M10C Exponents and Radicals

Lesson #4 - Working with Exponents (with Rational Bases)

1. Review Exponent Laws

2. Some New Concepts

Review: Exponent Laws

Handout: Laws of Exponents Review

Try:

1. Use the exponent laws to rewrite each expression as a single power.

(a) $(2^3)(2^5) = 2^8$ **(b)** $(3^2)^3 = 3^6$ **(c)** $\frac{4^5}{4^3} = 4^2$

2. Simplify each expression. You do not need to evaluate.

(a) $(3^2 \bullet 4^3)^2 = 3^4 \cdot 4^6$ **(b)** $\frac{2^7}{(2^2)^3} = \frac{2^7}{2^6} = 2$ **(c)** $(4^{\frac{1}{2}})^3 = 4^{\frac{3}{2}}$

Recall - Rational Exponents in the form $\frac{1}{n}$

$x^{\frac{1}{2}} = \sqrt{x}$ $\quad x^{\frac{1}{3}} = \sqrt[3]{x}$ $\quad x^{\frac{1}{4}} = \sqrt[4]{x}$ $\quad \cdots \quad x^{\frac{1}{n}} = \sqrt[n]{x}$

Rewrite the following as radicals, then evaluate.

a) $64^{\frac{1}{2}} = \sqrt{64}$
$= 8$

b) $(-27)^{\frac{1}{3}} = \sqrt[3]{-27}$
$= -3$

c) $\left(\frac{16}{25}\right)^{\frac{1}{2}} = \sqrt{\frac{16}{25}}$
$= \frac{4}{5}$

d) $16^{\frac{1}{4}} = \sqrt[4]{16}$
$= 2$

Working with Rational Exponents - Changing between Exponential and Radical form

To evaluate on the previous slides, we changed exponential expressions to radical form.

Examples: *a*) $49^{\frac{1}{2}} = \sqrt{49}$
$= 7$

b) $\left(\frac{8}{27}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{8}{27}}$
$= \frac{2}{3}$

Sometimes we can't evaluate, but we can simplify the radical.

Examples: a) $45^{\frac{1}{2}} = \sqrt{45}$
$= \sqrt{9}\cdot\sqrt{5}$
$= 3\sqrt{5}$

b) $16^{\frac{1}{3}} = \sqrt[3]{16}$
$= \sqrt[3]{8}\cdot\sqrt[3]{2}$
$= 2\sqrt[3]{2}$

Working with Rational Exponents

Rewrite the following as a radical and then evaluate:

$a)\ 4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = \left(\sqrt{4}\right)^3 = 2^3 = 8$

$b)\ 27^{\frac{4}{3}} = \left(27^{\frac{1}{3}}\right)^4 = \left(\sqrt[3]{27}\right)^4 = 3^4 = 81$

$c)\ 81^{\frac{3}{4}} = \left(81^{\frac{1}{4}}\right)^3 = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$

Powers with Rational Exponents

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

and

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

Practice

C4 - Powers with Rational Bases Assignment #1 - 3

Rational Exponents & Exponent Laws

The exponent laws also work for powers with rational exponents.

Examples:

$a)\ 9^{\frac{1}{4}} \cdot 9^{\frac{1}{4}} = 9^{\frac{1}{4}+\frac{1}{4}}$

$= 9^{\frac{1}{2}}$

$= \sqrt{9}$

$= \boxed{3}$

$b)\ \dfrac{8^{\frac{5}{6}}}{\sqrt{8}} = \dfrac{8^{\frac{5}{6}}}{8^{\frac{1}{2}}}$

$= 8^{\frac{5}{6}-\frac{1}{2}}$

$= 8^{\frac{5}{6}-\frac{3}{6}}$

$= 8^{\frac{2}{6}}$

$= 8^{\frac{1}{3}}$

$= \sqrt[3]{8}$

$= \boxed{2}$

$c)\ \left(3^{\frac{2}{5}}\right)^5 = 3^{\frac{2}{5}\cdot 5}$

$= 3^2$

$= \boxed{9}$

$d)\ \sqrt{2^8 3^6} =$

$= \left(2^8 3^6\right)^{\frac{1}{2}}$

$= 2^4 \cdot 3^3$

$= (16)(27)$

$\boxed{432}$

	10	6
20	200	120
7	70	42

432

Practice: C4 Asgn #4

Investigate: Integral Exponents

Powerpoint Investigation

Debrief: Investigate Integral Exponents

Integer Exponents Rule #1 ⟶ $a^{-n} = \dfrac{1}{a^n}$

Examples:

$a)\ 4^{-3} = \dfrac{1}{4^3}$

$= \boxed{\dfrac{1}{64}}$

$b)\ (-5)^{-2} = \dfrac{1}{(-5)^2}$

$= \boxed{\dfrac{1}{25}}$

$c)\ -8 \cdot 2^{-3} = \dfrac{-8}{1} \cdot \dfrac{1}{2^3}$

$= \dfrac{-8}{8}$

$= \boxed{-1}$

Investigate: Integral Exponents

Integer Exponents Rule #2 $\longrightarrow$ $\frac{1}{a^{-n}} = a^n$

Prove Rule #2 using Rule #1

$$\frac{1}{a^{-n}} = \frac{1}{\frac{1}{a^n}} = 1 \cdot \frac{a^n}{1} = a^n$$

Examples:

$a)\ \frac{1}{3^{-3}} = 3^3$
$= 27$

$b)\ \frac{6}{2^{-2}} = 6 \cdot 2^2$
$= 6 \cdot 4$
$= 24$

$c)\ \frac{4^{-2}}{2^{-3}} = \frac{2^3}{4^2} = \frac{8}{16}$
$= \frac{1}{2}$

Investigate: Integral Exponents

Integer Exponents Rule #3 $\longrightarrow$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Prove Rule #3 using Rule #1 & #2

$$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$$

Examples:

$a)\ \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2$
$= \boxed{\frac{9}{4}}$

$b)\ \left(\frac{1}{5}\right)^{-3} = \left(\frac{5}{1}\right)^3$
$= \boxed{125}$

$c)\ \left(-\frac{4}{3}\right)^{-2} = \left(-\frac{3}{4}\right)^2$
$= \boxed{\frac{9}{16}}$

Practice: Integral Exponents

The following three rules are useful for evaluating expressions with integral exponents.

Integer Exponents Rule #1 $\longrightarrow$ $a^{-n} = \dfrac{1}{a^n}$

Integer Exponents Rule #2 $\longrightarrow$ $\dfrac{1}{a^{-n}} = a^n$

Integer Exponents Rule #3 $\longrightarrow$ $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^{n}$

Practice: C4 Asgn #5

Integral Exponents & Exponent Laws

The exponent laws also work for powers with integral exponents.

Examples:

$a)\ 3^4 \cdot 3^{-6} = 3^{-2}$

$= \dfrac{1}{3^2}$

$= \boxed{\dfrac{1}{9}}$

$b)\ \dfrac{5^{-4}}{5^{-7}} = 5^{-4+(+7)}$

$= 5^3$

$= \boxed{125}$

$c)\ \left(7^{-1}\right)^3 = 7^{-3}$

$= \dfrac{1}{7^3}$

$= \boxed{\dfrac{1}{343}}$

$d)\ \left(\dfrac{5^{-1}}{3^{-2}}\right)^{-2} = \dfrac{5^2}{3^4}$

$= \dfrac{25}{81}$

Practice: C4 Asgn #6

Putting it Together - Integral & Rational Exps

Examples:

a) $8^{-\frac{1}{3}} = \dfrac{1}{8^{\frac{1}{3}}}$

$= \dfrac{1}{\sqrt[3]{8}}$

$= \boxed{\dfrac{1}{2}}$

b) $\left(\dfrac{9}{16}\right)^{-\frac{3}{2}} = \left(\dfrac{16}{9}\right)^{\frac{3}{2}}$

$= \left(\left(\dfrac{16}{9}\right)^{\frac{1}{2}}\right)^{3}$

$= \left(\dfrac{4}{3}\right)^{3} = \boxed{\dfrac{64}{27}}$

c) $5^{-\frac{2}{3}} \cdot 5^{-\frac{1}{3}} = 5^{-\frac{3}{3}}$

$= 5^{-1}$

$= \boxed{\dfrac{1}{5}}$

d) $\sqrt{2^{-6} \cdot 3^{-4}} = \left(2^{-6} \cdot 3^{-4}\right)^{\frac{1}{2}}$

$= 2^{-3} \cdot 3^{-2}$

$= \dfrac{1}{2^3} \cdot \dfrac{1}{3^2}$

$= \dfrac{1}{8} \cdot \dfrac{1}{9}$

$\boxed{= \dfrac{1}{72}}$

Practice: C4 Asgn #7-10