

## C3 - Determine Equation of a Line

### Warm Up - Power of the Equation

When you know the equation of a relation you can find ... anything!

**Example:**  
(Board Work)

The profit of a sales company may be given by  $P(n) = 20n - 600$ , where  $P$  is the profit in dollars and  $n$  is the number of items sold.

- Determine  $P(0)$  and explain what it means.
- Determine  $P(100)$  and explain what it means.
- Determine  $n$  when  $P(n) = 10\,000$  and explain what it means.
- How many items will the company have to sell in order to break even.
- Sketch a graph of Profit vs. items sold.

## Determine Equation of a Line

To determine the equation of a line we need: **SLOPE** and a **POINT**

**Easy Case:** Slope is given, Given point is the y-intercept  
Simply put slope and y-intercept into  $y = mx + b$

**Example:** Determine the equation for each of the following.

- a) To rent a hall for a wedding there is an initial cost of \$80 plus \$30 per person.

$$C = mn + b \quad C = 30n + 80 \quad C(n) = 30n + 80$$

- b) Submarines must withstand tremendous pressure exerted on all sides by the water. The table shows the linear relationship between pressure and water depth.

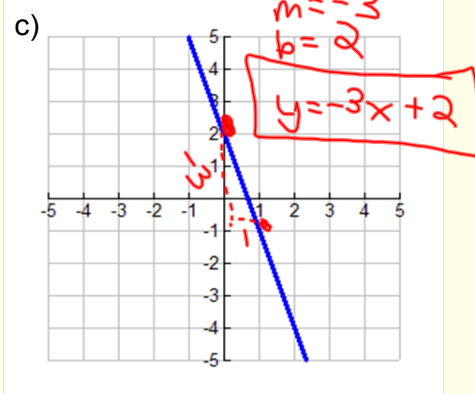
Depth (m)	Pressure (kPa)
0	100
25	350
50	600
75	850

$$m = \frac{\text{rise}}{\text{run}} = \frac{250}{25} = 10$$

$$y = mx + b$$

$$p = md + b$$

$$p = 10d + 100$$



Practice: pg. 143: 3ace, 5, 12

## Determine Equation of a Line

To determine the equation of a line we need: **SLOPE** and a **POINT**

**Hard Case:** Slope is given, Given point is the **not** the y-intercept  
Put slope and point (x,y) into  $y = mx + b$  and solve for b.

**Example:** Determine the equation for each of the following.

- a) To rent a hall for a wedding there is a cost of \$30 per person plus an initial fee. A friend used the same hall and paid \$2200 for a wedding with 70 guests.

$$m = 30$$

$$y = mx + b$$

$$y = 30x + b \rightarrow y = 30x + 100$$

$$C = 30n + 100$$

$$P(70, 2200)$$

$$(x, y)$$

$$b = 100$$

$$2200 = 30(70) + b$$

$$2200 = 2100 + b$$

$$100 = b$$

## Determine Equation of a Line

b) A line has a slope of  $-\frac{3}{5}$  and passes through the point (5,-1).

$$m = -\frac{3}{5}$$

$$P(5,-1)$$

$$b = 2$$

$$y = mx + b$$

$$y = -\frac{3}{5}x + b$$

$$-1 = -\frac{3}{5}\left(\frac{5}{1}\right) + b$$

$$-1 = -3 + b$$

$$2 = b$$

$$\rightarrow \boxed{y = -\frac{3}{5}x + 2}$$

## Practice (Board Work)

Ernesto needs to rent a paint sprayer. His friend Daniela rented one and paid \$15/h plus a fixed charge. Daniela could not remember the fixed charge, but remembered that she rented the sprayer for 4 hours and paid \$85.

- Determine an equation for the cost to rent the sprayer.
- How much would Ernesto have to pay if he rented the sprayer for 10 hours?

## Determine Equation of a Line

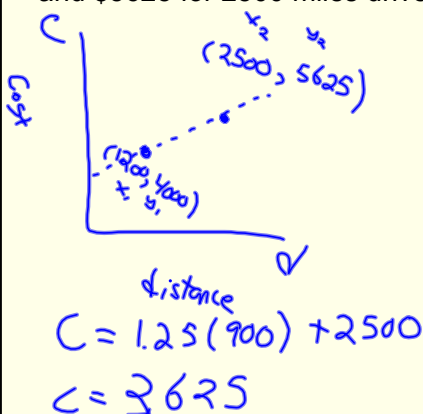
To determine the equation of a line we need: **SLOPE** and a **POINT**

**Harder Case:** Two points are given.

Calculate slope and put slope and a point (x,y) into  $y = mx + b$  and solve for b.

**Example:** Determine the equation for each of the following.

a) The annual cost of operating a snowmobile depends on the distance driven plus a fixed cost, which includes maintenance, depreciation, and trail fees. The cost is \$4000 for 1200 miles driven and \$5625 for 2500 miles driven. What is the cost of operating the snowmobile for 900 miles.



$$\begin{aligned} \text{calc } m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5625 - 4000}{2500 - 1200} \\ &= \frac{1625}{1300} \\ &= 1.25 \\ &= \$1.25/\text{mi} \end{aligned}$$

$$\begin{aligned} y &= mx + b \\ y &= 1.25x + b \\ 4000 &= 1.25(1200) + b \\ 4000 &= 1500 + b \\ 2500 &= b \end{aligned}$$

$$C = 1.25d + 2500$$

## Determine Equation of a Line

b) A line passes through the points (-6, 3) and (2, -3).

$$\begin{aligned} m &= \frac{(-3) - (3)}{2 - (-6)} \\ &= \frac{-6}{8} \\ &= -\frac{3}{4} \end{aligned}$$

$$\begin{aligned} m &= -\frac{3}{4} & y &= -\frac{3}{4}x + b \\ P(2, -3) & & -3 &= -\frac{3}{4}(2) + b \\ & & -3 &= -\frac{3}{2} + b \\ & & -3 + \frac{3}{2} &= b \\ & & -\frac{6}{2} + \frac{3}{2} &= b \\ & & -\frac{3}{2} &= b \end{aligned}$$

$$y = -\frac{3}{4}x - \frac{3}{2}$$

## Practice (Board Work)

A family drives at a constant speed from Wrigley, NT, to visit relatives in Fort Providence, NT. When they start driving at 9:00 a.m., they are 540 km from Fort Providence. At 12:30 p.m., they reach Fort Simpson, located 225 km from Fort Providence.



- Write an equation that describes their distance,  $d$ , in kilometres, from Fort Providence in terms of  $t$  hours past 9:00 a.m.
- What time will the family reach Fort Providence?

MHR pg. 376

Practice: CP pg. 160  
then Quick Check

## Solution in General Form

 $A, B, C \in \mathbb{I}$ 

Express the following linear equations in General Form ( $Ax + By + C = 0$ ).

 $A$  +ve

a)  $y = 5x + 9$

$$0 = 5x - y + 9$$

b)  $y = -2x - 12$

$$2x + y + 12 = 0$$

c)  $y = \frac{2}{3}x - 4$

$$3(y) = 3\left(\frac{2}{3}x\right) - 3(4)$$

$$3y = 2x - 12$$

$$0 = 2x - 3y - 12$$

d)  $y = -\frac{1}{4}x + \frac{2}{3}$

 $\times$  by LCM

$$12(y) = 12\left(-\frac{1}{4}x\right) + 12\left(\frac{2}{3}\right)$$

$$12y = -3x + 8$$

$$3x + 12y - 8 = 0$$

## Practice (Board Work)

Express the following linear equations in General Form ( $Ax + By + C = 0$ ).

a)  $y = 6x - 3$

b)  $y = -\frac{2}{3}x + 6$

c)  $y = \frac{1}{2}x - \frac{3}{2}$

d)  $y = -\frac{2}{3}x + \frac{4}{5}$

Determine Equations Assignment  
Extra Practice: pg. 152: 5

## Stacking Cups Activity

How many cups will it take to make a tower as tall as me?



**Step 1:** Get in your group and make a guess.  
Write your group number and your guess on a scrap of paper and hand it in to me.

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**Step 2:** Apply your knowledge to come up with a mathematical solution to the problem.  
You may use three cups and a ruler to help you.

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**Step 3:** Determine an equation for height vs. # of cups and sketch a graph.

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**Step 4:** How could you find the height of 326 cups?  
How many cups would you need to reach the ceiling?

## Determine Equation of a Line (General Form)

In problems where each variable is changing at a certain rate, **general form** is best used to determine the equation for the scenario.

Brooke wants to save \$336 to decorate her bedroom. She has two part-time jobs. On weekends, she works as a snowboard instructor and earns \$12 per hour. On weeknights, she earns \$16 per hour working as a high-school tutor.

- Write an equation to represent the number of hours Brooke needs to work as a snowboard instructor,  $S$ , and as a tutor,  $T$ .
- What is the  $S$ -intercept of a graph of the equation? What does the  $S$ -intercept represent?
- What would the  $T$ -intercept be? What does it represent?
- Suppose Brooke works 8 h as a snowboard instructor. How many hours will she need to work as a tutor?

$$a) 12S + 16T = 336$$

$$b) \text{S-int}(T=0)$$

$$12S = 336$$

$$S = 28$$

Work 28 hrs as snowboard instructor and 0 hrs tutoring to make \$336.

$$c) \text{T-int}(S=0)$$

$$16T = 336$$

$$T = 21$$

Work 21 hrs tutoring and 0 hrs as snowboard instructor to make \$336.

$$d) 12S + 16T = 336$$

$$12(8) + 16T = 336 \quad (S=8)$$

$$96 + 16T = 336$$

$$16T = 240$$

$$T = 15$$

15 hrs as a tutor and 8 snowboard



Practice: Text pg. 148: 6, 7

## Slope-Point Form

We have been using slope-intercept form to determine the equation of a line. An alternative method is to use slope-point form.

$$y - y_1 = m(x - x_1)$$



Derives from the slope formula:

where:

- $m$  is the slope of the line
- $(x_1, y_1)$  is a known point on the line
- $x$  and  $y$  represent points on the line  $(x, y)$ .

e.g.  $y - 2 = -\frac{2}{5}(x - (-4))$

$$m = -\frac{2}{5} \quad (x_1, y_1) = (-4, 2)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

$$y_2 - y_1 = m(x_2 - x_1)$$

## Slope-Point Form - Example

Determine the equation of a line that has a slope of  $-\frac{3}{5}$  and passes through the point (5,-1).

$$y - y_1 = m(x - x_1)$$

$$y + 1 = -\frac{3}{5}(x - 5)$$

$$5y + 5 = -3(x - 5)$$

$$5y + 5 = -3x + 15$$

$$3x + 5y - 10 = 0$$

$$5y = -3x + 10$$

$$y = -\frac{3}{5}x + 2$$

$$y = mx + b$$

$$y = -\frac{3}{5}x + b$$

$$-1 = -\frac{3}{5}(5) + b$$

$$-1 = -3 + b$$

$$2 = b$$

$$y = -\frac{3}{5}x + 2$$

$$5y = -3x + 10$$

$$3x + 5y - 10 = 0$$

Try some of these using slope-point form ...

Determine Equations Assignment  
Extra Practice: pg. 152: 5