A4 • Evaluating algebraic expressions

Mathematical goals

To enable learners to:

- distinguish between and interpret equations, inequations and identities;
- substitute into algebraic statements in order to test their validity in special cases.

Starting points

Learners often use letters in algebra without understanding what they mean. Common misconceptions include believing that:

- a letter can only stand for one particular number;
- different letters must stand for different numbers;
- letters can only stand for whole numbers.

Such misconceptions often arise when learners generalise from a restricted range of examples. This session will build on their knowledge of substitution to reconsider such interpretations.

Materials required

For each small group of learners you will need:

- Card set A – *Always, sometimes or never true?*;
- large sheet of paper for making a poster;
- glue stick;
- felt tip pen.

Time needed

About 1 hour.

Suggested approach

Beginning the session

Write the following statement on the board: $x + y = xy$

Ask learners to explain to you what this statement says in words and whether they think it is a true statement or not.

Typically, learners will begin by saying that this is clearly not true because 'add two numbers' does not mean the same as 'multiply two numbers'.

Ask learners for values of x and y which will prove that the statement is not always true. This is not usually difficult.

Now ask if learners can spot any values that make the statement true. Typically, they quickly spot $x = 2$, $y = 2$ and $x = y = 0$. Some learners may reject the case $x = 2$, $y = 2$ because "x and y are different letters so they cannot take the same value". This misconception needs to be discussed explicitly.

The statement is therefore sometimes true. If you wish to challenge learners further, you could ask them to see if they can think of any more cases for which the statement is true, e.g. when $x = 1.5$ or $\frac{3}{2}$, $y = 3$; $x = \frac{4}{3}$, $y = 4$; $x = \frac{5}{4}$, $y = 5$. This makes the point that the examples do not have to be integers.

Explain that in this session learners will be asked to consider a collection of statements in a similar way. The objective of the session is for each group of learners to produce a poster which shows each statement classified according to whether it is always, sometimes, or never true and furthermore:

- if it is sometimes true, then write examples around the statement to show when it is true and when it is not true;
- if it is always true, then to give a variety of examples demonstrating that it is true, using large numbers, decimals, fractions and negative numbers if possible;
- if it is never true, then to say how we can be sure of this.

Working in groups

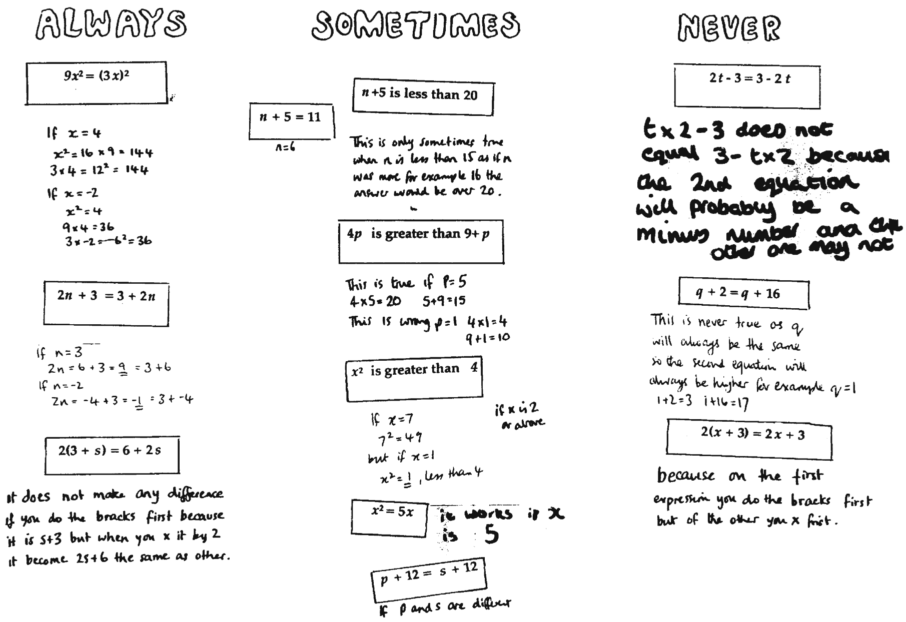
Ask learners to work in pairs or groups of three.

Give each group Card set A – *Always, sometimes or never true?,* a large sheet of paper and a glue stick.

Ask learners to divide their poster into three columns and head these with the words 'Always true', 'Sometimes true', 'Never true'.

Learners should now take it in turns to place a card in one of the columns and justify their answer to their partner(s). Their partner(s) must challenge them if the explanation has not been clear and complete. When the group is in agreement, they should stick the card down and write examples around it to justify their choice. This should include examples and counterexamples.

Learners should not need to rearrange the equations. Trial and error substitutions should prove adequate in most cases (although some may enjoy showing that statements which are always true reduce to 0 = 0).



When doing this activity, we often find that learners sort their cards quickly and superficially, at least to begin with. They often need prompting to try decimal and negative substitutions to check their assumptions. Learners may change their minds many times; arrows often appear all over the poster showing that a statement was initially classified incorrectly.

Many common beliefs and misinterpretations will surface. Some, for example, appear to believe that different letters have to stand for different numbers and classify $p + 12 = s + 12$ as 'never true'. Some combine letters and numbers inappropriately, thus believing that $3 + 2y = 5y$ is 'always true'. Some appear to believe that $x^2 > x$ is always true because multiplication always makes numbers bigger, and so on. These misconceptions should be picked up in the whole group discussion at the end.

Encourage learners to justify why some expressions are always true (and are therefore identities) using area diagrams (see **A1 Interpreting algebraic expressions**).

On the positive side, you may also notice learners beginning to interpret and reason, using the symbols confidently and meaningfully. In one session, we saw one small group immediately classify $2t - 3 = 3 - 2t$ as 'never true' because "one side is always the negative of the other side". We asked them to reflect on whether changing the sign of a number will always change its value and later returned to find that they had realised that the statement was true when both sides of the equation are zero. That led them to the solution $t = 1.5$.

Reviewing and extending learning

Conclude the session with whole-group questioning using mini-whiteboards.

Ask learners whether they believe that statements such as the following are always, sometimes, or never true, and then ask them to justify their answers. They should hold up their mini-whiteboard showing A, S or N, and give their reasons when challenged.

$x + 2 = 3$

$n + 12 = n + 30$

$x + 6 = y + 6$

$x + y + z = y + z + x$

$3n > n + 3$

$x^2 > 2x$

What learners might do next

Ask learners to create a further set of cards. Their set should contain examples that are as different from one another as possible ($2x + 3 = 5$; $2x + 3 = 6$ are too similar) and should contain an equal number of 'always', 'sometimes' and 'never true' statements.

Learners should produce complete solutions on a separate piece of paper. Then, at a later date, learners can exchange sheets and do each others' questions.

Further ideas

This activity is about examining a series of mathematical statements and deciding on their validity. This idea may be used in many other topics and levels. Examples in this pack include:

SS4 Evaluating statements about length and area;

S2 Evaluating probability statements.

A4 Card set A – *Always, sometimes or never true?*

1 $n + 5 = 11$	2 $q + 2 = q + 16$
3 $2n + 3 = 3 + 2n$	4 $2t - 3 = 3 - 2t$
5 $3 + 2y = 5y$	6 $p + 12 = s + 12$
7 $4p > 9 + p$	8 $n + 5 < 20$
9 $2(x + 3) = 2x + 3$	10 $2(3 + s) = 6 + 2s$
11 $x^2 > 4$	12 $x^2 = 5x$
13 $x^2 > x$	14 $9x^2 = (3x)^2$