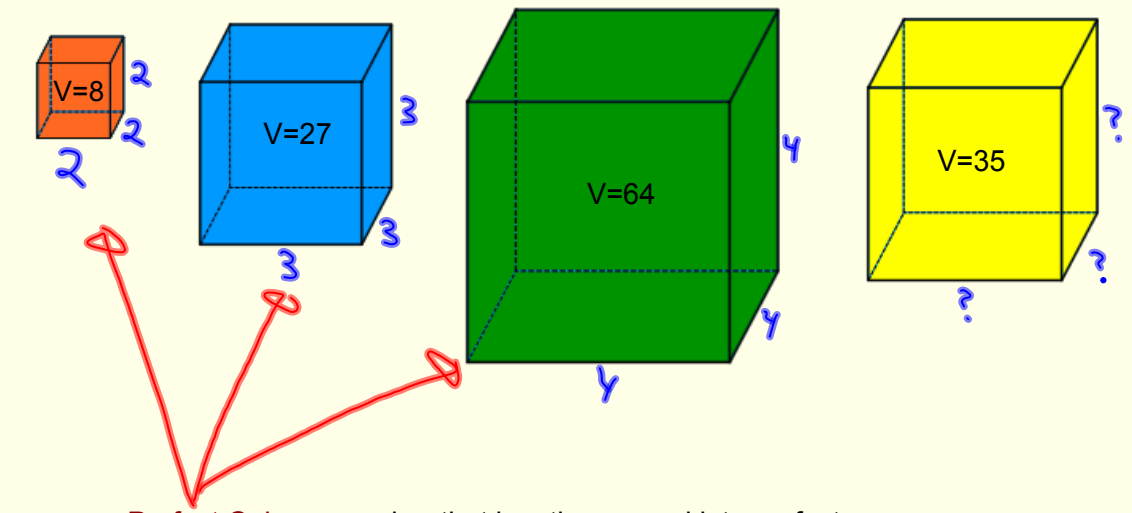
M10C E&R Ls 2: Perfect Cubes & Cube Roots

1. Define Perfect Cube
2. Determine if a Number is a Perfect Cube
3. Define Cube Root
4. Evaluate and Estimate Cube Roots

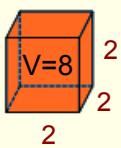
Define Perfect Cube

What is the side length of each cube?



Perfect Cube: a number that has three equal integer factors.

eg. 8 is a perfect cube



Think About it:
Are 2, 2 and 2 the only equal factors of 8?

Determine if a # is Perfect Cube

Example: Determine if 216 is a perfect cube.

$2 \mid 216$
$2 \mid 108$
$2 \mid 54$
$3 \mid 27$
$3 \mid 9$
$3 \mid 3$
1

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 216$

$(2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = 216$

$6 \cdot 6 \cdot 6 = 216$

$\therefore$ 216 is a perfect cube.

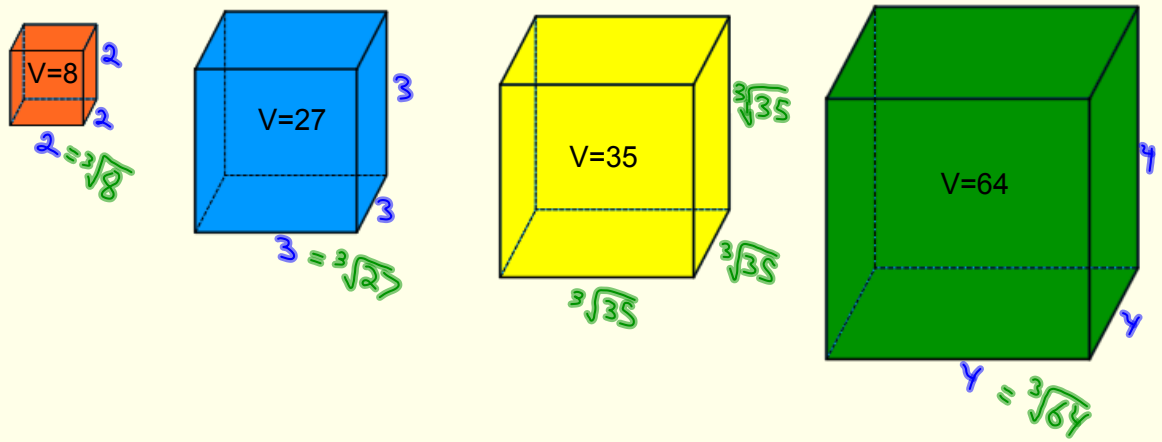
Skill
Use prime factorization.
If it is possible to make three equal groups out of the prime factors then the # is a perfect cube.

Practice: Determine if the following numbers are perfect cubes.

a) 512 ✓ b) 350 ✗ c) 729 ✓

Define Cube Root

What is the side length of each cube?



Cube Root: one of three equal factors of a number.

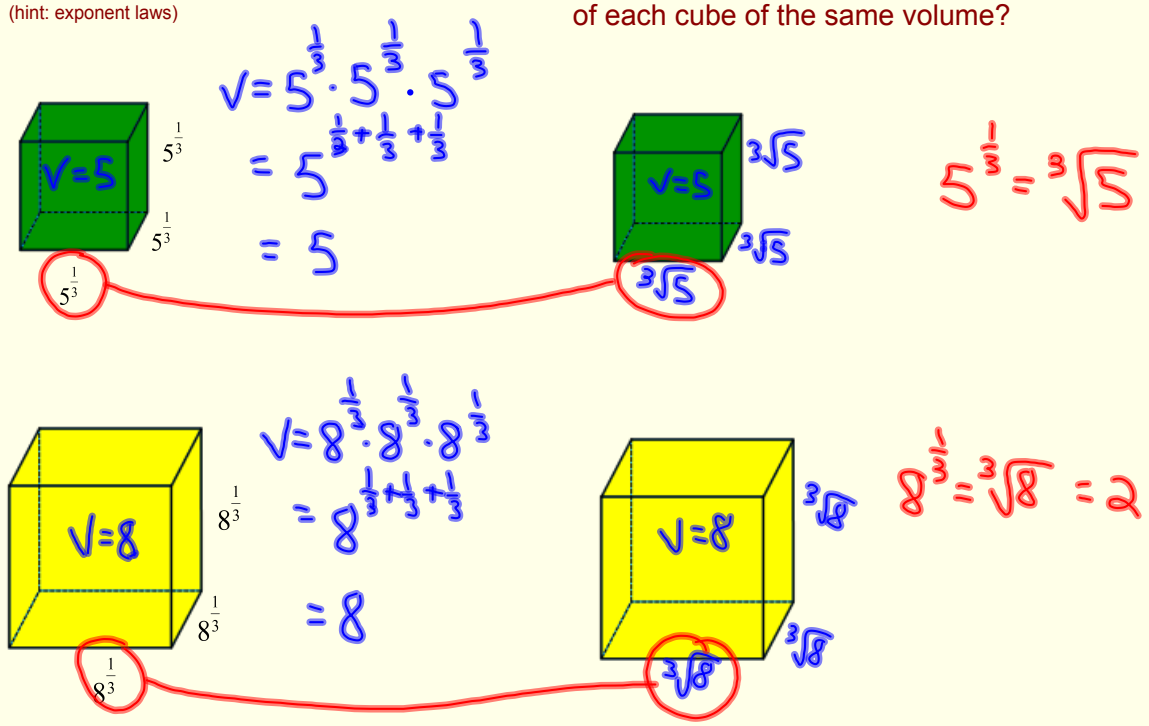
eg. $27 = 3 \cdot 3 \cdot 3$ therefore $\sqrt[3]{27} = 3$

eg. $35 = \sqrt[3]{35} \cdot \sqrt[3]{35} \cdot \sqrt[3]{35}$

Investigate Cube Roots and Exponents

Evaluate the Volume of each cube.
(hint: exponent laws)

Is there another way to write the dimensions of each cube of the same volume?



$V = 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}}$

$= 5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$

$= 5$

$V = 8^{\frac{1}{3}} \cdot 8^{\frac{1}{3}} \cdot 8^{\frac{1}{3}}$

$= 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$

$= 8$

$5^{\frac{1}{3}} = \sqrt[3]{5}$

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

Practice: Cube Roots in Exponent Form

Evaluate the following:

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$216^{\frac{1}{3}} = 6$

$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

$343^{\frac{1}{3}} = 7$

Evaluate Cube Roots

We can use our knowledge of Perfect Cubes to help us evaluate Cube Roots.

Perfect Cubes: (list as many as you can) 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

Evaluate the following:

a) $\sqrt[3]{216} =$ 6

b) $\frac{27^{\frac{1}{3}}}{\sqrt[3]{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$

c) $\sqrt[3]{8 \cdot 125} = \sqrt[3]{8} \cdot \sqrt[3]{125}$
$= 2 \cdot 5$
$= 10$

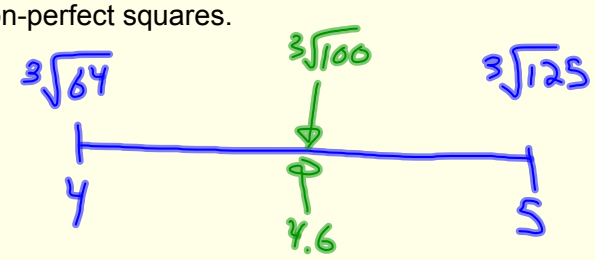
d) $\sqrt[3]{343x^3} =$ 7x

Practice: Text pg. 64: 4

Estimate Cube Roots

Examples of estimating square roots of non-perfect squares.

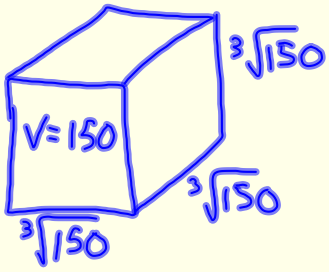
$\sqrt[3]{100} \approx$ 4.6



Problems Involving Cube Roots

Most problems involving cube roots relate to volume.

e.g. The volume of a cubic box is 150 in^3. Determine the boxes dimensions.

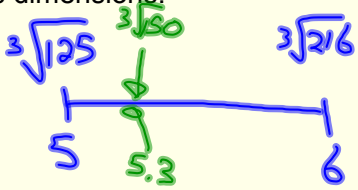


$V = s^3$

$\sqrt[3]{150} = \sqrt[3]{s^3}$

$\sqrt[3]{150} = s$

$s \approx 5.3$



Practice: Text pg. 64: 16, 17 (No Calculator - Evaluate or Estimate)