M10C Exponents and Radicals

Lesson #5 - Working with Exponents (with Variable Bases)

1. Any ?'s from Lesson 4
2. Lesson 4 Quiz
3. Lesson 5 - Variable Bases

Simplify vs. Evaluate

Most of the problems done so far we have evaluated to a numerical value.

Sometimes the solution is not a nice numerical value and we just want to simplify.

e.g. $(12^{-3} \cdot 12^{-2})^{-2}$

$= (12^{-5})^{-2}$

$= \boxed{12^{10}}$

e.g. $\dfrac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$

$= \dfrac{9^1}{9^{\frac{3}{4}}}$

$= \boxed{9^{\frac{1}{4}}}$ or $\boxed{\sqrt[4]{9}}$

Integer Exponents (Variable Bases)

We can simplify expressions with variables the same way we simplify expressions with rational bases.

Integer Exponents - Remember the three basic cases.

Rational Bases:

$3^{-2} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

$\frac{1}{4^{-3}} = 4^3 = \boxed{64}$

$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \boxed{\frac{16}{9}}$

Variable Bases:

$x^{-2} = \boxed{\frac{1}{x^2}}$

$\frac{1}{m^{-3}} = \boxed{m^3}$

$\left(\frac{a}{b}\right)^{-2} = \left(\frac{b}{a}\right)^2 = \boxed{\frac{b^2}{a^2}}$

Practice - Ls 5 Assignment #1

Exponent Laws - Integer Exponents

When simplifying, the exponent laws are often used.

e.g. $x^{-7} \times x^{5} = x^{-2} = \boxed{\frac{1}{x^2}}$

$m^{-4} \div m^{-3} = m^{(-4)-(-3)} = m^{-1} = \boxed{\frac{1}{m}}$

$(3c^{-2})^{-3} = 3^{-3} \cdot c^{6} = \frac{1}{3^3} \cdot c^6 = \boxed{\frac{c^6}{27}}$

$\frac{x^{-2}y^{-5}}{x^{-4}y^{-3}} = x^2 y^{-2} = \boxed{\frac{x^2}{y^2}}$

$(a^{-3}b^{2})^{3}(a^{2}b^{-2})^{2} = a^{-9} b^{6} a^{4} b^{-4} = a^{-5} \cdot b^{2} = \boxed{\frac{b^2}{a^5}}$

Practice - Ls 5 Assignment #2

Rational Exponents (Variable Bases)

Rational Exponents - Remember the basic cases

Rational Bases:

$4^{\frac{1}{2}} = \sqrt{4}$
$= 2$

$216^{\frac{1}{3}} = \sqrt[3]{216}$
$= 6$

$81^{\frac{1}{4}} = \sqrt[4]{81}$
$= 3$

$16^{\frac{3}{2}} = (\sqrt{16})^3$
$= 4^3$
$= 64$

Variable Bases:

$x^{\frac{1}{2}} = \sqrt{x}$

$x^{\frac{1}{3}} = \sqrt[3]{x}$

$x^{\frac{1}{4}} = \sqrt[4]{x}$

$x^{\frac{3}{2}} = (\sqrt{x})^3$

Exponent Laws - Rational Exponents

When simplifying, the exponent laws are often used.

e.g. $x^{\frac{5}{2}} \cdot x^{\frac{7}{2}}$
$= x^{\frac{12}{2}}$
$= x^6$

$\dfrac{a^{\frac{1}{2}}}{\sqrt{a}} = \dfrac{a^{\frac{1}{2}}}{a^{\frac{1}{2}}}$
$= a^0$
$= 1$

$\sqrt[3]{8a^3b^6} = (8a^3b^6)^{\frac{1}{3}}$
$= 2ab^2$

$(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$
$= x^{\frac{4}{2}}y^1$
$= x^2y$

$\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}} = 2a^{-4}b^{\frac{1}{3}}$
$= \dfrac{2b^{\frac{1}{3}}}{a^4}$

Practice - Ls 5 Assignment #3-5

Solving Problems

Example:

A culture of bacteria in a lab contains 2000 bacterium cells. The number of cells doubles every day. This relationship can be modelled by the equation $N=2000(2)^t$, where N is the estimated number of bacteria cells and t is the time in days from the moment the experiment began.

(a) How many cells were present for each amount of time?

i/ after two days.

$t=2$ $\quad N=2000(2)^2 = 8000$

ii/ after one week

$t=7$ $\quad N=2000(2)^7 = 256\,000$

iii/ two days ago

$t=-2$ $\quad N=2000(2)^{-2} = 2000(2^{-1})^2$

$= 2000\left(\frac{1}{2}\right)^2$

$= 500$

(b) What does $t = 0$ indicate?

Initial Moment

Try: Workbook Page 68 #11, 15, 16