Sample Questions

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the *standard of excellence*.

Please be aware that the worked solutions shown are possible strategies; there may be other strategies that could be used.

1. An expression that is equivalent to $\frac{x^2 + x}{x}$, $x \neq 0$, is

***A.** $x + 1, x \neq 0$
B. $x^2 + 1, x \neq 0$
C. $2x, x \neq 0$
D. $x^2, x \neq 0$

Numerical Response

2. The non-permissible value of $\frac{2x + 1}{3x - 9}$ is ________

Possible Solution:

$$3x - 9 \neq 0$$
$$3x \neq 9$$
$$x \neq 3$$

The non-permissible value for x is 3.

Use the following information to answer the next question.

Sanja and David both simplified the expression $\frac{x}{x^2 + x}$. Their work is shown below.

Sanja	**David**
$\frac{x}{x^2 + x}$	$\frac{x}{x^2 + x}$
$\frac{x}{x(x + 1)}$	$\frac{{}^{1}\cancel{x}}{x^{\cancel{2}} + {}^{1}\cancel{x}}$
$\frac{1}{x + 1}$	$\frac{1}{x + 1}$

Sanja stated that the non-permissible values of x for the equivalent rational expressions are –1 and 0.

David stated that the non-permissible value of x for the equivalent rational expressions is –1.

3. Which student is correct? Justify your choice.

Possible Solution:
Sanja is correct. Both students correctly simplified the expression, but David made an error in stating the non-permissible values. Non-permissible values of x must be identical for equivalent rational expressions. The non-permissible values for the original expression were –1 and 0 and they must be the same for the simplified expression.

4. Explain why the non-permissible value for the expression $\frac{3x}{(x + 2)}$ is –2.

Possible Solution:
The non-permissible value for any rational expression is the value that, when substituted for the variable, makes the denominator equal 0 and causes the rational expression to be undefined. In this case, if –2 is substituted in for x, the denominator will be 0. This is why the non-permissible value for x is –2.

5. When the rational expression $\frac{2x + 4}{x^2 - 4}$ is simplified, the equivalent expression is

*A. $\frac{2}{x - 2}$, $x \neq -2, 2$

B. $\frac{2}{x + 2}$, $x \neq -2$

C. $\frac{2}{x}$, $x \neq -2, 0, 2,$

D. $\frac{2}{x}$, $x \neq 0$

6. An expression with a non-permissible value of $x = 1$ has been simplified to x. Determine an equivalent rational expression for the simplified expression.

Possible Solution:
An equivalent expression could be $\frac{x(x - 1)}{(x - 1)} = \frac{x^2 - x}{x - 1}$. This expression has a non-permissible value for x of 1.

Use the following information to answer the next question.

The expression $\frac{ab}{c}$ can be simplified to $\frac{x+4}{x+3}$, $x \neq -3, 3$. Henry knows that one expression can be selected from each of the columns below to form a correct simplification.

Possibilities for *a*	**Code**	**Possibilities for *b***	**Code**	**Possibilities for *c***	**Code**
$(x - 3)$	**1**	$(2x + 8)$	**4**	$(3x^2 - 27)$	**7**
$(2x - 6)$	**2**	$(x + 4)$	**5**	$(2x^2 - 18)$	**8**
$(3x - 9)$	**3**	$\frac{1}{2}(x + 4)$	**6**	$(x^2 - 9)$	**9**

Numerical Response

7. One possible selection to form a correct simplification is $(x - 3)$, $(x + 4)$, and $(x^2 - 9)$, so Henry records the code 159. To form another correct simplification, a code for another possibility for

a is __________ (Record in the **first** column)

b is __________ (Record in the **second** column)

c is __________ (Record in the **third** column)

Possible Solutions:
2, 5, and 8
1, 4, and 8
2, 6, and 9
3, 5, and 7

Note: This question is intended to be an alternate digital-format item. Please consult the site https://questaplus.alberta.ca/ for more examples of this item type.

8. Simplify the following. State all non-permissible values.

a. $\dfrac{5}{3x^2} \bullet \dfrac{6x}{x+2}$

b. $\dfrac{x}{x+2} \bullet \dfrac{x+2}{x-3}$

c. $\dfrac{x+3}{5x-1} \div \dfrac{2x+6}{4x}$

d. $\dfrac{x^2+3x}{x^2-4} \div \dfrac{x+3}{x+2}$

Possible Solutions:

a. $\dfrac{5}{3x^2} \bullet \dfrac{6x}{x+2} = \dfrac{30x}{3x^2(x+2)}$

$= \dfrac{10}{x(x+2)}, x \neq -2, 0$

b. $\dfrac{x}{x+2} \bullet \dfrac{x+2}{x-3} = \dfrac{x(x+2)}{(x+2)(x-3)}$

$= \dfrac{x}{x-3}, x \neq -2, 3$

c. $\dfrac{x+3}{5x-1} \div \dfrac{2x+6}{4x} = \dfrac{x+3}{5x-1} \bullet \dfrac{4x}{2(x+3)}$

$= \dfrac{2x}{5x-1}, x \neq -3, 0, \dfrac{1}{5}$

d. $\dfrac{x^2+3x}{x^2-4} \div \dfrac{x+3}{x+2} = \dfrac{x(x+3)}{(x+2)(x-2)} \bullet \dfrac{x+2}{x+3}$

$= \dfrac{x}{x-2}, x \neq -3, -2, 2$

9. Simplify the following. State all non-permissible values.

a. $\frac{3}{5x}+\frac{7x}{4}$

b. $\frac{4x}{x+2}-\frac{5x+3}{x+2}$

c. $\frac{x}{3-x}-\frac{3}{x-3}$

SE **d.** $\frac{x}{x^2-4}+\frac{3x}{x^2+2x}$

SE **e.** $\frac{x^2+3x}{x^2-4}+\frac{x^2+5x}{x+2}$

Possible Solutions:

a.
$$\begin{aligned}\frac{3}{5x}+\frac{7x}{4}&=\frac{12}{20x}+\frac{35x^2}{20x}\\&=\frac{12+35x^2}{20x},\ x\neq 0\end{aligned}$$

b.
$$\begin{aligned}\frac{4x}{x+2}-\frac{5x+3}{x+2}&=\frac{4x-5x-3}{x+2}\\&=\frac{-1x-3}{x+2},\ x\neq -2\end{aligned}$$

c.
$$\begin{aligned}\frac{x}{3-x}-\frac{3}{x-3}&=\frac{x}{-(x-3)}-\frac{3}{x-3}\\&=\frac{-x-3}{x-3},\ x\neq 3 \quad \textbf{or} \quad \frac{-(x+3)}{x-3},\ x\neq 3 \quad \textbf{or} \quad \frac{x+3}{3-x},\ x\neq 3\end{aligned}$$

d.
$$\begin{aligned}\frac{x}{x^2-4}+\frac{3x}{x^2+2x}&=\frac{x}{(x+2)(x-2)}+\frac{3x}{x(x+2)}\\&=\frac{x}{(x+2)(x-2)}+\frac{3(x-2)}{(x+2)(x-2)}\\&=\frac{x+3x-6}{(x+2)(x-2)}\\&=\frac{4x-6}{(x+2)(x-2)},\ x\neq -2, 0, 2\end{aligned}$$

Possible Solutions:

e. $$\frac{x^2+3x}{x^2-4}+\frac{x^2+5x}{x+2}=\frac{x^2+3x}{(x+2)(x-2)}+\frac{(x^2+5x)(x-2)}{(x+2)(x-2)}$$
$$=\frac{x^2+3x+x^3+3x^2-10x}{(x+2)(x-2)}$$
$$=\frac{x^3+4x^2-7x}{(x+2)(x-2)},\ x\neq-2,2$$

10. Solve each equation.

a. $\frac{5x-1}{4x+11}=\frac{3}{4}$

b. $\frac{3}{x}+\frac{5}{3}=10$

c. $\frac{4}{x}+\frac{6x}{x+1}=6$

SE **d.** $\frac{2x}{x+3}+\frac{x}{x-3}=\frac{18}{x^2-9}$

Possible Solutions:

a. $\frac{5x-1}{4x+11}=\frac{3}{4}, x \neq \frac{-11}{4}$

$4(5x-1)=3(4x+11)$

$20x-4=12x+33$

$8x=37$

$x=\frac{37}{8}$

b. $\frac{3}{x}+\frac{5}{3}=10, x \neq 0$

$9+5x=30x$

$9=25x$

$x=\frac{9}{25}$

c. $\frac{4}{x}+\frac{6x}{x+1}=6, x \neq 1, 0$

$4(x+1)+6x(x)=6x(x+1)$

$4x+4+6x^2=6x^2+6x$

$4=2x$

$x=2$

d. $\frac{2x}{x+3}+\frac{x}{x-3}=\frac{18}{x^2-9}, x \neq -3, 3$

$2x(x-3)+x(x+3)=18$

$2x^2-6x+x^2+3x=18$

$3x^2-3x-18=0$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3, x=-2$

However, $x \neq 3$ so it is an extraneous solution and must be rejected. Therefore, the only solution is $x=-2$.

Use the following information to answer the next question.

A student solved a rational equation using the steps shown below.

$$\frac{x}{x+1}-\frac{3}{x-2}=-9,\ x\neq -1,2$$

Step 1 $x(x-2)-3(x+1)=-9$

Step 2 $x^2-2x-3x-3=-9$

Step 3 $x^2-5x+6=0$

Step 4 $(x-3)(x-2)=0$

Step 5 $x=3,2$

SE

11. Identify the errors made in the steps shown above, and justify the corrections necessary to obtain the correct solution.

Possible Solution:
Step 1: The student forgot to multiply the right hand side of the equation by the common denominator.

Steps 2, 3, and 4: The student has carried the error from Step 1 through (Note: If there was not an error in Step 1, these steps would be correct).

Step 5: Again the student has carried the error from Step 1 through and there is an additional error here. The student has failed to reject the extraneous solution of $x = 2$.

Use the following information to answer the next question.

Elliott Nicholls currently holds the world record for the fastest text messaging while blindfolded. He was able to text 160 characters in a time that was 40 seconds less than the previous world record holder's time. Elliott's average rate of texting was 1.6 characters/second faster than the previous world record holder's average rate of texting. The chart below summarizes this information.

	Number of Characters	**Time Taken (s)**	**Average Rate of Texting (characters/s)**
Previous record holder	160	x	$\frac{160}{x}$
Elliott	160	$x - 40$	$\frac{160}{x-40}$

12. The following equation models this information.

$$\frac{160}{x-40} - \frac{160}{x} = 1.6$$

a. State the non-permissible values of x for this equation.

Possible Solution:
$x \neq 0, 40$

b. Describe the values of x that must be rejected for this context.

Possible Solution:
The value of x represents the time required to text 160 characters. Since Elliot beat the previous world record holder's time, x, by 40 seconds, the stated solution for x must a positive value greater than 40 seconds.

c. The equation can be simplified to obtain $1.6x^2 - 64x - 6\,400 = 0$. Solve this equation algebraically. Express your solution to the nearest tenth of a second.

Possible Solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{64 \pm \sqrt{(-64)^2 - 4(1.6)(-6400)}}{2(1.6)}$$

$$x = \frac{64 \pm \sqrt{45056}}{3.2}$$

$$x \approx 86.3,\ x \approx -46.3$$

d. Do the solutions for x make sense in this context? Explain why or why not.

Possible Solution:
Of the two solutions, 86.3 seconds is reasonable. However, −46.3 seconds does not fit the context as it is a negative value for time, which has no logical meaning. Therefore, it must be rejected.

13. Write $4^2 = 16$ in logarithmic form.

Possible Solution:

$\log_4 16 = 2$

14. Evaluate $\log_2\left(\frac{1}{16}\right)$.

Possible Solution:

$\log_2\left(\frac{1}{16}\right) = -4$

15. Write each of the following logarithmic equations in exponential form.

a. $\log(100) = 2$
b. $\log_2 8 = 3$
c. $\ln(x) = 2$
d. $\log_a 5 = 2$

Possible Solutions:

a. $10^2 = 100$
b. $2^3 = 8$
c. $e^2 = x$
d. $a^2 = 5$

16. Use the laws of logarithms to determine the value of each of the following.

a. $\log_6 3 + \log_6 12$

b. $\log 520 - \log 52$

Possible Solutions:

a.
$$\begin{aligned}\log_6 3 + \log_6 12 &= \log_6 36\\ &= \log_6 6^2\\ &= 2\log_6 6\\ &= 2\end{aligned}$$

b.
$$\begin{aligned}\log 520 - \log 52 &= \log 10\\ &= 1\end{aligned}$$

17. Describe how to estimate the value of $\log_2 15$ without using technology.

Possible Solution:
The value of $\log_2 15$ is the exponent that must be applied to 2 to obtain a value of 15. Since $2^3 = 8$ and $2^4 = 16$, the value of $\log_2 15$ must be between 3 and 4. Since 15 is closer to 16 than it is to 8, the value of $\log_2 15$ would be closer to 4 than it is to 3. Therefore, an estimated value is 3.9.

SE

18. Express $2 \ln x - \ln y$ as a single logarithm.

Possible Solution:
$$\begin{aligned}2\ln x - \ln y &= \ln x^2 - \ln y\\ &= \ln\left(\frac{x^2}{y}\right)\end{aligned}$$

19. Express $\log 6$ in a different logarithmic form.

Possible Solutions:
$\log 2 + \log 3$
or
$\log 12 - \log 2$
or
$\frac{1}{2}\log 36$
or
$\frac{\ln 6}{\ln 10}$

20. Solve algebraically.

SE

a. $3 = 9^{2x}$

b. $2^{(x-1)} = 4^{(x-2)}$

c. $10 = 3^{x}$

d. $2^{(x-1)} = 3$

Possible Solutions:

a.

$$3 = (3^2)^{2x}$$
$$3^1 = 3^{4x}$$
$$1 = 4x$$
$$x = \frac{1}{4}$$

b.

$$2^{(x-1)} = 4^{(x-2)}$$
$$2^{(x-1)} = 2^{2(x-2)}$$
$$2^{(x-1)} = 2^{(2x-4)}$$
$$x - 1 = 2x - 4$$
$$x = 3$$

c.

$$10 = 3^x$$
$$x = \log_3 10$$
$$x = \frac{\log 10}{\log 3}$$
$$x \approx 2.1$$

or

$$10 = 3^x$$
$$\log 10 = \log 3^x$$
$$1 = x \log 3$$
$$x \approx 2.1$$

d.

$$2^{(x-1)} = 3$$
$$x - 1 = \log_2 3$$
$$x = \log_2 3 + 1$$
$$x = \left(\frac{\log 3}{\log 2}\right) + 1$$
$$x \approx 2.6$$

or

$$2^{(x-1)} = 3$$
$$\log 2^{x-1} = \log 3$$
$$(x-1)\log 2 = \log 3$$
$$x - 1 = \frac{\log 3}{\log 2}$$
$$x = \frac{\log 3}{\log 2} + 1$$
$$x \approx 2.6$$

21. Describe how to determine the solution of $2^{(x-1)} = 3^{(x-2)}$ graphically.

Possible Solution:
Using a graphing calculator, graph $y_1 = 2^{(x-1)}$ and $y_2 = 3^{(x-2)}$, and use a window setting that shows both graphs and their intersection point. A possible window setting would be x:[-10, 10, 1] y:[-10, 10, 1]. Then determine the intersection point. The x-coordinate of this point will be the solution to the equation.

Use the following information to answer the next question.

> Sam deposits $100 into a savings account that pays 2.4%/a, compounded monthly. A function that models the growth of the deposit is
>
> $$y = 100\left(1 + \frac{0.024}{12}\right)^x$$
>
> where x = number of months and y = value of investment, in dollars.

22. a. Determine how long it will take for the investment to be worth at least $150 at 2.4%/a, compounded monthly

Possible Solutions:

$$y = 100\left(1 + \frac{0.024}{12}\right)^x$$

$$150 = 100(1.002)^x$$
$$1.5 = (1.002)^x$$

$\ln(1.5) = \ln(1.002)^x$	**or**	$\log(1.5) = \log(1.002)^x$
$\ln(1.5) = x\ln(1.002)$		$\log(1.5) = x\log(1.002)$
$x = \dfrac{\ln(1.5)}{\ln(1.002)}$		$x = \dfrac{\log(1.5)}{\log(1.002)}$
$x \approx 203$ months		$x \approx 203$ months

203 months is approximately 16.9 years

Therefore, it will take 16 years and 11 months for the investment to be worth at least $150.

SE

b. Modify the exponential function to reflect an interest rate of 4%/a, compounded quarterly.

Possible Solution:
The value of the base of the exponent will become $\left(1 + \frac{0.04}{4}\right)$ or the value of b in $y = a \cdot b^x$ will become 1.01 and x will represent the number of quarters instead of the number of months.

Use the following information to answer the next question.

The intensity of an earthquake can be calculated using the formula

$$I = I_o(10)^M$$

where I represents the intensity of an earthquake, M is the magnitude of the earthquake on the Richter scale, and I_o represents the intensity of an earthquake with a magnitude of 0.

23. Explain why an earthquake with a magnitude of 8.5 is almost 40 times as intense as an earthquake with a magnitude of 6.9.

Possible Solution:

The earthquake with a magnitude of 8.5 has an intensity of $I = I_o(10)^{8.5}$, while the earthquake of magnitude 6.9 has an intensity of $I = I_o(10)^{6.9}$. When the ratio of these two intensities, $\dfrac{I_O(10)^{8.5}}{I_O(10)^{6.9}} = 10^{1.6}$, is calculated, we see that the stronger quake is $10^{1.6}$ or approximately 40 times as intense as the weaker quake.

Use the following information to answer the next question.

A researcher discovered mould growing in a Petri dish in her laboratory. When first observed, the mould covered only 3% of the dish's surface. Every 24 hours, the surface area of the mould doubles in size, as shown in the table below.

Time (h)	Area covered (%)
0	3
24	6
48	
72	

24. **a.** Complete the table above and then write an exponential function to model the growth of the mould over time.

Possible Solutions:
$y = 3(2)^x$, where x represents the number of 24-hour periods and y represents the percentage of the area covered.

Time (h)	Area covered (%)
0	3
24	6
48	12
72	24

or

Using regression, $y = 3(1.029302237\ldots)^x$, where x represents the number of hours since the first discovery and y represents the percentage of the area covered

b. Use your function from part (a) to determine the approximate length of time, to the nearest tenth of an hour, it will take for the Petri dish to be completely covered with mould.

Possible Solutions:

$$y = 3(2)^x$$
$$100 = 3(2)^x$$
$$\frac{100}{3} = 2^x$$
$$x = \log_2\left(\frac{100}{3}\right)$$
$$x = 5.058893689 \ldots \text{ 24-hr periods}$$
$$x \approx 5.058893689 \cdot 24 \approx 121.4 \text{ hr}$$

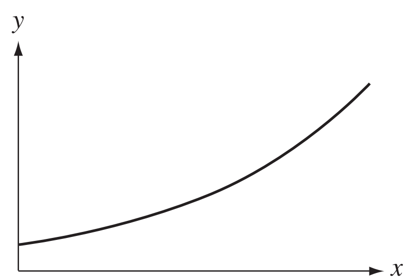
or

$$y = 3(1.029302237...)^x$$
$$100 = 3(1.029302237...)^x$$
$$\frac{100}{3} = (1.029302237...)^x$$
$$\log\left(\frac{100}{3}\right) = x \log(1.029302237...)$$
$$x \approx 121.4 \text{ hr}$$

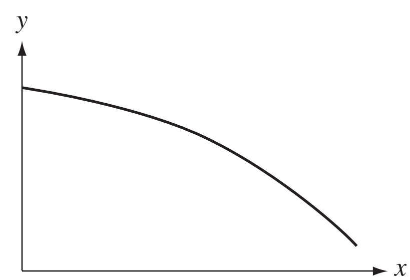
It will take approximately 121.4 hours for the petri dish to be completely covered with mould.

25. A painting was purchased in 2012 for $10 000. If the painting appreciates in value at 5%/a, then which of the following graphs best models the appreciated value of this painting for the next 40 years?

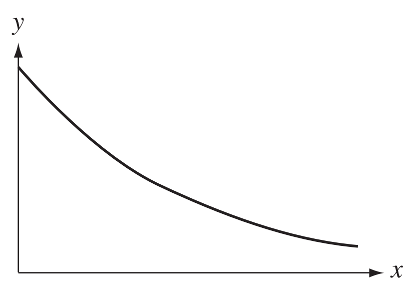
***A.**



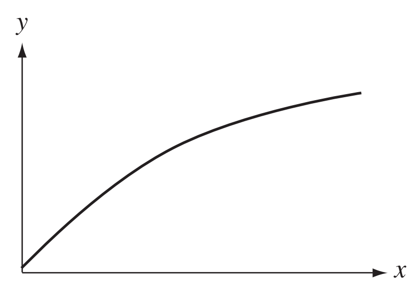
B.



C.



D.



Use the following information to answer the next question.

The pH of a solution can be determined using the formula

$$\text{pH} = -\log_{10}(C)$$

where C is the concentration of hydrogen ions in the solution. The pH of a particular solution is 6.6.

Numerical Response

26. To the nearest tenth, if the concentration of hydrogen ions in the solution is doubled, the new pH of the solution will be __________.

Possible Solution:
$6.6 = -\log_{10}(C)$
$10^{-6.6} = C$
If the concentration is doubled, the new concentration would be $2 \cdot 10^{-6.6}$.

$\text{pH} = -\log_{10}(2 \cdot 10^{-6.6})$
$\text{pH} \approx 6.29897$
$\text{pH} \approx 6.3$

Use the following information to answer the next question.

Corlene invested money in a GIC that pays interest compounded annually, as shown in the table below.

Year	Value of Investment
0	$1 000.00
1	$1 020.00
2	$1 040.40
3	$1 061.21

27. **a.** To model the investment's growth and predict its future value, Corlene has chosen to use an exponential model. Discuss the effectiveness of her model.

Possible Solution:
Corlene made an effective choice of model because the growth rate of the investment is a constant percent as time passes. This increasing function from a set starting point is characteristic of an exponential function.

b. Write an exponential function that Corlene could use to predict the future value of her investment. Explain what the numerical values in your function represent in the context of this problem.

Possible Solution:

Using an exponential function in the form $y = a \bullet b^x$, a possible function is $y = 1000(1.02)^x$, where x is the year and y is the value of the investment. In this function, a represents the initial value of the investment, which was $1 000, and b represents the yearly rate of increase in value. In this case, the investment is growing at 2%/a compounded annually, so the yearly rate of increase can be represented by 1.02.

SE

c. If Corlene invested in a GIC that paid 1.40%/a compounded annually, how would this affect the value of the investment over time?

Possible Solution:
If the interest rate was 1.40%/a compounded annually, the value of the investment over time would increase at a slower rate.

SE

d. If Corlene invested in a GIC that paid 1.40%/a compounded annually, how would this affect the function found in Part b?

Possible Solution:
If the interest rate was 1.40%/a compounded annually, the function would change to $y = 1000(1.014)^x$, where x is the year and y is the value of the investment.

Use the following information to answer the next question.

When objects of different mass are compared without a scale, to be perceived the difference in mass must be large enough. For example, when held in a person's hands, masses within 5 g of 100 g will seem to be the same. The 5 g difference is known as the Minimum Perceivable Difference.

For heavier objects, the Minimum Perceivable Difference increases. The Minimum Perceivable Difference for various masses is shown in the table below.

Mass (g)	**Minimum Perceivable Difference (g)**
100	5
200	10
400	15
800	20

These data can be modelled by a logarithmic regression function of the form

$$y = a + b\ \ln(x)$$

where x is the mass of the object, in grams, and y is the Minimum Perceivable Difference in mass, in grams.

28. a. Determine a logarithmic regression function of the form $y = a + b\ \ln(x)$, to model these data. Round values of a and b to the nearest tenth.

Possible Solution:
$y = -28.2 + 7.2\ \ln(x)$

b. Based on the regression equation, determine the Minimum Perceivable Difference for a 2100 g object, to the nearest whole gram.

Possible Solution:
Let $x = 2100$

$y = -28.2 + 7.2\ \ln(2\ 100)$

$\ \ = 26.8778$

The minimum noticeable change in mass would be approximately 27 g.

29. Describe the graph of $f(x) = -(x + 1)(x - 2)^2$. Include the intercepts, minimum, maximum, domain, and range in your description.

Possible Solution:
This polynomial function has a domain of $\{x \mid x \in R\}$, a range of $\{y \mid y \in R\}$, two distinct x-intercepts and one y-intercept. The y-intercept is at the point (0, –4) and the x-intercepts are at (–1, 0) and (2, 0). Viewed on a graphing calculator, this graph has a minimum turning point at (0, –4) and a maximum turning point at (2, 0).

Use the following information to answer the next two questions.

> A 15-gallon tank is being filled with water and has a pump that will cause it to drain when the amount of water inside the tank hits a certain volume. The volume of water in the tank over a 3-hour period can be modelled by the function
>
> $$y = -2t^2 + 5t + 6$$
>
> where y represents the volume of water in the tank in gallons and t represents the time in hours after noon on a particular day.

30. To determine the volume of water in the tank at noon, the characteristic of the function that should be analyzed is the

*__A.__ y-intercept
B. positive t-intercept
C. t-coordinate of the vertex
D. y-coordinate of the vertex

Numerical Response

31. The maximum volume of liquid in the tank, to the nearest tenth of a gallon, is __________ gallons.

Possible Solution:
Using a graphing calculator, the vertex is found to be approximately (1.25, 9.13). Therefore, the maximum volume of 9.1 gallons is reached after 1.25 hours.

Use the following information to answer the next question.

The rate at which snow fell on a driveway on a particular day can be modelled by

$$y = -3x^2 + 6x$$

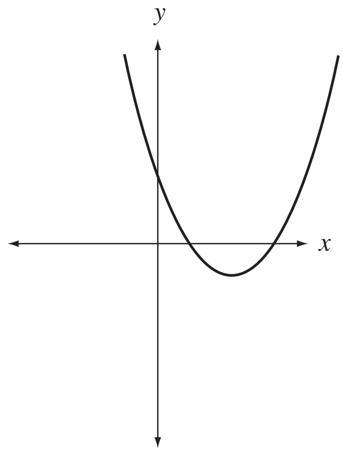
where y represents the rate of snowfall in ft^3/hr, and x represents the time in hours after midnight.

32. To estimate the length of time that snow fell on this particular day, a student should determine the

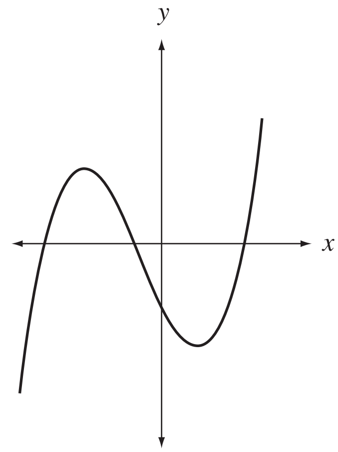
A. y-intercept
B. x-coordinate of the vertex
C. y-coordinate of the vertex
***D.** difference between the x-intercepts

33. Which of the following graphs would **most likely** represent the graph of a cubic function?

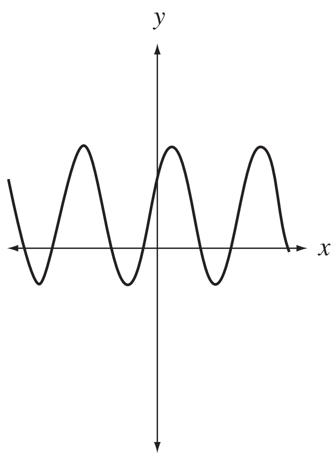
A.



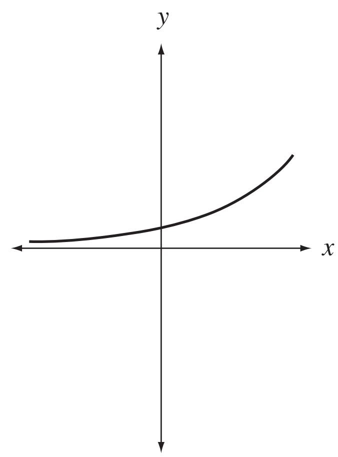
***B.**



C.



D.



Use the following information to answer the next question.

A hockey arena seats 1 600 people. The cost of a ticket is \$10. At this price, every ticket is sold. To obtain more revenue, the arena management plans to increase the ticket price. A survey was conducted to estimate the potential revenue for different ticket prices, as shown below.

Ticket price (\$)	Potential Revenue (\$)
10	16 000
15	19 500
20	20 300
25	14 750
30	5 500

The data above can be modelled by a quadratic regression function of the form

$$y = ax^2 + bx + c$$

where x is the ticket price, in dollars, and y is the potential revenue, in dollars.

34. Determine the ticket price that would maximize the revenue.

Possible Solution:
The quadratic regression function that models the given data is
$y = -91x^2 + 3125x - 6340$

Using a window of x:[0, 40, 1], y:[0, 25000, 1000], the maximum value of the function can be located at (17.17, 20 488.64). Therefore, the maximum potential revenue will be realized when the ticket price is approximately \$17.20.

Use the following information to answer the next question.

A juice box measures 5.0 cm × 4.0 cm × 12.0 cm and contains 240 mL of juice. The manufacturer wants to design a larger box by increasing each dimension of the juice box by the same amount.

The volume of the larger box can be modelled by the function

$$V = (5 + x)(4 + x)(12 + x)$$

where V represents the volume, in mL, and x represents the increase in the length of each dimension, in cm.

Note: 1 cm^3 = 1 mL

Numerical Response

35. If the larger box must hold a maximum of 1 000 mL of juice, the amount, x, by which each dimension of the juice box must be increased, to the nearest tenth of a centimetre, is __________ cm.

Possible Solution:
$V = (5 + x)(4 + x)(12 + x)$
The volume must be 1 000 cm^3; therefore
$1000 = (5 + x)(4 + x)(12 + x)$

This cubic equation can be solved using technology by sketching y_1=1000 and $y_2 = (5 + x)(4 + x)(12 + x)$. Using the window x:[0, 10, 1], y:[0, 1500, 100] the intersection point is (3.54, 1 000). The x-coordinate of the intersection point is the solution. Therefore, each dimension must be increased by 3.6 cm to obtain the necessary volume.

Use the following information to answer the next question.

The height of a pendulum, h, in inches, above a table top t seconds after the pendulum is released can be modelled by the sinusoidal regression function

$$h = 2\sin(3.14t - 1) + 5$$

Numerical Response

36. To the nearest tenth of an inch, the height of the pendulum at the moment of release is __________ in.

Possible Solution:
The h-intercept represents the starting moment, so let $t = 0$.
$h = 2\sin(3.14(0) - 1) + 5$
$h = 3.317 \ldots$

The height of the pendulum at the moment of release is approximately 3.3 in above the table.

Use the following information to answer the next two questions.

The height of a rider on a Ferris wheel can be modelled by the sinusoidal regression function

$$h = 6 \sin(1.05t - 1.57) + 8$$

where h is the height of the rider above the ground, in metres, and t is the time in minutes after the ride starts.

37. According to the sinusoidal regression function, the maximum height of the rider above the ground is

A. 2 m
B. 6 m
C. 8 m
***D.** 14 m

Numerical Response

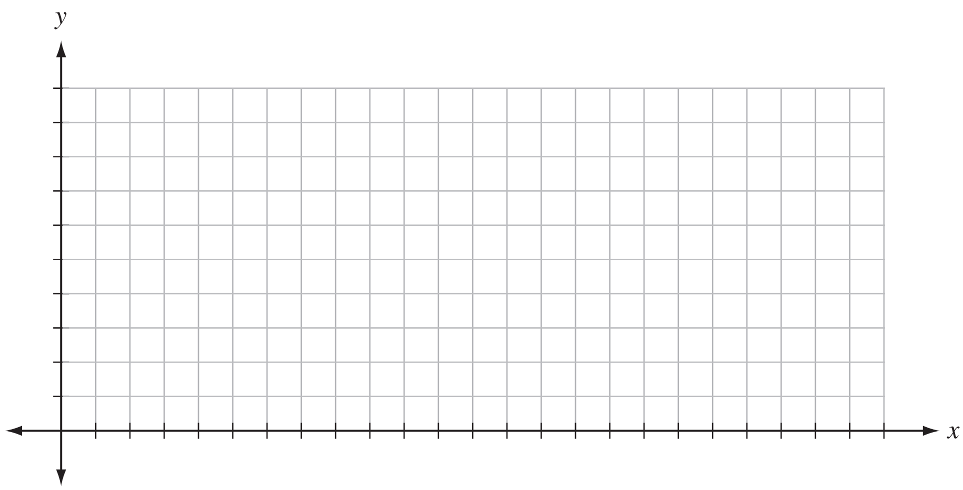
38. When the rider is at least 11.5 m above the ground, she can see the rodeo grounds. During each rotation of the Ferris wheel, the length of time that the rider can see the rodeo grounds, to the nearest tenth of a minute, is __________ min.

Possible Solution:
By sketching $y_1 = 11.5$ and $y_2 = 6 \sin(1.05t - 1.57) + 8$ using a window of x:[0, 7, 1], y:[0, 15, 1] and finding the intersection points between the graphs, it can be determined that the rider can see the rodeo grounds between approximately 2.09 min and 3.89 min on the first rotation. This means the rider sees the rodeo grounds for approximately 1.8 min on each rotation.

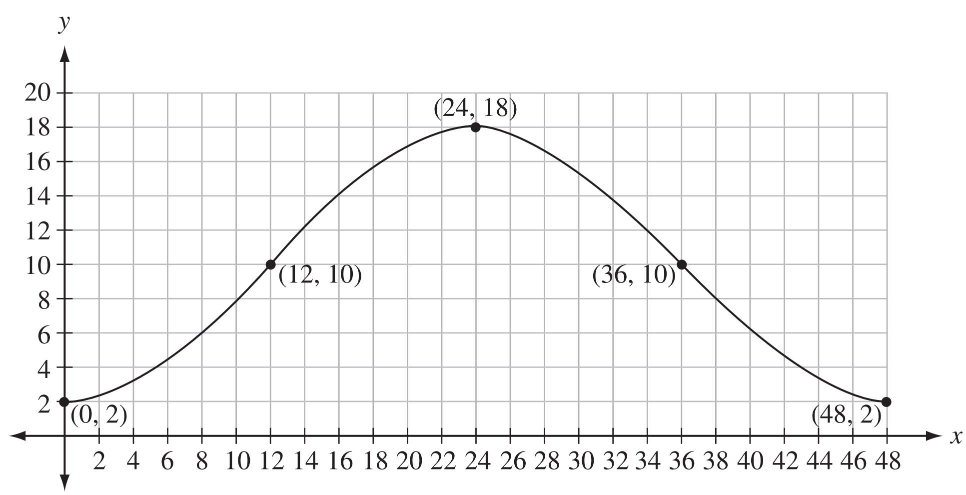
Use the following information to answer the next question.

A Ferris wheel has a radius of 8 m and its centre is 10 m above the ground. A rider gets on a chair of the Ferris wheel at its lowest point and completes one full revolution in 48 seconds.

SE **39.** **a.** Sketch a graph of the rider's height above the ground over the first 48 seconds on the grid below and label key points on the graph.



Possible Solution:



b. State the amplitude, period, and equation of the midline for the function sketched in part a, above.

Possible Solution:
The amplitude, a, will represent radius of the Ferris wheel, which is 8 m. The value of b can be determined by the formula $\frac{2\pi}{b}$ = period. $\frac{2\pi}{b} = 48$, so $b \approx 0.13$. The midline will represent the height of the centre of the Ferris wheel above the ground. The midline is $y = 10$.

c. Determine a function of the form $h = a \sin(bt - 1.57) + d$, where h represents the height of a rider above the ground and t represents the time after the ride has started that could be used to model the height of a rider on the Ferris wheel described above.

Possible Solution:
The function that models the height of a rider on this Ferris wheel is
$h = 8 \sin(0.13t - 1.57) + 10$

Use the following information to answer the next question.

The average daily high temperature of Montreal, in °F, for each of the months of the year is shown in the table below. January is month 1, February is month 2, etc..

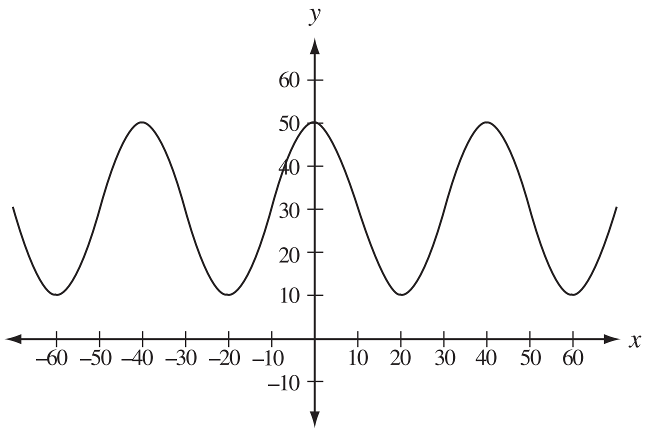
Month	**Average Daily High Temperature in °F**	**Month**	**Average Daily High Temperature in °F**
1	22	7	80
2	25	8	77
3	36	9	67
4	52	10	51
5	66	11	41
6	75	12	28

40. Write a sinusoidal regression function of the form $y = a \bullet \sin(bx + c) + d$, where x is the month number and y is the average daily high temperature, that could be used to model these data. Round the values of a, b, c, and d to the nearest hundredth.

Possible Solution:
$y = 29.08 \sin(0.51x - 2.02) + 50.77$

Use the following information to answer the next question.

The graph of a sinusoidal function is shown below.



41. *The amplitude of the sinusoidal function is* ___***i***___ *units and the midline is* *y* = ___***ii***___ *units.*

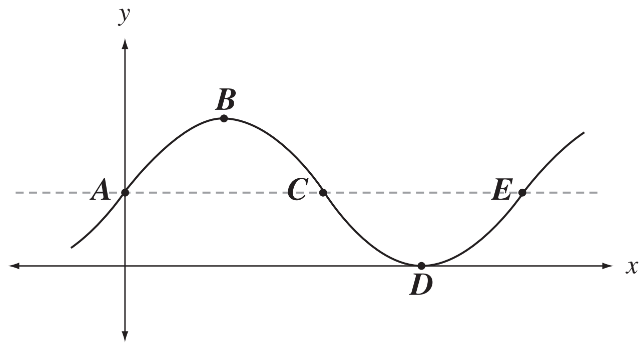
Pick one of the following choices to fill in each blank in the statement above:
Choices for amplitude in blank ***i*** are: 10, 20, 30, 40, or 50
Choices for midline in blank ***ii*** are: 10, 20, 30, 40, or 50

Possible Solution:
i = amplitude = 20
ii = midline = 30

Note: This question is intended to be an alternate digital format item. Please consult the site https://questaplus.alberta.ca/ for more examples of this item type.

Use the following information to answer the next question.

The graph of a sinusoidal function is shown below. The points ***A***, ***B***, ***C***, ***D***, and ***E*** are labelled.



42. **a.** Mary says that in order to find the period of the function, she would need to know the coordinates of points *A* and *E*. Bill says that he could find the period using the coordinates of *B* and *D*. Both Mary and Bill are correct. Explain why.

Possible Solution:
Mary is correct because the horizontal distance between *A* and *E* would be the period, as it is the length of time it takes the function to complete one cycle. Bill is also correct as the horizontal distance between *B* and *D* represents half the period. He would need to remember to double this result to determine the period

b. Select all the points that represent the *x*-intercepts of the function.

Possible Solution:
D is the only *x*-intercept visible on the graph above

c. Select all the points that represent the minimum value of the function.

Possible Solution:
D is also the only minimum point visible on the function above

d. Select two points that could be used to determine the amplitude of the function. Explain a process that could be used to determine the amplitude using the two selected points.

Possible Solution:
Answers may vary. One possible solution would be to select *B* and *C* and find the vertical distance between them. This distance would represent the amplitude.

Note: This question is intended to be an alternate digital-format item. Please consult the site https://questaplus.alberta.ca/ for more examples of this item type.