the possible outcomes when two fair dice are thrown.



(a) Explain clearly what outcome is represented by the cross P.

(2 marks)

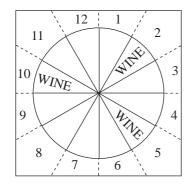
(b) On the diagram ring each of the crosses representing those outcomes with a total score on the two dice 8.

(1 mark)

- (c) Find the probability, as a fraction in its lowest terms, that
 - (i) the two dice will show a total score of 8,
 - (ii) the two dice will show the same score as each other,
 - (iii) the two dice will not show the same score as each other.

(4 marks) (MEG)

10. Peter and Jennifer go to their school's Autumn Fair. Peter has a go on the 'Spinning Wheel'. After every spin, three numbers are exactly next to the words 'WINE'.



WIN A BOTTLE OF WINE

Choose a number on the board
If your number is next to
'WINE' you will win a bottle!

(a) Peter says "I'll have one go. I have a good chance of winning."

Is Peter right? Give a reason for your answer.

(2 marks)

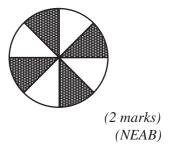
- (b) Peter chooses the number 8,
 - (i) What is the probability that he will win a bottle of wine? Give your answer as a fraction in its simplest form.
 - (ii) What is the probability that he does not win a bottle of wine?

(2 marks)

(c) The next stall they visit is a dart stall.

The probability that Jennifer hits the shaded area is 0.45 and the probability that she hits the unshaded area is 0.35.

What is the probability that she misses the dartboard altogether?



- 11. A coin is tossed three times. Find the probability of 'heads' coming up
 - (a) at all three throws,
- (b) not even once,
- (c) at least once.

(4 marks)

12. Leroy is taking two exams, Biology and Chinese.

The probabilities of passing these are as follows:

Biology:

Chinese: $\frac{4}{5}$

- (a) Construct a tree diagram, showing all the probabilities. (2 marks)
- (b) Calculate the probability that
- (i) he passes both subjects,
- (ii) he passes one subject and fails one.

(5 marks) (SEG)

Tests 10.1 and 10.2

Answers

Test 10.1

1. 5

2. 0.8

3. HH, HT, TH, HH

4. $\frac{1}{3}$

 $5. \quad \frac{5}{100} = \frac{1}{20}$

6. $\frac{1}{6}$

7. $\frac{4}{10} = \frac{2}{5}$

8. $\frac{3}{6} = \frac{1}{2}$

9. $\frac{6}{20} = \frac{3}{10}$

10. $\frac{1}{4}$

Test 10.2

1. 25

2. 0.7

3. 1H, 2H, 3H, 4H, 5H, 6H; 1T, 2T, 3T, 4T, 5T, 6T

4. 0.4

 $5. \quad \frac{10}{500} = \frac{1}{50}$

6. $\frac{1}{6}$

7. $\frac{9}{15} = \frac{3}{5}$

8. $\frac{2}{6} = \frac{1}{3}$

9. $\frac{5}{50} = \frac{1}{10}$

10. $\frac{1}{36}$

(TOTAL MARKS 50)

Test 10.3 Answers

- 1. (b) Impossible (c) Certain B1 B1 B1 B1 B1 (5 marks) (Responses to (a), (d), and (e) will vary.)
- 2. (a) About 45 times. (b) About 15 times. B1 M1 A1 (3 marks)
- 3. $\frac{1}{3}$ B1 (1 mark)
- 4. (1, 1), (1, 2) (also (2, 1) but not need to list both), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4) (for completely correct answer)
- 5. (a) 0.35 B1
 - (b) (i) Probabilities are not equal (ii) each = 0.25 B1 B1
 - (c)

 4

 3

 2

 1

 1

 1

 2

 3

 B2 B1 (6 marks)
- 6. $\frac{15}{40} = \frac{3}{8}$ M1 A1 (2 marks)
- 7. (a) $\frac{1}{2}$ (b) About 300 B1 B1 (2 marks)
- 8. (a) $\frac{1}{100}$ (b) $\frac{5}{100} = \frac{1}{20}$ (c) (i) $\frac{17}{20}$ (ii) 15 B1 B1 B1 M1 A1 (5 marks)
- 9. (a) 1st die 6, 2nd die 2 (b) cross on diagram B2 B1

 (c) (i) $\frac{5}{36}$ (ii) $\frac{1}{6}$ (iii) $\frac{5}{6}$ B1 B2 B1 (7 marks)
- 10. (a) No. Whichever number is chosen, the probability of winning is less than $\frac{1}{2}$. B1 B1
 - (b) (i) $\frac{1}{4}$ (ii) $\frac{3}{4}$ (c) 0.2 B1 B1 M1 A1 (6 marks)
- 11. (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ B1 B1 B2 (4 marks)
- 8 8 8
 12. (a) tree diagram B2
 - (b) (i) $\frac{3}{5}$ (ii) $\left(\frac{3}{4} \times \frac{1}{5}\right) + \left(\frac{1}{4} \times \frac{4}{5}\right) = \frac{7}{20}$ M1 A1 M2 A1 (7 marks)